On a Procedure of Response Analysis and Its Response Observation of Three-dimensional Piping System to Earthquake

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Summary

This paper deals with three dimensional response analysis of piping systems in nuclear power plants and other industrial facilities. At first, the authors mention that their vertical response should be analyzed as well as the horizontal one based on the data observed on the chemical engineering plant model in Chiba Field Station. Then, they discuss how to decide two components of design basis horizontal ground acceleration in relation to vertical component.

In the latter part of the paper, they discuss on the procedure of identifying the vibration characteristics, damping coefficient, mode distribution, exciting coefficient and so on, of piping systems from the records of their response to natural earthquakes.

The way of evaluation of the most suitable pickup arrangement is discussed. And then the trial to find the parameters based on the response record of the model pipings to a recent natural earthquake is explained. The damping characteristics of the model is so low as to applying locally stationarity assumption to the analysis, they only tried a moving-window type fitting technique.

1. Introduction

In Japan, most of design analysis of piping systems in nuclear power plants are made in the worst one horizontal axis, and summing up its result with that of vertical load effect which is evaluated under an assumption where the system is rigid. As already well-known, most of pipings in industrial facilities, such as nuclear power plants, conventional steam power plants, petro-chemical industries, oil refinaries, are flexible in three directions more or less. There is no big difference between their vibration characteristics in vertical direction and those in horizontal directions. Such data can be obtained only through a response observation or a very large-amplitude excitation test. The former method is more economical in the frequent-earthquake occuring countries like Japan. In some nuclear power plants, response recording systems were mounted on their piping systems and other equipment. Environment of such systems is usually noisy to record their earthquake response, and also they have non-linear characteristics as mentioned-above. So the data treatment of records of their response to estimate their vibration characteristics is rather difficult. The authors made a trial by using their model plant. The development of the procedure is still on the way.

2. Concept of Three-dimensional Piping Analysis

Three-dimensional analysis of piping systems seems to be easier than those of other types of structure. Its design analysis involves no difficult problem, if it remains in elastic range. The modal analysis technique is usually employed both for a time history analysis and a response spectrum analysis. The time history analysis is quite simple, except multi-input problem. On the other hand, the response analysis always brings some difficulty to select the schemes of summing-up the results of analysis of each mode. Times of reaching the maximum response are different mode by mode. The map of the maximum reaching time T_p can be drawn from several papers as shown in Fig. 1, such as Sato's.[1] For flexible, light damped and multi-degrees-of-freedom systems, their maximum response times are delayed from the time of the peak ground acceleration. Our image of piping system is that of flexible system like this. The actual system in a nuclear power plant becomes rather rigid by the regulatory requirement through the world at the present time.

In some sense, the too severe rigid design scheme is more dangerous than the flexible design scheme. On the other hand, dynamic analysis of flexible systems has some uncertainties as one of the authors mentioned very often,[2][3] however, we can overcome them with our adequate design judgements.

3. Response of a Piping System in Field Station

The authors have been observing the responses of a model piping system to natural earthquakes in Chiba Field Station,[2][4] Institute of Industrial Science. The histograms of response factors both of horizontal and vertical ground motions show their fluctuations in the same order (in Fig. 1). This model consists of two L-shape pipings connected to vessels at the both ends. Their critical damping ratios are not so high, and their computed eigen-frequencies are 4.73 Hz in horizontal and 5.50 Hz in vertical. The mean response factors are 22.6 in horizontal and 36.6 in vertical, and their dispersion factors are 34% and 32% respectively. This fact obtained by the author's observation shows that the response analysis to vertical ground motions is necessary as well as that to horizontal ground motions.

4. Three Components of Design Basis Earthquake

The studies on input ground motions have been made by various groups. One of the ways, which was shown by Penzien and Watabe,[5] is very effective. This method gives us time-dependent three principal values of the ground motions. An example is that of a moderate earthquake observed in the field station. This earthquake consisted of three successive shocks. The larger two axes σ_1 , σ_2 may be said to be shear waves, and the smallest one σ_3 to be P-wave and the vertical component of surface waves. There is not exact correspondence, and it is only analogical one based on their incident angles. Anyway, the angle of the third axis is usually near to vertical, 80°∿90° to the horizontal plane. We can find two facts which we should mention. One is the fact that the ratio of vertical component σ_3 to the horizontal component σ_1 and σ_2 is lower than that of peak vertical ground acceleration to those of horizontal ones. The second one is the fact that the peaks of σ_3 occured in the almost same times of those of σ_1 and σ_2, even though the axis of σ_3 , is the largest in the initial phase of an earthquake.

The next important discussion obtained from such type of analysis is on the relation of horizontal records of two components to the actual horizontal ground motion. Is the value of a recorded peak ground acceleration the actual maximum ground acceleration? It is always an awkword question to decide the design basis earthquake. Two axes of recording devices usually are not placed so as to coincide to the directions of two horizontal principal axes. The relation of those values can be shown by Mohr's circle diagram used for the stress relation. If we assume that the waves of the first principal axis would be independent to those of the second axis or that at least the peak value of σ_1 would be far greater than that of σ_2, a design basis acceleration or velocity may be decided based on the peak value of the first principal component σ_1. The observed peak values of two orthogonal components usually do not coincide with those of the principal axes. Therefore, we need to modify the observed values to the principal values or the design basis value.

The example shown in Fig. 2 is an unusual case, and the ratio of σ_2 to σ_1 is rather high. It is more than 70% in the periods of three peaks. The response of a low damping system like piping systems is resonative, and its maximum response value is determined by total input energy. Therefore, vector sum of these components σ_1 and σ_2 is reasonable in this view point. If we assume the peak value of σ_2 is 80% of that of σ_1, then the vector sum is 128% of σ_1. Then if we can assume that σ_1 and σ_2 depend on each other completely, then the recorded peak value may reach to 128% of the peak value of σ_1, which is equal to the vector sum, and 64% in the orthogonal axis as shown in Fig. 3(a). On the other hand, we may say that the peak values are 91% of the peak value in the σ_1 axis, if two peak values are equal as shown in Fig. 3(b). Usually the ratio of σ_2 to σ_1 is approximately one third, then these values are 120%, 53% and 85% respectively. Therefore, if the recorded peak values of two components are nearly equal, then the peak values in the first principal axis may be 110% of the recorded peak value. And the vector sum may estimated as the $\sqrt{2}$ times of the recorded peak value as shown in Fig. 3(b) independently to their ratio. In the opposit case shown in Fig. 3(a), the larger peak is equal to the vector sum. Therefore, the author dare to say that the effective peak horizontal ground acceleration for the design is $\sqrt{2}$ times of the maximum recorded peak value, if two recorded values are near to equal each other. Otherwise, if the one peak value is less than two thirds of the other peak value, then it is almost sure that the effective peak value is equal to the maximum peak value.

5. 3-D Response Analysis of Piping Systems

Under an assumption that the motions of supporting points are synchronized, then the very common modal analysis can be introduced.

$$\mathbf{U}(t)=\sum_{j=1}^{n} \mathbf{B} \int_{-\infty}^{\infty} h_j(t-\xi)\,\mathbf{A}(\xi)\,d\xi \tag{1}$$

Vectors $\mathbf{U}$ and $\mathbf{A}$ are transforms of 3-D response in displacement and 3-D ground acceleration respectively. The function $h_j(t)$ is the initiatory response function of a single-degree-of-freedom system. If we assume local stationarity, then we can apply the transfer function method.

$$S_U(\omega,t)=\left\{\sum_{j=1}^{n} H_j(\omega) B_j P\right\} S_A(\omega,t) \left\{\sum_{k=1}^{n} \overline{H_k(\omega)}(B_k P)^T\right\} \tag{2}$$

$$= G(\omega)\, S_A(\omega, t)\, \overline{G(\omega)} \tag{2}$$

Spectra $S(t)$ and $S_A(t)$ are the functions of time.

The Equation (2) contains the rotation matrix P. Although we can compute S from eq. (2) without introducing P, this rotation matrix works to convert S_A into a diagonal matrix whose axes coincide with the principal axes of the input ground motions. Through this conversion, it becomes easier to understand the relations between three components of ground motions than those defined of UD, NS and EW. By introducing the principal value method, it makes easier the analysis than the direct analysis.

Next problem is the estimation of the peak value of the response. The procedure described in their chapter seems to be applicable for the estimation of the response spectra. There have been many discussions on the relation of the spectrum of response to the peak value since 1950's. Tajimi [6] mentioned that the response factor is proportional to the ratio of the total power of input ground acceleration waves to that of response waves. Sato [1] analyzed the distribution of extreme values based on the random process theory originated by S.O. Rice. These studies gave us the concept of their relation, however, they can give us no practice of response analysis as required by the regulatory authorities. Because they have much interests on the upperbound of peak values of response in deterministic sense rather than stochastic approaches.

6. Optimum Pickup Arrangement for Parameter Estimation

The latter part of this paper was associated with the response observation and data analysis of actual piping systems in fields. The study has not completed yet. At first, the author discuss on the observability, and will show an example of the data analysis using the recorded data on the newly equiped piping model in next chapter.

What is the observability of responding modes of a piping system? If we use one set of three acceleration pickups prepares for 3-D response observation, then we can observe only one mode of the system under excitation of single sinusoidal motions in an arbitrary angle. If two modes are excited by this single sinusoidal input, we can not separate these two modes, because two modes vibrate in the same frequency. To identify two modes excited by a single sinusoidal motions, we need two sets of pickups. One set should be equiped on the hoop of the fundamental mode, and another set on the hoop of the second mode; this is an answer.

$$\left.\begin{aligned} U(r,t) &= a_1(r)\,u_1(t) + a_2(r)\,u_2(t) \\ U(s,t) &= a_1(s)\,u_1(t) + a_2(s)\,u_2(t) \end{aligned}\right\} \tag{3}$$

From records $U(r)$ and $U(s)$, and mode shape functions $A(\xi)$, we can solve this eq. (3) and obtain the generalized displacement function of these two modes $u_1(t)$ and $u_2(t)$. The scheme described above is to find the condition that $a_1(r)$ and $a_2(s)$ would be as large as possible. Another scheme is to choose their arrangement as to be $a_2(r)=0$, that is, the position of a pickup is the nodal point of the second mode. Then, the upper equation becomes simpler to solve it. If the number of sets of pickups is equal to that of modes in the frequency range which we are observing, the procedure of solving the equation is simple. If we put

$$u(\xi_j, t)+\nu_j(t)=\sum_{i=1}^{n}\phi_i(\xi_j)\left[q_i(t)+\varepsilon_i(t)\right] \tag{4}$$

where $\nu_j(t)$ is recording error at the point ξ_j and $\varepsilon_i(t)$ is estimation error in the generalized response function of the i th mode $q_i(t)$. Then

$$\nu(t)=\Phi\,\varepsilon(t) \tag{5}$$

where elements of vectors ν and ε are formed by the errors on recording points and generalized functions. Here we should notice those numbers are equal to each other. Of course Φ is the matrix of modal functions along the recording points and the concerning modes and consists of elements ϕ_{ji}. If we put Ψ_{ji} as the cofactor of the ith mode function at point j, the error $\varepsilon_i(t)$ can be solved as

$$\varepsilon_i(t)=\{\det\Phi\}^{-1}\sum_{j=1}^{n}\nu_j(t)\,\Psi_{ji} \tag{6}$$

Then we try to put the expected value of $\varepsilon_i(t)$ is minimized under the assumption that each recording noise is independent to others. The evaluation function may be written as

$$J(\xi_1,\xi_2,\cdots,\xi_n)=\{\det\Phi\}^{-2}\sum_{i=1}^{n}\sum_{j=1}^{n}\Psi_{ji}^{2} \tag{7}$$

then the arrangement of pickups (ξ_1, ξ_2, $\cdots$ ξ_n) can be obtained by minimizing J. If a number m of recording points is larger than that of concerning modes, then we should apply the least square method. The optimum solution of the error vector be ($\hat{\varepsilon}_1$, $\hat{\varepsilon}_2$, $\cdots$ $\hat{\varepsilon}_n$), then total error K may be written as

$$K=\sum_{k=1}^{m}\left\{\sum_{j=1}^{n}\phi_j(\xi_k)(\hat{\varepsilon}_j-\varepsilon_j)\right\}^2 \tag{8}$$

that means, the sum of the squares of errors on each recording point is minimized by selecting $\hat{\varepsilon}_j$. The best estimation can be written $\hat{\varepsilon}_i(t)$ can be solved easily, then by using them the evaluation function is

$$J=\{\det\hat{\Phi}\}^{-2}\sum_{i=1}^{n}\sum_{k=1}^{m}\left\{\sum_{j=1}^{n}\hat{\Psi}_{ji}\phi_j(\xi_k)\right\}^2 \tag{9}$$

In Fig. 4 one of the examples of the value of J on a model piping system described in next chapter.

7. Parameter Estimation Procedure from Recorded Responses

The purpose of study in this chapter is development of parameter estimation procedures from time history record of piping systems of nuclear power plants, chemical engineering plants and so on. We do not expect to obtain their parameters under the condition of large amplitude corresponds to natural earthquake events. By using a model piping systems, whose vibration characteristics are known, estimation procedure is checked.

In general, the equation for the generalized function $q_i(t)$ is described as

$$\ddot{q}_i(t)+2\zeta_i\omega_i\dot{q}_i(t)+\omega_i^2 q_i(t)=\sum_{r=1}^{3}\beta_{ir}\alpha_r(t) \tag{10}$$

Based on recorded values $\ddot{q}_i'(t)$ and $\alpha_r'(t)$, we try to estimate $\hat{q}(t)$ or the parameters $\hat{\zeta}$ and $\hat{\beta}_r$ in eq. (10). The recorded values usually contain errors, so it is a parameter identification problem. The error function

$$\hat{e}(t) = \ddot{q}'(t) - \hat{\ddot{q}}(t) \tag{11}$$

should be minimized through the duration of observation for the set of $\hat{\zeta}$ and $\hat{\beta}_r$ as $min.[\sigma_i^2]$, where

$$\sigma_i^2 = T^{-1}\int_0^T [\hat{e}(t)]^2 dt \tag{12}$$

There are several methods. If there is not recording noise, fitting in time history is the best. To overcome noise, the window in time domain can be used. If we assume that the input earthquakes would be locally stationary, then correlation function or, direct Fourier spectrum can be used as well as the response spectrum.

The i th mode generalized function can be separated into three components $\ddot{q}_{ir}(t)$ responding to three components $\alpha_r(t)$ of input acceleration time histories, then the peak values of each generalized function are described by the response spectrum $S_{\alpha r}(\zeta_i, T_i)$ to $\alpha_r(t)$. Therefore $\beta_{ir} S_{\alpha r}(\zeta_i, T_i)$ is a parameter which we want to estimate instead of a set of ζ_i and ω_i in some cases.

In this paper, the authors show only one of various approaches, that is, the moving-window averaged response fitting method. By applying the numerical filter of the peak shape of each mode obtained by direct Fourier analysis to response time histories, we can separate them to each modal response, and this can be compared to response time histories of a single-degree-of-freedom system whose parameter are ζ_i and ω_i . This comparison was made by the rms value of some certain length of duration, that is, the window T_w .

The schematic drawing of the model is shown in Fig. 5, and its vibration characteristics are also shown in Table 1. Their boundary structures were rather flexible at first. After several records were obtained, the boundary structures were improved to satisfy the rigid support condition. This model is equiped in Chiba Field Station with other models.

This system has very light damping at this moment compare to ordinary nuclear pipings. So the methods under an assumption of local stationarity are not suitable to the analysis. Therefore, the authors employed the moving-window averaged response fitting method is employed for the trial. By assuming $\hat{\zeta}_i$ at first, and to adjust the relative ratio of ω_i to minimize the sum of rms values from the beginning to the end of the earthquakes. This procedure was made on the wave forms to which the numerical filter decided by the power spectral density (Fig. 6) as mentioned before. The filtered wave forms of the 1st, 2nd and 3rd modes are shown in Fig. 7. The relation between the width of the moving window and the relative error is shown in Table 2. In this case, the width of the moving window is the best around T_w =2 sec for the 1st and 2nd modes. The theoretical reason to decide this width has not established yet.

The response observation is being continued. The authors feel the necessity of more records which have various features, even though the earthquake analyzed in this chapter is one of typical ones in the field station area.

8. Concluding Remarks

The 3-D analysis of piping systems is very significant for their anti-earthquake design. Although its fundamental procedure is clear, the details of them, for example, how to decide the two axial design basis horizontal ground motion, how to verify the adequacy of the result of their response analysis on actual pipings in nuclear power plant and so on. The authors tried to discuss such problems very briefly in this paper. We should study these subject more deeply, especially on the practical procedure of analyzing recorded responses of piping systems to decide their parameters.

One of the authors, Shibata has been working for the sub-committee on the standardization of anti-earthquake design of equipment and pipings, the Ministry of Internal Trade and Industry, and also the research group on damping coefficient of piping systems, which sponsored by the electric power companies. He realized the importance of the study in this field through his activities in those committees. The authors wish to express their gratitude to the cooperation of the members of those committees.

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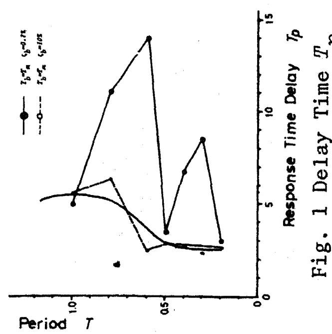

Fig. 1 Delay Time T_p of Response Peak to El Centro NS, 1941 [*Data after Sato*(1)]

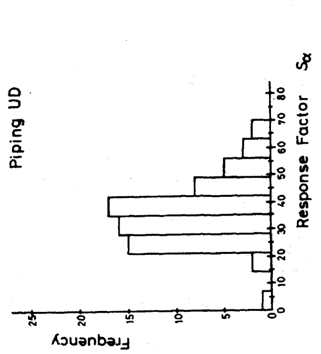

Fig. 2 Frequency of Occurrence of Vertical Response of Piping

Table 2 An Example of Estimated Parameters and Width of Lag-window T_ω

1st Mode

Lag-Window T_ω [sec]	Freq. $\hat{f}$	Damp. Ratio $\hat{\zeta}$ *	Exciting Coef.* $\hat{\beta}_{NS}$	$\hat{\beta}_{EW}$	$\hat{\beta}_{UD}$	Residual $\bar{R}$
0.5	3.360	0.125	-0.0398	-1.3388	-0.8576	0.128
1.5	3.360	0.135	0.0282	-1.3905	-0.8724	0.099
2.0	3.360	0.140	0.0643	-1.3924	-0.7784	0.078
3.0	3.370	0.110	0.1341	-1.3172	-0.2630	0.128

* These values are not normalized

Table 1 Vibration Characteristics of Model Piping

Mode	Observed Freq. with Flex. Bound. [Hz]	Compt'd Freq. with Complete Bound. [Hz]	Critical Damping Ratio [%]	Exciting Coef. β_{NS}	β_{EW}	β_{UD}
1	3.28	4.55	0.10	0.1406	-0.5801	-0.4534
2	4.40	5.41	0.23	0.5790	0.3497	-0.2230
3	5.15	6.44	0.16	0.1844	-0.3082	0.0736

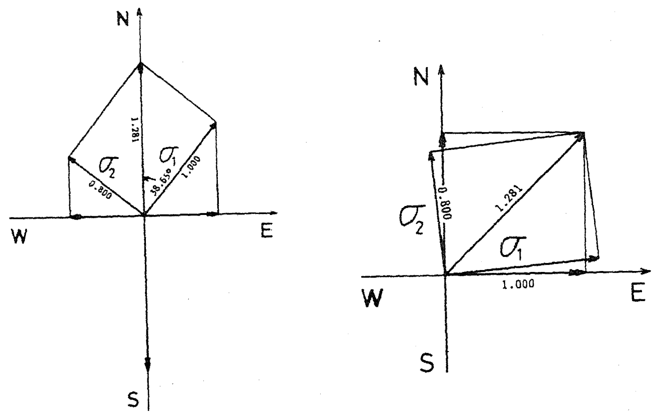

Fig. 3(a),(b) Relation between Two Principal Axes of Ground Motions and Recorded Ground Motions

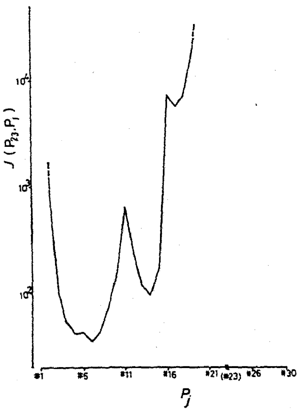

Fig. 4 Evaluation Function J for Optimum Pickup Arrangement

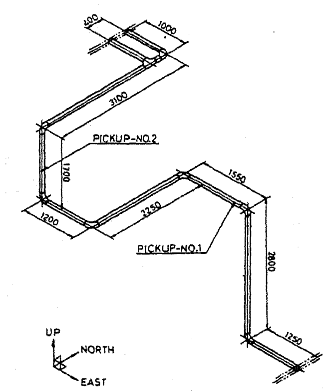

Fig. 5 Configuration of Model Piping

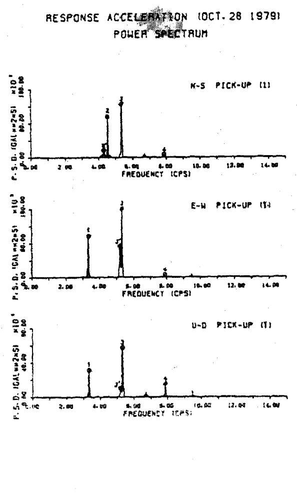

Fig. 6 Power Spectra of 3-D Piping Response

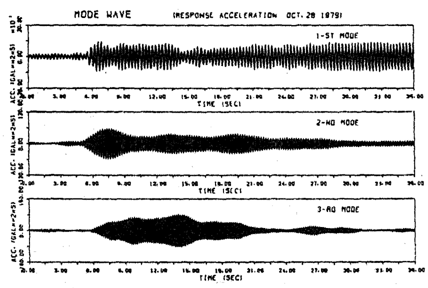

Fig. 7 Modal Response Time Histories of Piping