LIFELINE RESPONSE ANALYSIS UNDER
NONSTATIONARY TRAVELING SEISMIC WAVE LOADING

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SUMMARY

A random vibration model has been developed in this study to examine the effects of nonstationarity and the traveling characteristics of a seismic surface wave on above-the-ground lifeline structures. Computer simulation of the model has been performed on a simple single degree of freedom structure having two support motions. The results show that out-of-phase support motions produce smaller peak responses than when the supports are moving in-phase. In addition the assumption of stationarity of ground motion is conservative because it resulted in larger responses than when the ground motion is nonstationary.

THEORETICAL MODEL

Lifeline Structure

The above-the-ground lifeline structure has been modeled as one long configuration with many supports. For a system with M support motions and N responses, the equation of motion can be written as

$$\begin{bmatrix} \underset{\sim}{M}_s & \underset{\sim}{M}_g \\ \underset{\sim}{M}_g{}^T & \underset{\sim}{M}_{gg} \end{bmatrix} \begin{bmatrix} \ddot{\underset{\sim}{x}}_s \\ \ddot{\underset{\sim}{x}}_g \end{bmatrix} + \begin{bmatrix} \underset{\sim}{C}_s & \underset{\sim}{C}_g \\ \underset{\sim}{C}_g{}^T & \underset{\sim}{C}_{gg} \end{bmatrix} \begin{bmatrix} \dot{\underset{\sim}{x}}_s \\ \dot{\underset{\sim}{x}}_g \end{bmatrix} + \begin{bmatrix} \underset{\sim}{K}_s & \underset{\sim}{K}_g \\ \underset{\sim}{K}_g{}^T & \underset{\sim}{K}_{gg} \end{bmatrix} \begin{bmatrix} \underset{\sim}{x}_s \\ \underset{\sim}{x}_g \end{bmatrix} = \underset{\sim}{0} \qquad (1)$$

where $\underset{\sim}{x}_g$ and $\underset{\sim}{x}_s$ are vectors of absolute displacements of order M and N of support motions and structural responses respectively, and

$\underset{\sim}{M}_s, \underset{\sim}{C}_s, \underset{\sim}{K}_s$ = structural properties associated with $\underset{\sim}{x}_s$

$\underset{\sim}{M}_{gg}, \underset{\sim}{C}_{gg}, \underset{\sim}{K}_{gg}$ = foundation properties related to $\underset{\sim}{x}_g$

$\underset{\sim}{M}_g, \underset{\sim}{C}_g, \underset{\sim}{K}_g$ = terms indicating the interactions between support motions and structural responses.

The first set of equations in eq. 1 corresponds to the equation of motion for the response degrees of freedom,

$$\underset{\sim}{M}_s \ddot{\underset{\sim}{x}}_s + \underset{\sim}{C}_s \dot{\underset{\sim}{x}}_s + \underset{\sim}{K}_s \underset{\sim}{x}_s = -\underset{\sim}{M}_s \ddot{\underset{\sim}{x}}_g - \underset{\sim}{C}_s \dot{\underset{\sim}{x}}_g - \underset{\sim}{K}_s \underset{\sim}{x}_g \qquad (2)$$

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The absolute displacements of the responses. x_s can be written in terms of pseudo-static displacements x_{sd} which are the responses to the static motions of supports, and the relative or dynamic displacements u_s (ref. 1),

$$\underline{x}_s = \underline{x}_{sd} + \underline{u}_s = \underline{r}\ \underline{x}_g + \underline{u}_s \tag{3}$$

where $\underline{r} = \underline{K}_s^{-1}\ \underline{K}_g$ is commonly referred to as the influence coefficient matrix. Insertion of eq. 3 into eq. 2 and rearrangement for u_s would result in the equation of motion in terms of relative displacement of lifeline structure with $\underline{x}_g$ support motions.

$$\underline{M}_s\ \ddot{\underline{u}}_s + \underline{C}_s \dot{\underline{u}}_s + \underline{K}_s\ \underline{u}_s = -\ [\underline{M}_s\ \underline{r} + \underline{M}_g]\ \ddot{\underline{x}}_g - [\underline{C}_s\ \underline{r} + \underline{C}_g]\ \dot{\underline{x}}_g \tag{4}$$

Because the input motions $\{\ddot{\underline{x}}_g\}$ and $\{\dot{\underline{x}}_g\}$ are generally nonstationary processes for a typical earthquake, the responses $\{\underline{u}_s\}$ are also nonsationary processes.

For a lifeline with large number of supports and response degrees of freedom, a time-history analysis for a nonstationary loadings would be impractical and costly. A frequency-domain analysis, however, has the computational advantages based on product terms instead of correlation integrals as required in time-history analysis. As a result, eq. 4 can be converted into

$$\underline{S}_u\ (\omega_1,\ \omega_2) = \underline{H}\ (\omega_1)\ \underline{S}_g\ (\omega_1,\ \omega_2)\ \underline{H}^T(\omega_2) \tag{5}$$

where $\underline{S}_u\ (\omega_1,\ \omega_2)$ and $\underline{S}_g\ (\omega_1,\ \omega_2)$ are the matrices of cross-power spectral density of the dynamic responses and support motions with orders NxN and MxM respectively. Each element of $\underline{S}_u\ (\omega_1,\ \omega_2)$ is defined as the double Fourier Transform of the cross-correlation matrix $\underline{R}_u\ (t_1,\ t_2)$ obtained for the nonstationary processes $\{\underline{u}_s\ (t)\}$,

$$S_{u_{ij}}(\omega_1,\ \omega_2) = \iint_{-\infty}^{\infty} R_{u_{ij}}(t_1,\ t_2)\ e^{-i(\omega_1 t_1 + \omega_2 t_2)}\, dt_1\ dt_2 \tag{6a}$$

$$R_{u_{ij}}(t_1,\ t_2) \triangleq E\ [u_i\ (t_1)\ u_j\ (t_2)] \tag{6b}$$

where $R_{u_{ij}}(t_1,\ t_2)$ is the cross-correlation function between response degrees of freedom i and j.

The transfer function matrix $\underline{H}\ (\omega)$ of the lifeline system can be obtained from eq. 4 under the steady state conditions,

$$\underset{\sim}{H}(\omega) = \frac{\omega^2 [\underset{\sim}{M}_s \underset{\sim}{r} + \underset{\sim}{M}_g] - i\omega [\underset{\sim}{C}_s \underset{\sim}{r} + \underset{\sim}{C}_g]}{-\omega^2 \underset{\sim}{M}_s + i\omega \underset{\sim}{C}_s + \underset{\sim}{K}_s} = \int_{-\infty}^{\infty} \underset{\sim}{h}(t)\, e^{i\omega t} dt \qquad (7)$$

where $\underset{\sim}{h}(t)$ is the impulse response matrix of the system described by eq. 4. The lifeline structure can be visualized as a multiple input/output filter with a transfer function matrix $\underset{\sim}{H}(\omega)$ acting on a set of input processes $\{\underset{\sim}{x}_g\}$ to produce output responses $\{\underset{\sim}{u}_s(t)\}$. It can be seen that the calculations in eq. 5 for a moderate size lifeline and a sufficient numbers of discrete frequency values for ω_1 and ω_2, would require an enormous amount of computations. For this reason, the transfer function matrix has been calculated for the steady-state conditions in eq. 7 instead of the more general term $H(\omega,t)$ which in addition includes the transient responses by the definition (ref. 2),

$$\underset{\sim}{H}(\omega,t) = \int_{-\infty}^{t} \underset{\sim}{h}(t) e^{i\omega t} dt \qquad (8)$$

Ground Motion

The principal objective of this paper is to show the effects of nonstationarity and the phase differences between support motions on above-the-ground lifeline structures. For this reason and to avoid unnecessary complications, the ground motion at a site close to the structure and far enough from the epicenter is assumed to be a nonstationary gaussian surface wave traveling at a constant horizontal velocity V. The nonstationary process $\{x_0(t)\}$ at site 0 is assumed to be the commonly used model of the ground motion which is the product of an envelope function $I(t)$ representing the strong motion part of an earthquake and a stationary process $\{\alpha(t)\}$ (ref. 3),

$$x_0(t) = I(t)\,\alpha(t) = I(t) \sum_k A_k \sin(\omega_k t + \phi) \qquad (9)$$

where A_k is a set of harmonic amplitudes obtainable from a prescribed power spectral density $S_0(\omega)$ of the stationary process $\{\alpha(t)\}$, so that $A_k^2 = 2 S_0(\omega_k)\,\Delta\omega$ with $\Delta\omega$ as the frequency resolution desired to discretize the spectrum. The phase angle ϕ is taken as a random variable with a uniform distribution between 0 and 2π.

The motions at the supports are modeled as the passage of motion at the site through a set of independent path filters. If the impulse response functions of the path filters from the site to the supports i and j are assumed to be a_i and a_j respectively, the ground motions at support i and j would therefore be

$$\begin{aligned} x_i(t) &= x_0(t) * a_i(t) \\ x_j(t) &= x_0(t) * a_j(t) \end{aligned} \qquad (10)$$

Figure 1 describes how the paths to individual supports are determined.

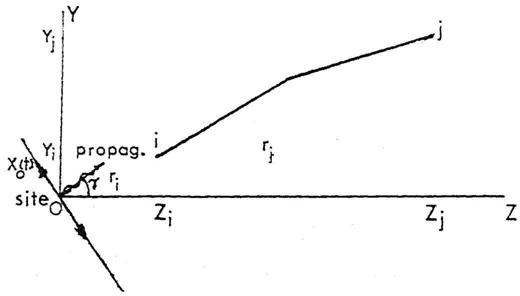


Figure 1 - Travel Path of the Surface Wave to Supports i and j

The surface wave would travel the paths

$r_i = Z_i \cos \gamma + Y_i \sin \gamma$ and
$r_j = Z_j \cos \gamma + Y_j \sin \gamma$ respectively

the travel times are $t_i = r_i/V$ and
$t_j = r_j/V$ where V = surface wave velocity

The cross-spectral density function between the processes $\{x_i (t)\}$ and $\{x_j (t)\}$ can be formulated similar to eq. 5,

$$S_{g_{ij}}(\omega_1, \omega_2) = A_i (\omega_1) G_o (\omega_1, \omega_2) A_j (\omega_2) \qquad (11)$$

where

$G_o (\omega_1, \omega_2)$ = the auto power spectral density function of the ground displacement at the site O

= double Fourier Transform of $\{R_o(t_1, t_2) = E [x_o (t_1) x_o(t_2)]\}$

$$= \iint_{-\infty}^{\infty} e^{-i (\omega_1 t_1 + \omega_2 t_2)} dt_1, dt_2 \{1/2 \; I (t_1) \; I (t_2) \sum_k A_k^2 \cos\omega_k(t_1 - t_2)\} \qquad (12)$$

Here, $A_i (\omega)$ and $A_j (\omega)$ are the transfer functions for the filters representing paths r_i and r_j respectively. For the path filter, a constant Q-filter has been assumed (ref. 4), so that

$$A_j (\omega) = \exp [- \frac{t_j\omega}{2Q}] \; \exp [-i \; t_j\omega \; (1- \frac{1}{\pi Q} \ln | \frac{\omega}{\omega_0} | \;)] \qquad (13)$$

The first term represents ground attenuation, and the second describes the effect of the path on the phase of the ground motion; Q denotes the quality factor which is inversely proportional to the damping of the ground madium; and ω_o is the frequency for centering the path filter.

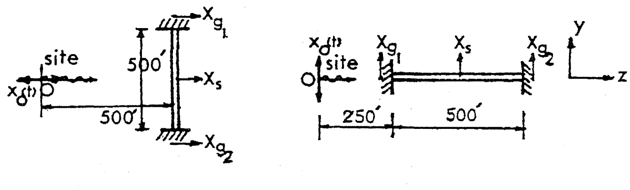
Inserting eq. 13 into eq. 11 one can obtain

$$S_{g_{ij}}(\omega_1, \omega_2) = G_o(\omega_1, \omega_2) \exp\left[\frac{-t_i\omega_1 - t_j\omega_2}{2Q}\right] \exp\left[-i(t_i\omega_1 + t_j\omega_2) - \frac{i}{\pi Q}\left(t_i\omega_1 \, \ell n \left|\frac{\omega_1}{\omega_o}\right| + t_j\omega_2 \, \ell n \left|\frac{\omega_2}{\omega_o}\right|\right)\right] \tag{14}$$

which is the expression for the cross-power spectral density function between the support motions i and j. An MxM matrix is calculated for all the pairs of support motions to yield the matrix $\underset{\sim}{S}_g(\omega_1, \omega_2)$. The resulting values are used in eq. 5 to calculate the responses. For a gaussian process $\{\underset{\sim}{u}_s(t)\}$ the knowledge of spectral density $\underset{\sim}{S}_u(\omega_1, \omega_2)$ would completely define the process. The calculation of $\underset{\sim}{S}_u(\omega_1, \omega_2)$ for all pairs of responses would require enormously costly computations; as a result, special algorithems and simplified assumptions are necessary for large size problems.

EXAMPLE PROBLEMS

A computer program called LIFELINE has been developed during this study to calculate the cross-correlation function between any two degrees of freedom of the responses for an above-the ground lifeline structure subjected to a nonstationary gaussian traveling S-wave. To examine the traveling characteristics and nonstationarity of the seismic wave, a simple SDOF pipe structure is considered as shown in Figure 2. The structure has two support motions and one response at the midspan. To simplify, only the transverse motions are considered in the analysis.



(a) Case 1 (b) Case 2

Pipe element properties:

r_o = outside radius = 6' E = 30×10^6 psi
r_i = inside radius = 5.5' damping ratio = 5 percent
$\overline{m}$ = mass per unit length = 275 lb. sec^2/ft^2

Figure 2. Example Problems. (a) in-phase support motions, (b) out-of-phase support motions.

In Figure 2a the structure is perpendicular to the direction of wave propagation and, therefore, the supports experience in-phase motions. In Figure 2b, the structure is parallel to the direction of the propagation and the supports are subjected to out-of phase motions. The motion at site 0 is considered as a nonstationary gaussian process according to eq. 9. The envelope function I (t) is assumed to be a "Boxcar" time function with a duration of 5 seconds. It is further assumed that the prescribed power spectral density $S_0(\omega)$ of the ground displacement is $S_0(\omega) = S_a(\omega)/\omega^4$ where $S_a(\omega)$ is the power spectral density of the ground acceleration suggested by Tajimi (ref. 5.).

$$S_a(\omega) = \frac{1 + 4\,\xi_g^{\,2}\,(\omega/\omega_g)^2}{[1 - (\omega/\omega_g)^2]^2 + 4\,\xi_g^{\,2}(\omega/\omega_g)^2} \tag{15}$$

The values of $\xi_g = 0.6$ and $\omega_g = 4\pi$ rad/sec have been chosen for the soil medium. To avoid the singularity of $S_0(\omega)$ as $\omega \to 0$, the calculations are carried on for $\omega > 0.01$ rad/sec.

For the two cases considered in Fiqures 2a and 2b, the resulting values of the autocorrelation function of the displacement at midspan are plotted in Figures 3 and 4. The wave velocity is assumed as 500 ft/sec; therefore, there is a 1-second time delay between the support motions of Figure 2b. The peak value of the autocorrelation for Figure 2a is shown to be higher than in Figure 2b by a factor of 2.5, which indicates that the phase difference between the support motions reduced the peak response.

To better examine the phase effect, the same problem is solved when the ground motion and the output response are assumed to be stationary. The variation of wave velocity from 40 to 770 ft/sec has changed the mean square value of the response by a factor of 6 (Fig. 5), thus showing the significance of the phase effect on the response values. In addition, a comparison between Figures 4 and 5 reveals that the assumption of stationarity has provided a larger response value for $V = 500$ ft/sec by a factor of 3.

CONCLUSIONS

The main objective of this paper is to examine the effects of the traveling characteristics and nonstationarity of the ground motion on the responses of above-the-ground lifeline structures. As a first result, it has been observed that computation of statistical averages of seismic response of lifelines under random excitations is less expensive in the frequency-domain analysis compared to the time domain analysis, because of less complicated formulations. The results of the analysis of a simple SDOF system subjected to a nonstationary gaussian S-wave indicates that the assumption of stationarity overestimates the peak response value by several factors. In addition, neglecting the phase difference between the support motions of a lifeline system would result in even more conservative response values. The degree of conservatism depends upon the problem, and it should be examined for different realistic lifelines. In conclusion, the nonstationary and traveling characteristics of ground motion are important assumptions that should be included in the analysis of lifeline systems under seismic loadings.

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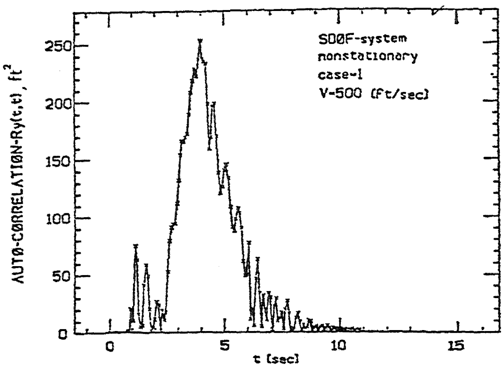

FIGURE 3. Nonstationary Response of SDOF Lifeline Structure with In-Phase Support Motions

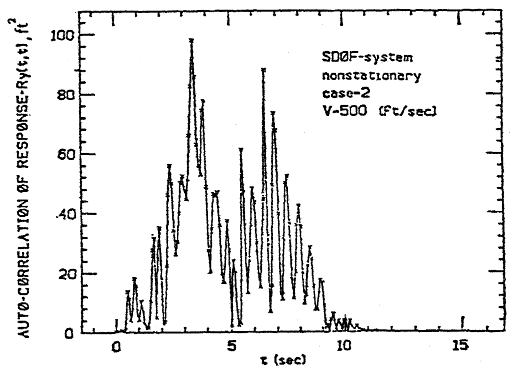

FIGURE 4. Nonstationary Response of the SDOF Structure with Out-of-Phase Support Motions

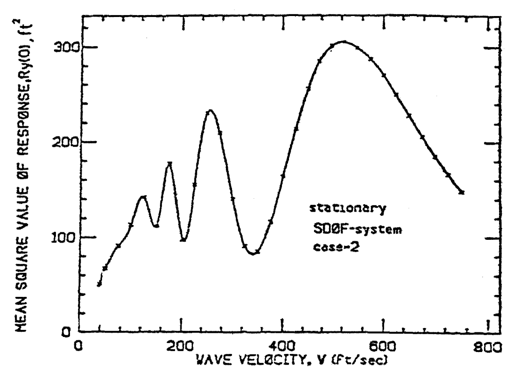

FIGURE 5. Effect of Variation of Surface Wave Velocity on the Stationary Response of the SDOF Lifeline Structure with Out-of-Phase Support Motions