NONLINEAR ANALYSIS OF REINFORCED CONCRETE CONTAINMENTS CONSIDERING THE ROTATION OF DIAGONAL REINFORCEMENT

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SUMMARY

The paper presents a procedure for nonlinear analysis of reinforced concrete structures for extreme loads based on a layering concept and includes the consideration of rotational strains in diagonal reinforcement. The basic layering approach is first described, followed by a formulation for the rotational strains for diagonal reinforcement. Conclusions are then presented.

INTRODUCTION

Analysis of reinforced concrete structures under extreme loads, such as earthquakes, aircraft impacts, tornadoes, and safety relief valve discharges, etc., becomes very difficult due to the complex nonlinear behavior of the reinforced concrete material. The complexities involve the cracking as well as the crushing of concrete, the yielding of reinforcing steel, the nonlinear interaction between steel and concrete, and the nonhomogeneity and anisotropism of the composite materials. It therefore becomes necessary to develop special anisotropic models, based on appropriate assumptions and limited test data, by means of which the diverse failure possibilities can be described and converted into mathematical formulations.

The standard nonlinear finite elements and the available analytical approaches do not completely take into consideration the above described complex nonlinear behavior of reinforced concrete cross-sections and are also extremely expensive to use, especially for dynamic analyses. Most of the finite element solutions use standard nonlinear flow rules and yield surfaces for isotropic materials (applicable mainly to metal structures). They also do not take into account the strains related to the overall rotation of the diagonal reinforcement, which may change the stresses by as much as 100 percent, as shown by the results of some recent analyses (1). An injudicious application of such standard approaches to reinforced concrete structures can sometimes provide inaccurate and unconservative results.

This paper presents a procedure for nonlinear analysis of reinforced concrete structures for extreme loads based on a layering concept for modeling the cracked concrete and the yielded steel, in conjunction with

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a technique for properly taking into consideration the strains due to overall rotation of the diagonal reinforcement.

The layering approach presented in the paper is mainly used to form the composite stiffness matrix of various layers at element level, so that the behavior of a reinforced concrete cross-section is realistically modeled by a single element rather than several elements representing the concrete and steel separately. The loading and unloading of the different layers and the resulting cracking and crushing of concrete and the yielding of steel through the thickness of the wall are easily monitored. Very few degrees of freedom, corresponding to those of a single element (rather than several layers of finite elements) are needed, resulting in considerable reduction of computer time.

The technique for the inclusion of the rotation of diagonal reinforcement presented in the paper provides a solution to the problem of the treatment of incompatibility of deformation between uniaxial strains in the reinforcing bars and multiaxial strains in the concrete during and after cracking. A derivation is presented in this paper for constitutive equations including rotation of diagonal reinforcement. The calculation procedure presented here can either be used with hand calculations or coded into a computer program.

THE BASIC APPROACH

Reinforced concrete is an anisotropic material due to the orientation and distribution of reinforcing steel bars and the crack formation in concrete. However, if a cross-section can be subdivided into layers of cracked and uncracked concrete and yielded and unyielded steel, the investigation of its real behavior becomes more direct and much easier to perform.

The basic approach presented here makes use of this layering concept, where each layer is used as a sub-element in developing a realistic element stiffness for a segment of the reinforced concrete structure (this layering concept is similar to the standard substructure approach). Similar layering concepts have been utilized in different forms in the analyses of fiber composite material in the aerospace industry, as well as for reinforced concrete plate and shell structures of the Civil Engineering type mainly for static analyses (Refs. 2, 3, and 4). The approach described in this paper was developed for dynamic analyses.

The approach consists of dividing the elements into as many layers as needed depending on the variation of material properties within the element.

The composite material matrix of the element is computed as follows:

$$[D] = \sum_{i=1}^{N_c} \int \phi_i \, [D]_{ci} \, dv + \sum_{i=1}^{N_s} \int \phi_i \, [D]_{si} \, dv$$

where

ϕ_i = Spatial function relating the displacements at the reference plane and each layer i
$[D]$ = Composite material matrix of the element
$[D]_{ci}$ = The material matrix of the i^{th} concrete layer
$[D]_{si}$ = The material matrix of the i^{th} steel layer
N_c = The total number of concrete layers
N_s = The total number of steel layers

The stiffness matrix of the element is then formed using the above material matrix in terms of nodal points located along a reference plane arbitrarily selected within the element, as shown in Figures 1 and 2. The stiffness matrix for the complete structure is then obtained using the standard direct stiffness assembly procedure.

The behavior of each layer under loading and unloading is monitored mainly by investigating the state of stresses in these layers. A "layer state" matrix is used to keep track of the states in these layers regarding cracking of concrete, yielding of steel, and closing of cracks, etc.

The following are the main advantages in using the layered formulation:

(a) Efficency in Computation: As mentioned above, the layered formulation is essentially a substructure technique which does not require the internal nodes of the layers during the assembly of the whole structure. Only one layer of external nodes, usually along a reference plane, is needed. The overall structural stiffness and mass matrices are relatively small in size. The scheme provides a considerable saving in the solution of equilibrium equations at the expense of a few additional manipulations carried out at the element level (Ref. 5).

(b) Accurate Formulation of Anisotropic Material Matrix: The material matrix of the layered element is formed at element level by considering variation of material properties through the thickness. The uncracked concrete, the cracked concrete, the crushed concrete, the yielded and unyielded steel can all be taken into consideration properly in such a formulation. The anisotropic characteristics of the composite material are therefore accounted for naturally and logically without introducing unusual and severe assumptions.

(c) Easy Monitoring of Loading and Unloading: The monitoring of loading and unloading is fairly easy to perform through the concept of a "layer state" matrix, resulting in savings in computer time and a realistic evaluation of the behavior of the reinforced concrete cross-section.

Figures 1 and 3 show the application of the layering concept to a beam and a containment structure, respectively.

CONSIDERATION OF ROTATION OF DIAGONAL REINFORCEMENT

The use of diagonal reinforcement in massive reinforced concrete structures, such as nuclear containments, is very common and is found to be very effective in resisting lateral forces (Ref. 6). Although significant progress has been made in treating some of the major problems associated with nonlinear analysis of reinforced concrete structures, such as those described in References 3 and 4, the behavior of diagonal steel in the post-cracking concrete zone is still not properly treated in general; and, in most cases, it is outright neglected.

In the theory of continuum mechanics, the rotation of a line element in two- or three-dimensional domains can be decoupled into a pure rigid body rotation and a shear deformation (Refs. 7 and 8). The reinforcement, being one-dimensional physically, is considered to have uniaxial strains only. However, the rotation of the reinforcing bars must be included in the derivation of constitutive relations to ensure complete compatibility between concrete and reinforcing steel, as described below.

Let us consider an element of concrete, ABCD, of unit thickness, reinforced with diagonal steel bars AC and BD, and consider it to be deformed into the new position A'B'C'D' under vertical stress σ_y, as shown in Figure 4. It is assumed that point A' is translated back to coincide with point A. The deformation of reinforcing steel bar AC can be divided into the following two stages using the theory of small strains:

(1) The end C is stretched to position C" under stress σ_s, the stress in steel.

(2) Then, the stretched steel AC" is rotated into the final position A'C' through an infinitesimal angle, α.

The constitutive equation for the steel along its axis is given by:

$$\varepsilon_s = \frac{\sigma_s}{E_s} \qquad (1)$$

where E_s and ε_s are the modulus of elasticity and strain in the steel, respectively.

The vertical deformation of steel, d_{uy}, using geometric compatibility in conjunction with equation (1), is then given by

$$d_{uy} = \alpha \cdot d_x + \frac{\sigma_s}{E_s} \cdot d_y \quad \text{for a small angle } \alpha \qquad (2)$$

Neglecting the concrete stress for simplification, and considering equilibrium in Y direction, the following relationship can be obtained for unit thickness,

$$2\cdot A_s\cdot\sigma_s\cdot\sin\theta = \sigma_y\cdot d_X \tag{3}$$

where A_s is the area of diagonal reinforcing bars AC and BD, respectively, within the distance d_X, and σ_y is the element stress in the Y direction:

Introducing the reinforcing ratio for the diagonal bars, P, given by

$$P = \frac{2A_s}{d_X} \tag{4}$$

the equation (3) becomes

$$\sigma_s = \frac{\sigma_y}{P\cdot\sin\theta} \tag{5}$$

Substituting equation (5) into equation (2), and dividing both sides by d_Y, we obtain

$$\frac{d_{uy}}{d_Y} = \alpha\cdot\frac{d_X}{d_Y} + \frac{\sigma_y}{P\cdot E_s\cdot\sin\theta} \tag{6}$$

$$\text{or } \sigma_y = (\varepsilon_y - \alpha\cdot\cot\theta)\cdot P\cdot E_s\cdot\sin\theta \tag{7}$$

where ε_y is the strain in the Y direction.

Therefore, it can be seen that the rotational deformation, α, now appears in equation (7), which describes the modified stress-strain relationship.

Based on equation (7), a set of equations was developed for the analysis of containment structures with diagonal reinforcement for various loading conditions. For problems with less than 15 layers, hand calculations can be easily performed using this approach. A general purpose computer program is currently being developed to include all types of structures and loading conditions.

CONCLUSIONS

A procedure was presented for nonlinear analysis of reinforced concrete structures for extreme loads, such as earthquakes, aircraft impacts, tornadoes, safety relief value discharges, etc. The procedure is based on a layering concept for modeling the cracked concrete and the yielded steel, in conjunction with a technique for the consideration of strains due to rotation of the diagonal reinforcement.

The main advantages of the layering concept proposed include efficiency in computation, accurate formulation of anisotropic material matrix, and easy monitoring of loading and unloading. The procedure can be easily programmed (currently in progress), but lends itself very well to hand computations for certain loading conditions.

The use of diagonal reinforcement in massive reinforced concrete structures, such as nuclear containments, is very common and is found to be very effective in resisting lateral forces. However, the treatment of diagonal reinforcement is not usually properly taken into consideration in nonlinear analyses. A formulation was presented showing an improved technique for the inclusion of rotational deformations in diagonal reinforcement. Recent analyses (Ref. 1) have indicated that a proper inclusion of such deformations can change the stresses in the diagonal reinforcement by as much as 100 percent.

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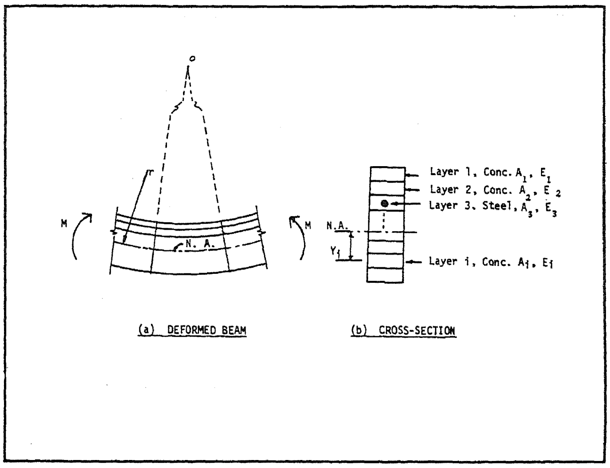


(a) DEFORMED BEAM

(b) CROSS-SECTION

FIGURE 1. Use of Layering Concept for a Beam Cross-Section Under Pure Bending

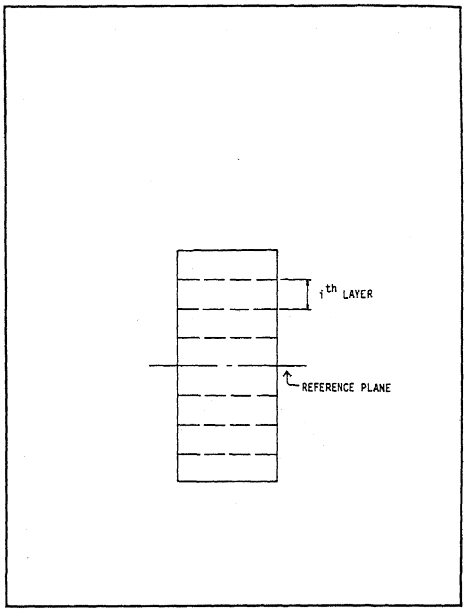
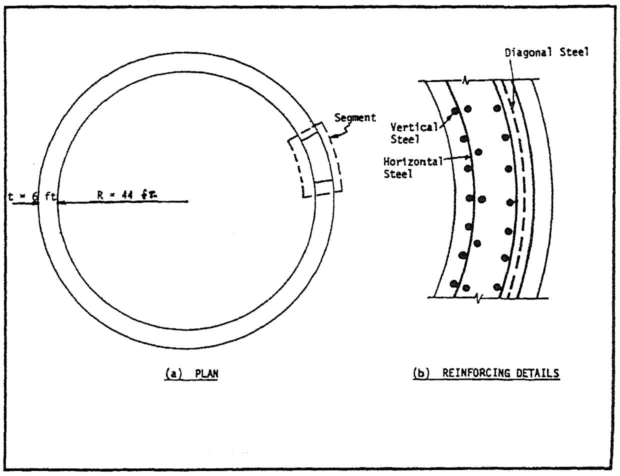


FIGURE 2. Cross-Section of a Layered Element



(a) PLAN

(b) REINFORCING DETAILS

FIGURE 3. Use of Layering Concept for a Containment Structure

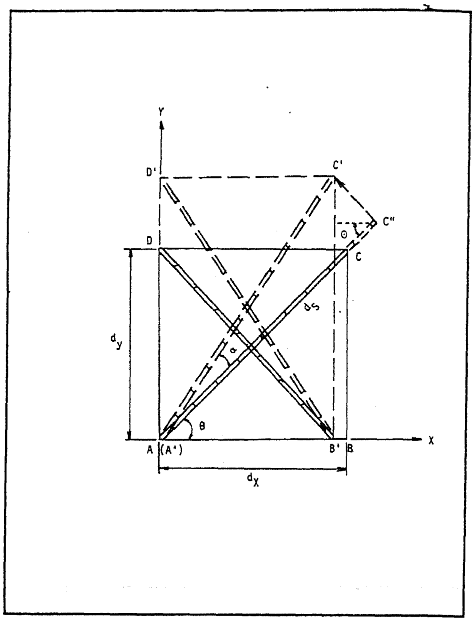


FIGURE 4. Rotation of Diagonal Bar