RESPONSE OF A BURIED PIPE TO SEISMIC WAVES

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SUMMARY

The lifeline earthquake engineering problem of dynamic response of a buried pipe in an elastic medium due to incident seismic waves has been studied in this paper. The pipe has been modelled as a thin elastic circular cylindrical shell of infinite length. It is assumed to be embedded in a semi-infinite elastic homogeneous medium. The response of the pipe has been shown to be greatly influenced by the depth from the free surface of the half-space which should be taken into account in actual design of buried pipes.

INTRODUCTION

In recent years much attention has been focused on the seismic response of buried pipelines, which have been found to be extensively damaged due to earthquakes. Recent works in this area have been reviewed in References 1-3. It appears that none of these works has treated the problem of the interaction of a buried pipe with its surrounding semi-infinite elastic medium.

The object of the present paper is to analyze the motion of a pipe buried at a finite depth in an elastic half-space. For simplicity attention has been focused here on the plane strain problem. The more general three dimensional problem will be discussed in further communications.

FORMULATION

Consider a thin circular shell of mean radius a and thickness $\tilde{h}$ embedded with its axis at a depth h from the free surface of an elastic isotropic homogeneous half-space. Cartesian coordinates x, y, z are chosen so that the axis is parallel to the z-axis and $y = 0$ is the free surface.

The equations of motion of the medium are, in non-dimensional form,

$$\underset{\sim}{\nabla}\underset{\sim}{\nabla}\cdot\underset{\sim}{u}(\bar{x},\bar{y}) - \tau^{-2}\underset{\sim}{\nabla}\wedge\underset{\sim}{\nabla}\wedge\underset{\sim}{u}(\bar{x},\bar{y}) = -\epsilon^2\underset{\sim}{u}(\bar{x},\bar{y}) \tag{1}$$

where

$$\bar{x} = x/a\,,\quad \bar{y} = y/a\,,\quad \tau = c_1/c_2\,,\quad \epsilon = \omega a/c_1\,,$$

$$c_1/c_2 = \sqrt{\frac{2(1-\sigma)}{1-2\sigma}}$$

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σ is the Poisson's ratio and ω the circular frequency. The factor $e^{-i\omega t}$ has been suppressed.

The equations of motion of the shell are taken as (see Flügge [4])

$$Bu = -\epsilon^2 Ku \quad \text{at} \quad \bar{r} = 1 \tag{2}$$

where

$$B = \begin{bmatrix} K_3 \{\frac{\partial}{\partial\theta} + [1 + K_4 (1 + \partial^2/\partial\theta^2)^2] + \partial/\partial\bar{r} + \sigma^* \frac{1}{\bar{r}} & \frac{1}{\bar{r}} \frac{\partial}{\partial\theta} \\ K_1 \frac{\partial}{\partial\theta} + \frac{1}{\bar{r}} \frac{\partial}{\partial\theta} & K_1 \frac{\partial^2}{\partial\theta^2} + \frac{\partial}{\partial\bar{r}} - \frac{1}{\bar{r}} \end{bmatrix}$$

$$u = \begin{bmatrix} u_r \\ u_\theta \end{bmatrix}, \quad K = \begin{bmatrix} K_5 & 0 \\ 0 & K_2 \end{bmatrix}, \quad K_1 = \frac{D}{a\mu}$$

$$K_2 = -\frac{\rho_s \tilde{h} c_1^2}{a\mu}, \quad K_3 = -\frac{D(1-\sigma^{*2})}{aE^*}, \quad K_4 = \frac{1}{12}(\frac{\tilde{h}}{a})^2$$

$$\sigma^* = \frac{\sigma}{1-\sigma}, \quad E^* = \frac{2\mu}{1-\sigma}, \quad D = \frac{E_s \tilde{h}}{1-\sigma_s^2}$$

$\mu \equiv$ the rigidity of the medium; E_s, ρ_s, σ_s are the Young's modulus, density and Poisson's ratio of the shell, respectively.

Here polar coordinates r, θ have been used with origin at the center of the shell and θ measured from the negative y-axis.

METHODS OF SOLUTION

The solution of Eq. 1 has to be found subject to Eq. 2, the stress-free boundary conditions at $\bar{y} = 0$, and appropriate radiation conditions as $\bar{r} \to \infty$. In an infinite medium the solution is easily obtained by decomposing $\underset{\sim}{u}$ into its angular modes, which are uncoupled. However, these modes are coupled to a bounded medium. For this reason two methods of solution were used in this paper.

Successive Reflections

Let $\underset{\sim}{u}^{(i)}$ be some incident disturbance, be it due to body or surface waves. Then $\underset{\sim}{u}$ can be written as

$$\underset{\sim}{u} = \underset{\sim}{u}^{(i)} + \underset{\sim}{u}^{(s)}, \quad \underset{\sim}{u}^{(s)} = \underset{\sim}{\nabla}\Phi^{(s)} + \underset{\sim}{\nabla}\wedge(\Psi^{(s)}\underset{\sim}{e_z}) \tag{3}$$

$\Phi^{(s)}$ and $\Psi^{(s)}$ are now expanded as

$$\Phi^{(s)} = \Phi_1^{(s)} + \Phi_2^{(s)} + \ldots \quad , \quad \Psi^{(s)} = \Psi_1^{(s)} + \Psi_2^{(s)} + \ldots \tag{4}$$

where $\Phi_1^{(s)}$, $\Psi_1^{(s)}$ are the solutions in an infinite medium. These can be written as

$$\Phi_1^{(s)} = \sum_{n=-\infty}^{\infty} A_n^{(1)} H_n(\epsilon\bar{r})\, e^{in\theta} \quad , \quad \Psi_1^{(s)} = \Sigma B_n^{(1)} H_n(\tau\epsilon\bar{r})\, e^{in\theta} \tag{5}$$

Writing

$$u_r = \Sigma u_{nr}\, e^{in\theta} \quad ; \quad u_\theta = \Sigma u_{n\theta}\, e^{in\theta} \tag{6}$$

one can easily solve for $A_n^{(1)}$, $B_n^{(1)}$ in terms of the u_{nr} , $u_{n\theta}$ and their derivatives (via the application of matrix B) at $\bar{r} = 1$. Since $u_1^{(s)}$ does not satisfy the stress-free condition at $\bar{y} = 0$, a displacement $\underset{\sim}{u}_2^{(s)}$ is added. This is obtained in terms of $\Phi_2^{(s)}$ and $\Psi_2^{(s)}$, which are

$$\Phi_2^{(s)} = \Sigma\, [A_n^{(1)} E_n + B_n^{(1)} G_n] \, , \quad \Psi_2^{(s)} = \Sigma\, [A_n^{(1)} F_n + B_n^{(1)} H_n] \tag{7}$$

The potentials E_n and F_n represent the dilatational and shear waves created in a half-space due to a dilatational source represented by $H_n(\epsilon\bar{r})\, e^{in\theta}$. Similarly G_n, H_n are the potentials due to a shear source represented by $H_n(\tau\epsilon\bar{r})\, e^{in\theta}$.

Since $\underset{\sim}{u}_2^{(s)}$ does not satisfy Eq. 2, a displacement field $\underset{\sim}{u}_3^{(s)}$ is added. This will be given in terms of potentials $\Phi_3^{(s)}$ and $\Psi_3^{(s)}$, which have the same forms as given in Eq. 5 with different constants.

This process can be continued until the desired degree of accuracy is attained. It is found that the two series given in Eq. 4 converge rapidly if $\epsilon h/a >> 1$. For $\epsilon h/a = O(1)$, however, these converge very slowly and then the following alternative method was used.

Matched Asymptotic Expansions

The displacement $\underset{\sim}{u}$ is expanded in two asymptotic series, one valid near the shell and the other far away. The former is called the 'inner expansion' and the other the 'outer expansion.'

The inner expansion is assumed to be

$$\underset{\sim}{u} = \underset{\sim}{u}_o + \mu_1(\epsilon)\,\underset{\sim}{u}_1 + \ldots \tag{8}$$

where $\lim_{\epsilon\to 0} \mu_{n+1}/\mu_n = 0$. Assuming that the pipeline is disturbed by an incident wave which originates outside the pipe the incident field, $\mathbf{u}^{(i)}$, can be expanded as

$$\mathbf{u}^{(i)} = \mathbf{u}_o^{(i)} + \epsilon\,\mathbf{u}_1^{(i)} + \epsilon^2\,\mathbf{u}_2^{(i)} + \ldots \tag{9}$$

where $\mathbf{u}_o^{(i)}$ is a rigid body displacement.

Comparison of Eqs. 9 and 10 gives

$$\mathbf{u}_o = \mathbf{u}_o^{(i)} \tag{10}$$

and $\mu_1(\epsilon) = \epsilon$. Then from Eq. 1 $\mathbf{u}_1$ satisfies the equation

$$\mathcal{L}\mathbf{u}_1 \equiv \nabla\nabla\cdot\mathbf{u}_1 - \tau^{-2}\nabla\wedge\nabla\wedge\mathbf{u}_1 = 0 \tag{11}$$

Further, the inner boundary conditions satisfied by $\mathbf{u}_1$ is obtained from substituting Eq. 8 in Eq. 2 and equating the like ordered terms, i.e.

$$B\,\mathbf{u}_1 = 0 \tag{12}$$

This determines $\mathbf{u}_1$ completely.

Once $\mathbf{u}_1$ is known one may now proceed to determine the outer expansion.

The outer variables are defined as

$$x' = \epsilon\bar{x}\;, \quad y' = \epsilon\bar{y}\;, \quad r' = \epsilon\bar{r} \tag{13}$$

and the outer expansion of $\mathbf{u}$ is taken as

$$\mathbf{u} = \mathbf{u}^{(i)} + \nu_1(\epsilon)\,\mathbf{u}_1' + \ldots \tag{14}$$

where $\lim_{\epsilon\to 0} \nu_{n+1}/\nu_n = 0$. It can be shown by the process of matching that $\nu_1(\epsilon) = \epsilon^2$ and

$$\mathbf{u}_1' = \nabla'\,\Phi_1 + \nabla'\wedge(\Psi_1\,\mathbf{e}_z) \tag{15}$$

where

$$\begin{aligned} \Phi_1 &= \sum_{n=-2}^{2} [a_n\,\Phi_n^{\,p} + b_n\,\Phi_n^{\,s}] \\ \Psi_1 &= \sum_{n=-2}^{2} [a_n\,\Psi_n^{\,p} + b_n\,\Psi_n^{\,s}] \end{aligned} \tag{16}$$

where $\Phi_n^{\,p} = H_n(\epsilon\bar{r})\,e^{in\theta} + E_n$, $\Phi_n^{\,s} = G_n$, $\Psi_n^{\,p} = F_n$, $\Psi_n^{\,s} = H_n(\tau\epsilon\bar{r})\,e^{in\theta} + H_n$.

Both expansions 8 and 14 were carried out to $0(\varepsilon^4)$. Expansion 8 was found to be (for details see Ref. [5])

$$\underset{\sim}{u} = \underset{\sim}{u}_o + \varepsilon \underset{\sim}{u}_1 + \varepsilon^2 \ln\varepsilon \underset{\sim}{u}_2 + \varepsilon^2 \underset{\sim}{u}_3 + \varepsilon^3 \ln\varepsilon \underset{\sim}{u}_4 + \varepsilon^3 \underset{\sim}{u}_5 + \varepsilon^4 (\ln\varepsilon)^2 \underset{\sim}{u}_6 + \varepsilon^4 \ln\varepsilon \underset{\sim}{u}_7 + \varepsilon^4 \underset{\sim}{u}_8 + 0(\varepsilon^5) \quad (17)$$

NUMERICAL RESULTS AND DISCUSSION

Expansions 17 and 4 were used to compute the displacement of the shell and hoop stress in it. The shell was taken to be of steel and two types of surrounding medium, one rocky (σ = 0.2, ε = 20.6 GPa, ρ = 2424 Kg/m^3) and the other soft clay (σ = 0.4, ε = 0.01 GPa, ρ = 2500 Kg/m^3). Some representative results are shown in Figs. 1-4.

It is generally observed that decreasing h/a results in larger displacements and hoop stress. Also, decreasing the stiffness of the surrounding medium increases the hoop stress and decreases the radial displacement. Increasing ϵ initially increases the hoop stress but then decreases it. Note that for small ϵ Eq. 17 were used whereas, for large ϵ use was made of Eq. 4.

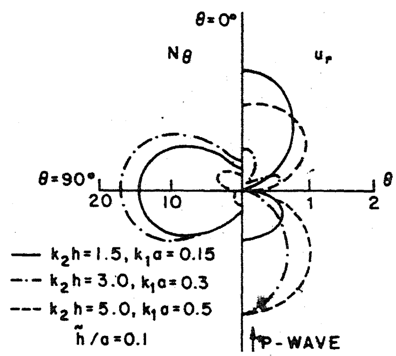

Fig. 1

Plot of u_r and N_θ with θ for various ϵ for incident P-wave. (Rocky Material)

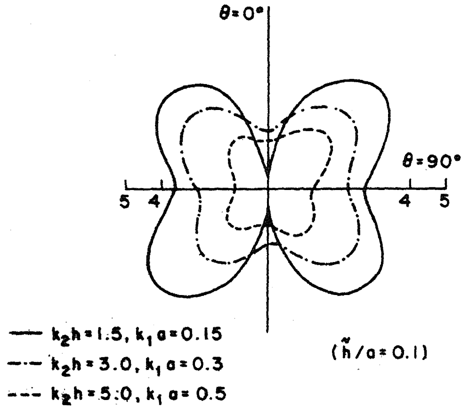

Fig. 2
Plot of N_θ for incident Rayleigh Wave. (Rocky Material)

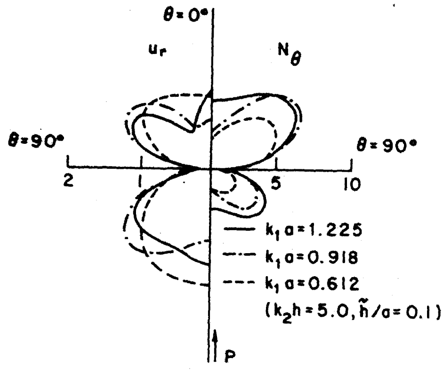

Fig. 3
Plot of u_r and N_θ for incident P-wave for large ϵ. (Rocky Material)

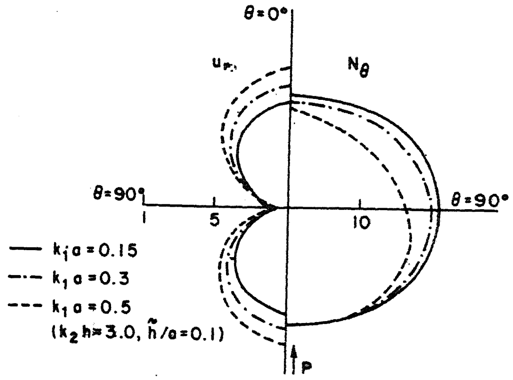

Fig. 4

Plot of u_r and N_θ for incident P-wave. (Soft Clay)

Varying the angle of incidence of the incident wave was found to have strong influence on the response of the shell. (This is not shown here.)

ACKNOWLEDGMENT

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