BUCKLING ANALYSIS OF BURIED PIPELINES UNDER SEISMIC LOADS

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SUMMARY

The Donnell and Flügge forms of the stability equations of cylindrical shells are employed to analyze the axisymmetric, elastic quasi-static buckling of buried pipelines subject to seismic excitations. Using shell dimensions and the stiffness of the soil medium surrounding the pipe as parameters, a series of numerical results are obtained, which show that no significant half sine wave occurring in the circumferential direction for relatively long pipes when axial buckling load reaches its minimum. It also is shown that for a given pipeline an increase in soil stiffness causes a decrease in wavelength of the critical-load instability mode and, consequently, causes an increase in axial critical load.

INTRODUCTION

Buckling failure of long buried pipelines during a severe seismic excitation has been recognized as an important problem in the pipeline industry. It has recently attracted the attention of earthquake engineers and researchers i.e., [1]. These studies, however, have concentrated on a long circular cylinder or tube surrounded by uniform soil medium, and subjected to uniform, static, radial pressure. Studies on the critical buckling load of buried pipelines due to axial compressive loading are rarely found in the literature. To the best of the authors' knowledge, the study on this subject was usually simplified and restricted to the model of a beam on an elastic foundation [2].

The purpose of this work is to investigate the buckling failure mode of buried pipelines under seismic excitations, using a model of a cylindrical shell surrounded by uniform springs. In the analysis, Donnell shell equations are used for quasi-shallow shells, and Flügge shell equations are employed when the shallow shell equations are not applicable. Attention is focused on the parametric studies that concern the effects on dimensions of the pipe itself as well as the stiffness of the soil medium surrounding the pipe.

FORMULATION

Consider a thin-walled circular cylindrical shell of finite length L, wall thickness h and mean radius a, with $h \ll a$, surrounded by a uniform soil medium with extensional stiffness k, in the radial direction. On the middle surface of the cylinder the cylindrical coordinate system x and θ is placed. Distances from the middle surface are measured by a coordinate

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z, positive outward. Displacement components in the x, θ and z directions are denoted by u,v and w, respectively. The cylindrical shell is considered to be subjected to edge loading and to static axial compressive loading P.

Donnell's form of the stability equations

Using the stationary potential energy criterion, the nonlinear equilibrium equations of the Donnell's form are written as follows [4].

$$a\,N_{x,x} + N_{x\theta,\theta} = 0$$

$$a\,N_{x\theta,x} + N_{\theta,\theta} = 0 \qquad (1)$$

$$D\nabla^4 w + \frac{1}{a} N_\theta - (N_x w_{,xx} + \frac{2}{a} N_{x\theta} w_{,x\theta} + \frac{1}{a^2} N_\theta w_{,\theta\theta}$$

$$+ kw + P\,w_{,xx} = 0$$

where N_x and N_θ are in-plane normal force intensities, $N_{x\theta}$ is in-plane shear force intensity, ∇ is the Laplace operator and $(\ )_{,x_i} = \frac{\partial}{\partial x_i}(\ )$. The linear equilibrium equations can easily be obtained by the omission of all quadratic and high-order terms in u, v, and w from Eq. (1). Introduction of the constitutive and kinematic relations to these linear equilibrium equations yields

$$a^2 u_{,xx} + \frac{1}{2}(1-\mu)u_{,\theta\theta} + \frac{1}{2}(1+\mu)a\,v_{,x\theta} + a\mu w_{,x} = 0$$

$$\frac{1}{2}(1+\mu)a u_{,x\theta} + \frac{1}{2}(1-\mu)a^2 v_{,xx} + v_{,\theta\theta} + w_{,\theta} = 0 \qquad (2)$$

$$D\nabla^4 w + \frac{1}{a^2} C(v_{,\theta} + w + a\mu u_{,x}) + kw + Pw_{,xx} = 0$$

In Eq. (2), μ is the Poisson's ratio and C and D are termed extensional and bending stiffness parameters, respectively, and are given by

$$C = \frac{Eh}{1-\mu^2}$$

and (3)

$$D = \frac{Eh^3}{12(1-\mu^2)}$$

in which E is Young's modulus.

With the aid of linear equilibrium equations given in Eq. (2), the linear stability equation of the Donnell's form can be obtained by either the adjacent-equilibrium or the minimum potential energy criterion and written in the form

$$
\begin{aligned}
&a^2u^*_{,xx}+\frac{1}{2}(1-\mu)u^*_{,\theta\theta}+\frac{1}{2}(1+\mu)a\,v^*_{,x\theta}+a\,\mu w^*_{,x}=0\\
&\frac{1}{2}(1+\mu)a\,u^*_{,x\theta}+\frac{1}{2}(1-\mu)a^2v^*_{,xx}+v^*_{,\theta\theta}+w^*_{,\theta}=0\\
&D\nabla^4w^*+\frac{1}{a^2}C(v^*_{,\theta}+w^*+a\mu\,u^*_{,x})-(N^*_xw^*_{,xx}+\frac{2}{a}N^*_{x\theta}w^*_{,x\theta}+\frac{1}{a^2}N^*_\theta w^*_{,\theta\theta})\\
&\qquad+kw^*+Pw^*_{,xx}=0
\end{aligned}
\tag{4}
$$

In Eq. (4), u^*, v^*, and w^* are the displacements of the buckled shell and N^*_x, N^*_θ, and $N^*_{x\theta}$ are governed by the linear form of Eqs. (1) and can be obtained for a given edge loading condition.

It is to be noted that Eqs. (4) represent a coupled set of three equations in the variables u^*, v^*, w^*. These equations may also be partially uncoupled and written as follows.

$$
\begin{aligned}
&\nabla^4u^*=-\frac{\mu}{a}w^*_{,xxx}+\frac{1}{a^3}w^*_{,x\theta\theta}\\
&\nabla^4v^*=-\frac{2+\mu}{a^2}w^*_{,xx\theta}-\frac{1}{a^4}w^*_{,\theta\theta\theta}\\
&D\nabla^8w^*+\frac{1-\mu^2}{a^2}Cw^*_{,xxxx}-\nabla^4(N^*_xw^*_{,xx}+\frac{2}{a}N^*_{x\theta}w^*_{,x\theta}+\frac{1}{a^2}N^*_\theta w^*_{,\theta\theta})\\
&\qquad+k\nabla^4w^*+P\nabla^4w^*_{,xx}=0
\end{aligned}
\tag{5}
$$

Flügge's form of the stability equations

In a sense, the Flügge's stability equations can be regarded as Donnell's stability equations plus a "modifying" part.

The Donnell's stability equations given in Eqs. (4) in the absence of extensional stiffness of soil medium k and axial compressive force P can be written in a matrix form as follows.

$$
[L_D]\{x^*\}=\{0\} \tag{6}
$$

where $[L_D]$ and $\{x^*\}$ are a matrix differential operator and a displacement vector respectively and are given by

$$
[L_D]=\begin{bmatrix}
a^2\frac{\partial^2}{\partial x^2}+\frac{1-\mu}{2}\frac{\partial^2}{\partial\theta^2} & \frac{1+\mu}{2}a\frac{\partial^2}{\partial x\partial\theta} & \mu a\frac{\partial}{\partial x}\\
 & \frac{1-\mu}{2}a^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial\theta^2} & \frac{\partial}{\partial\theta}\\
\text{Sym.} & & 1+\frac{a^2h^2}{12}\nabla^4
\end{bmatrix}
\tag{7}
$$

and

$$\{x^*\} = \begin{Bmatrix} u^* \\ v^* \\ w^* \end{Bmatrix} \tag{8}$$

The matrix differential operator according to Flügge's theory can be obtained and treated as the sum of two operators,

$$[L_F] = [L_D] + \xi\ [L_{MOD}] \tag{9}$$

where $[L_{MOD}]$ is a "modifying" operator and ξ is the nondimensional shell parameter defined by

$$\xi = \frac{h^2}{12a^2} \tag{10}$$

The modifying operator in Eq. (9) takes the form

$$[L_{MOD}] = \begin{bmatrix} \frac{1-\mu}{2}\frac{\partial^2}{\partial\theta^2} & 0 & -a^3\frac{\partial^3}{\partial x^3} + \frac{1-\mu}{2}\,a\,\frac{\partial^3}{\partial x\partial\theta^2} \\ & \frac{3(1-\mu)}{2}\,a^2\frac{\partial^2}{\partial x^2} & -\frac{3-\mu}{2}\,a^2\frac{\partial^3}{\partial x^2\partial\theta} \\ \text{Sym.} & & 1+2\frac{\partial^2}{\partial\theta^2} \end{bmatrix} \tag{11}$$

Including both the soil medium effects plus the axial compressive load in Eq. (9), the matrix differential operator given in Eq. (7) becomes [3]

$$[L_D] = \begin{bmatrix} (1-\frac{P}{C})a^2\frac{\partial^2}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2}{\partial\theta^2} & \frac{1+\mu}{2}\,a\,\frac{\partial^2}{\partial x\partial\theta} & \mu a\frac{\partial}{\partial x} \\ & (\frac{1-\mu}{2}-\frac{P}{C})a^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial\theta^2} & \frac{\partial}{\partial\theta} \\ \text{Sym.} & & (1+\frac{a^2}{C}k)+\frac{P}{C}\,a^2\frac{\partial^2}{\partial x^2}+\frac{a^2h^2}{12}\nabla^4 \end{bmatrix} \tag{12}$$

APPLICATIONS

In the following, parametric studies on the critical load analysis for a long simply supported cylindrical shell are considered. Since the axial buckling is the main concern in this study, only the seismically induced axial load is considered in the analysis

Donnell's axial critical load

In the absence of edge loading, the stability equation governing the deformation in the radial direction becomes

$$D\nabla^8 w^* + \frac{1-\mu^2}{a^2}\,Cw^*_{,xxxx} + k\nabla^4 w^* + P\nabla^4 w^*_{,xx} = 0 \tag{13}$$

Solutions of the form

$$w^* = A \sin mx \cdot \sin n\theta \tag{14}$$

where A is an arbitrary constant, $m = \hat{m}\pi/L$, and $\hat{m},n=1,2,3...$, are seen to satisfy both the differential equation and the assumed simply supported boundary conditions. It is noted that the quantities $\hat{m}$ and n represent the number of half sine waves in the axial and circumferential directions respectively.

Substitution of Eq. (14) into Eq. (13) gives

$$P = \frac{1}{m^2}\left[k + D\left(m^2 + \frac{n^2}{a^2}\right)^2\right] + \frac{C(1-\mu^2)}{a^2} \cdot \frac{m^2}{\left(m^2 + \frac{n^2}{a^2}\right)^2} \tag{15}$$

A distinct eigenvalue P corresponds to each element of the set $(\{\hat{m}\} x \{n\})$. A preliminary study in this work has shown that, when the axial compressive load P reaches its minimum, the cross section remains circular. It is therefore reasonable to assume that the shell deforms in such a way that the shape is totally governed by the number of axial half sine waves. Assuming

$$w^* = A \sin \frac{m\pi x}{L} \tag{16}$$

and substituting Eq. (16) into (13), the counterpart of Eq. (15) becomes

$$P = D\frac{m^2\pi^2}{L^2} + \left(k + \frac{1-\mu^2}{a^2} C\right)\frac{L^2}{m^2\pi^2} \tag{17}$$

The axial critical load P_{cr} can then be obtained by minimizing Eq. (17) with respect to m as follows:

$$P_{cr} = \frac{Eh^2}{a}\left[\frac{1+\alpha}{3(1-\mu^2)}\right]^{\frac{1}{2}} \tag{18}$$

where α is the nondimensional parameter,

$$\alpha = \frac{k}{(Eh/a^2)} \tag{19}$$

The number of half sine waves in the axial direction is

$$m = \left[\frac{L}{\pi}\ \frac{12(1-\mu^2)(1+\alpha)}{\alpha^2 h^2}\right]^{\frac{1}{4}} \tag{20}$$

The Batdorf parameter Z [4] is given by

$$Z = \frac{L^2}{ah}(1-\mu^2)^{\frac{1}{2}}\ . \tag{21}$$

Then, substitution of Eqs. (3) and (21) into Eqs. (17) and (20) gives

$$P = \frac{1}{2\sqrt{2}}\left[\frac{\pi^2}{Z} m^2 + \frac{12(1+\alpha)}{\pi^2} \cdot \frac{Z}{m^2}\right] \cdot P_{cr} \tag{22}$$

and

$$m = \frac{Z}{\pi}\left[12(1+\alpha)\right]^{\frac{1}{2}} \tag{23}$$

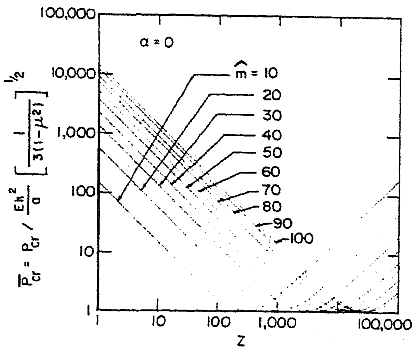

Fig. 1 Nondimensional critical axial load $\bar{P}_{cr}$ versus Z.

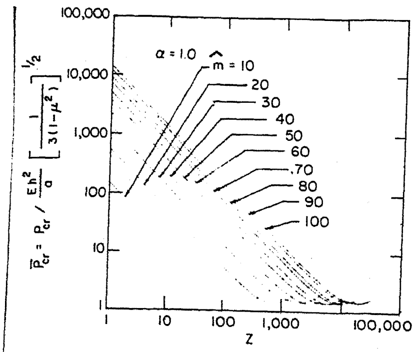

Fig. 2 Nondimensional critical load $\bar{P}_{cr}$ versus Z.

Figures (1) and (2) show the variation of critical axial load, P_{cr}, as a function of the Batdorf parameter Z for various values of $\hat{m}$, the number of half sine waves. In Fig. (1) α is zero while in Fig. (2) α is unity. Similar curves for other relevant values of α would be obtained and utilized in the determination of the critical load. Therefore, this procedure could be useful in the design of buried pipelines.

Flügge's Axial critical load

Due to the shallowness limitation, in the case of moderate or short cylindrical shells, Flügge's stability equations should be used rather than Donnells'.

For a simply supported shell, the displacement $\{x^*\}$ has the components

$$u^* = A^* \sin n\theta \cos \frac{\lambda x}{a}$$

$$v^* = B^* \cos n\theta \sin \frac{\lambda x}{a} \qquad (24)$$

$$w^* = C^* \sin n\theta \sin \frac{\lambda x}{a}$$

where A*, B*, C* are arbitrary constants and

$$\lambda = \frac{m\pi a}{L} \qquad (25)$$

with m being the number of half sine waves in the axial direction of the shell. Thus Eqs. (9)-(11) yield

$$A^*[\lambda^2 + \frac{1-\mu}{2} n^2(1+\xi) - \Phi\lambda^2] + B^*[-\frac{1+\mu}{2}\lambda n] + C^*[-\lambda\mu - \xi(\lambda^3 - \frac{1-\mu}{2}\lambda n^2)] = 0$$

$$A^*[-\frac{1+\mu}{2}\lambda n] + B^*[n^2 + \frac{1-\mu}{2}\lambda^2(1+3\xi) - \Phi\lambda^2] + C^*[n - \frac{3+\mu}{2}\xi\lambda^2 n] = 0 \qquad (26)$$

$$A^*[-\lambda\mu - \xi(\lambda^3 - \frac{1-\mu}{2}\lambda n^2)] + B^*[n + \frac{3-\mu}{2}\xi\lambda^2 n] + C^*[1 + \xi(\lambda^4 + 2\lambda^2 n^2 + n^4 - 2n^2 + 1) + (1-\mu^2)\alpha - \Phi\lambda^2] = 0$$

where

$$\Phi = \frac{P}{C} = \frac{P(1-\mu^2)}{Eh} \tag{27}$$

The above represents three linear equations with the buckling amplitudes A*,B*,C* as the unknowns. For a non trivial mode, the determinant of Eq. (26) must be zero. For small values of the parameters ξ and Φ, it is sufficient to retain only linear terms. Thus the determinant becomes,

$$\begin{aligned}&ABC+ABc\xi-AB\lambda^2\Phi+3AbC\xi-A\lambda^2C\Phi+aBC\xi-B\lambda^2C\Phi\\&+2DEF+2DEf\xi+2DeF\xi-BE^2-2BEe\xi-3bE^2\xi\\&+\lambda^2E^2\Phi-AF^2-2AFf\xi-aF^2\xi+\lambda^2F^2\Phi-DC^2-cD^2\xi+\lambda^2D^2\Phi=0\end{aligned} \tag{28}$$

where $A=\lambda^2+a,\quad B=n^2+b,\quad C=1+(1-\mu^2)\alpha,\quad D=-\frac{1+\mu}{2}\lambda n,\quad E=-\lambda u,\quad F=n$ (29)

with $a = \frac{1-\mu}{2} n^2,\quad b = \frac{1-\mu}{2}\lambda^2,\quad c=\lambda^4+2\lambda^2n^2+n^4-2n^2+1$ (30)

$$e = -\lambda(\lambda^2-a), \qquad f = \frac{3-\mu}{2}\lambda^2 n$$

Solving for Φ in Eq. (28) yields

$$\Phi = \frac{R}{S} \cdot \frac{1}{\pi^2}\left(\frac{L}{ma}\right)^2 \tag{31}$$

where $R=(ABC+2DEF-BE^2-AF^2-CD^2)+(ABc+3AbC+aBC+2DEF+2DeF-2BFe$

$-3bE^2-2AFf-aF^2-cD^2)\,\xi,\quad S = AB+AC+BC-D^2-E^2-F^2$ (32)

Figures (3) and (4) give the variation of Φ which is related to the axial buckling load, as a function of the non dimensional parameter L/ma. Both figures are for a ductile iron pipe of L=40 ft, a=24 in, h=0.51 in and $\xi=3.763 \times 10^{-5}$. They differ in that the higher value of α in Fig. (3) is 0.01 while in Fig. (4), it is 0.5. Both figures also give the case of no soil medium around the pipe (α=0).

The stability of the above pipe under seismically induced axial loads can be examined using Figures (3) and (4) for selected values of L/ma (m=1,2,3,etc.) and the corresponding values of Φ. As long as all the points for specific values of L/ma and Φ lie below the heavier dashed lines (no soil) or the solid lines (with soil) the pipe is stable. When the axial load P and hence Φ increases, all the points move upward. As soon as any point reaches one of the curves, the pipe is in neutral equilibrium and about to buckle. Figs. (3) and (4) show that the soil medium has an important effect in the critical axial load by causing an appreciable increase in P_{cr}.

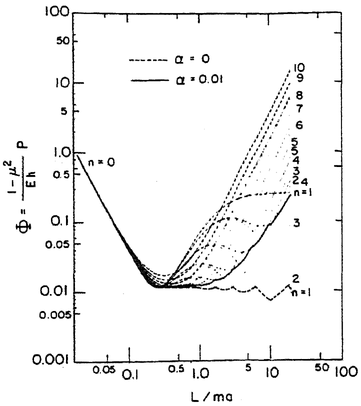

Fig. 3 Buckling diagram for axial compression.

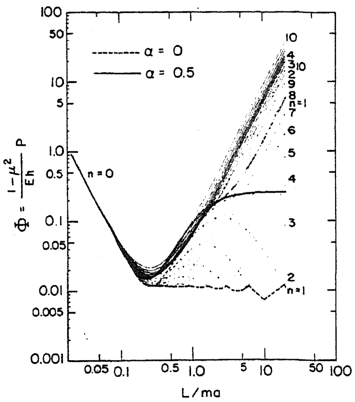

Fig. 4 Buckling diagram for axial compression.

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