EARTHQUAKE LOAD ANALYSIS OF TUNNELS AND SHAFTS

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SUMMARY

Long embedded structures such as tunnels or shafts may experience bending moments as well as axial and transverse loads during an earthquake. This paper presents a closed-form solution for determining bending moments and axial loads for tunnels and shafts under earthquake load, while considering soil-structure interaction as a pseudostatic problem. The results are presented graphically in terms of nondimensional parameters, such that these curves can be used directly to aid in design-load calculations. The solutions are particularly useful for relatively rigid structures embedded in soft soils, or for tunnels/shafts with small segment lengths.

INTRODUCTION

Previous methods for earthquake load analysis of long embedded structures such as tunnels, shafts or pipelines have generally been based on equating the deformation of the structure to that of the surrounding soil medium (e.g. Newmark and Rosenblueth, 1971). As further confirmed in this paper, this method, ignoring soil-structure interaction, appears to be appropriate for the analysis of flexible structures embedded in a soft medium; but it may be overly conservative for more rigid structures, and/or structures with small segment lengths. This paper extends the previous approach by considering soil structure interaction as a pseudostatic problem and presents solutions for determining maximum bending moment and longitudinal tensions for varying stiffnesses of the structures and the soils, as well as for various structure segment lengths.

As shown in Fig. 1, passage of seismic waves around a long embedded structure can impose: transverse stresses in a section perpendicular to the axis of the structure, longitudinal tensile loading parallel to the axis of the structure, and bending loading. Transverse stresses have previously been estimated assuming deformation compatibility, however there is some recent evidence (Constantopoulos, et al 1980) that these stresses are approximately equal to those experienced in the free field. Since methods for estimating transverse stresses are available elsewhere (Constantopoulos, 1980; Newmark and Rosenblueth, 1971), this paper deals only with the longitudinal tension and bending of structures.

Dilatational waves traveling parallel to the axis of an embedded structure will create tensile loading in the structure (Fig. 1). For

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instance, vertically traveling compression waves will create tension in a shaft, and the dilatational component of horizontally traveling Rayleigh waves will generate tensile loading in tunnels. Waves traveling parallel to the axis of the structure, but with a soil particle movement transverse to the wave propagation direction, will generate bending in the structure. Since analyses of both tunnels and shafts are similar, no distinction is made between them. The terms "embedded structure" and "structure" are generically used to describe either a tunnel segment or a shaft segment. The tunnels or shafts are presumed to consist of several segments, each of which has free ends -- a typical segment is analyzed here.

The following sections present: a model for tensile loading, a model for bending loading, discussion of results, a method for determining the inputs to models, and an illustrative example.

TENSILE MODEL

Previous practice for determining the tensile loading has generally been based on equating soil strain to structure strain ignoring soil-structure interaction (Newmark and Rosenblueth, 1971). To model soil structure interaction in this mode of loading a structure segment of length 2L is presumed connected to soil particles through distributed soil springs (Fig. 2). The soil particle movement is approximated by:

$$u_s = s_t \sin \alpha x \qquad (1)$$

where: x = distance from the center of the structure; $-L \leq x \leq L$; s_t = maximum amplitude of dilatational wave -- this term is related to maximum ground acceleration as discussed later; α = 2 /(wave length of the dilatational wave); u_s = displacement of soil particle which is connected to point x on the structure.

The governing equation for the structure can be established as:

$$\frac{d^2u}{dx^2} = \beta^2(u-u_s) \qquad (2)$$

where: u = displacement of the structure at x; $\beta^2 = k_t/EA$, where k_t is the spring constant, E is Young's modulus of the structure, and A is its area of cross section.

The above equation can be solved with the boundary conditions that the ends of the structure are free; i.e: Tension (at x = + L or -L) = 0. With these above boundary conditions the solution to Eq. 1 is obtained as:

$$\frac{u}{s_t} = \beta^2(\sin \alpha x)/(\alpha^2 + \beta^2) - \alpha\beta \cos \alpha L \sinh \beta x/\{(\alpha^2 + \beta^2) \cosh \beta L\} \qquad (3)$$

Note that αL ,and βL are dimensionless quantities.

Quantities of significant design interest include -- maximum tension and the maximum deformation at the edge of the structure.

Maximum tension is represented in Fig. 3, in nondimensional form as the ratio SR, between maximum structure strain to maximum soil strain, the latter of which is αs_t. SR is thus a direct representation of the modification of strain compatibility due to soil structure interaction. Knowing SR, the maximum tension is conveniently obtained as (SR αs_t EA). However, since the soil cannot transmit loads to a structure beyond the frictional capacity between the soil and the structure, an upper bound tensile loading must be established based on friction capacity at the soil-structure interface.

The results for maximum deformation of the structure are presented in Fig. 4, as the nondimensional ratio, TDR, maximum structure deformation to maximum soil deformation, the latter of which equals $s_t \sin \alpha L$. These results are further discussed and illustrated in latter sections of this paper.

BENDING MODEL

To consider soil structure interaction in bending mode, a structure of length 2L is considered as supported by a set of soil springs connected to far field soil deforming according to free field motion. The structure bending will be maximum when the crest of a passing wave with maximum curvature is at the middle of the structure segment. The boundary soil deformation is (Fig. 2) approximated as:

$$y_s = s(1 - \cos \alpha x) \quad \text{for } -L < x < +L \qquad (4)$$

where: y_s = deformation of soil at x; s = maximum transverse soil displacement amplitude of the earthquake wave.

Considering the soil structure interaction model shown in Fig. 2, the governing equation for the displacement of a structure with bending stiffness EI can be obtained as:

$$\frac{d^4 y}{dx^4} = \frac{k}{EI}(y_s - y) = 4\beta^4 (y_s - y) \qquad (5)$$

where: y = displacement of the structure at x. It is presumed that initially both the beam and soil deformations are zero, with no loads transmitted through the springs, i.e. $y = y_s = 0$; and $4\beta^4 = k/EI$

The boundary conditions applicable to the above model are: moment (at x = + L)= moment (at x = -L) = shear (at x = + L)= shear (at x = -L) = 0.

Considering the soil displacement given in Eq. 4, and the symmetry of the boundary conditions the solution to Eq. 5 is obtained as:

$$\frac{y}{s} = 1 - 4\beta^4 \cos \alpha x / (\alpha^4 + 4\beta^4) + A \cos \beta x \cosh \beta x + B \sin \beta x \sinh \beta x \qquad (6)$$

where the integration constants A and B are to be obtained by implementing the boundary conditions: moment (at x = L) = shear (at x = L) = 0.

Results for the maximum bending curvature, maximum deformation, and maximum rotation are presented in Figs. 5, 6 and 7, in nondimensional forms which are convenient to use by a designer. Fig. 5 plots the ratio, CR, of the maximum curvature of a structure to that of the soil. The ratio allows an easy estimation of maximum structure curvature and hence of the maximum bending moment. Fig. 6 presents the displacement ratio, DR, defined as the ratio between the maximum difference between displacements of the structure at x = + L and x = 0, and the maximum soil displacement at x = L.Finally, Fig. 7 presents the ratio, THR, between the maximum rotations of the structure (at x = L) and that of the soil (at x = L).

DISCUSSION OF THE RESULTS

Figs. 3 through 7 have been designed to present nondimensional results for use by a designer. They afford significant flexibility and convenience. The independent nondimensional parameters are αL and βL.

The term β represents the ratio of the stiffnesses of the soil springs and the structure. If the springs are considerably softer than the structure or if the segment length L is small, βL will be near zero and the soil displacement will not be transmitted to the structure. Thus all of the nondimensional parameters plotted in Figs. 3 through 7 start with a near zero value when βL is near zero (<0.4). On the other hand, if βL is high (say greater than 3.0), the structure is considerably softer than the soil springs and the structure deforms with the soil. In such cases the normal practice of soil-structure deformational compatibility is valid. All of the nondimensional ratios in Figs. 3 through 7 reach the value of 1.0 for βL between 3 to 10.

The parameter αL represents the ratio between structure length and the wave length. The absolute minimum value of αL is zero. The maximum value of αL is obtained by the assumption that the structure segment is small enough not to experience double curvature, i.e. the structure length (2L) is smaller than one half the wave length. Thus the upperbound αL must be much less than $\pi/2$ or about 1.6, and a realistic upper bound value of 1.0 has been adopted. However, as all of the plots in Figs. 3 through 7 indicate, the derived nondimensional parameters are not much affected by αL.

Results for SR and CR (Figs. 3 and 5 respectively) are useful in obtaining the tension and bending moment for a structure. Results for TDR, DR, THR, aid in determining the maximum deformation at the edge of a structure, and thus can be used to determine the maximum deformational capacity required of the flexible joints connecting two adjacent structure segments.

ESTABLISHMENT OF INPUTS TO THE ABOVE MODELS

The spring coefficient k for the bending model is obtained as cG, where G is the shear modulus of the soil surrounding the structure, and c is a coefficient varying between 2 and 4 (Richart et al, 1970 and Constantopoulos et al 1979). A value of near 3.0 for c has been found applicable in most cases. The spring constant k_t for the tensile model would be one half of that for the bending model; i.e. k_t=0.5cG (Richart et al, 1970). It is noted that since β is proportional to the one fourth power of k and β_t is proportional to the square root of k_t, an error in selection of k and k_t will modify β and β_t only slightly.

The soil displacement amplitudes are related to peak soil particle acceleration and velocity, as well as to the wave propagation velocity (Newmark, 1968). For the tensile model the maximum soil strain $s_t\alpha$ will equal the peak soil velocity divided by the wave propagation velocity. Newmark et al. (1973) have shown that the peak velocity is approximately 48 in/sec(1.2m/sec) for each 1.0g peak ground acceleration. Similarly, for the bending model maximum soil curvature $S\alpha^2$ will equal the peak soil acceleration divided by the square of the wave propagation velocity. From these relationships one can establish s_t and s values.

ILLUSTRATIVE EXAMPLE

Suppose a tunnel with 120 ft (36.6m) segments and area of cross section of 5 square ft. (1.86m^2), is buried in a soil with a density of 115 lb/ft^3 (1.8gm/c.c.) and with shear and Rayleigh wave velocities of approximately 800 ft/sec (244m/sec). The design peak horizontal accceleration at the tunnel level is 0.1g. The tunnel is made of concrete with a Young's modulus of 4E+6 psi (28E+3 MPa). Of interest are: (a) maximum tensile stress without considering the soil-structure interaction model, (b) maximum tensile stress if the interaction is considered, and (c) the maximum tunnel segment length if the stress due to tensile loading must be limited to 700 psi (4.8MPa).

For the above example the maximum soil strain will be 0.1x4/800, or 0.0005. The shear modulus for the soil is (115/32.2) (800)^2 in psf, or 15,900 psi (109MPa). Thus for a 'c' value of 3.0, k_t is obtained as 0.5x3.0 x 15,900 or 23,900 psi (164MPa); resulting in a value for β of $\sqrt{23,900/(5 \times 4E+6)}$ or 0.0346 per ft (0.113 per m). Solutions to the items of interest are then obtained as follows -- (a) for no interaction there will be a strain compatibility (equality) between soil and tunnel and thus the tunnel stress will be 0.0005(4E+6) or 2,000 psi (13.8MPa); (b) βL is 0.0346x60 or 2.1, for which the value of SR from Fig. 3 is 0.72, and thus the tunnel stress will be 0.72x0.0005x4E+6 or 1440 psi (9.9Mpa); (c) to have 700 psi stress SR must be 700/(4E+6x0.0005) or 0.35. From Fig. 3 for SR of 0.35, βL should be approximately 1.0, i.e. L should be 1.0/0.0346 or about 29 ft, and thus the segment length should be 58 ft(17.7m).

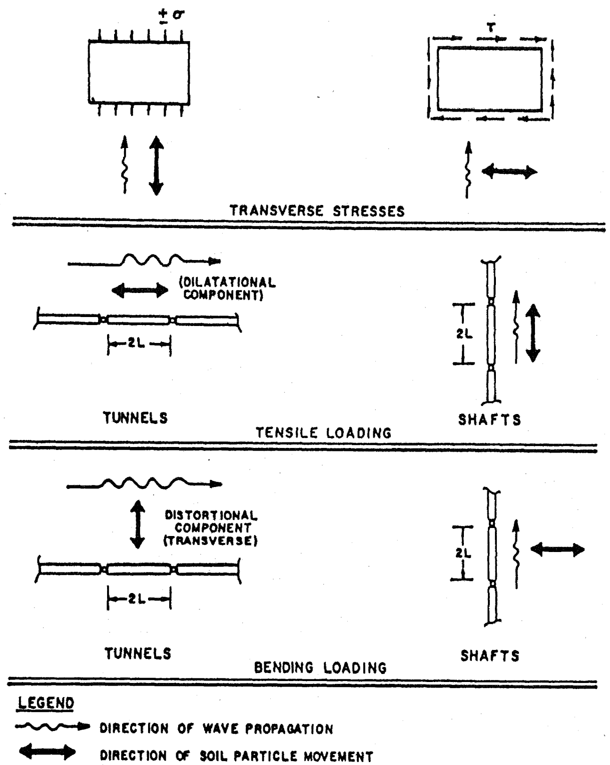
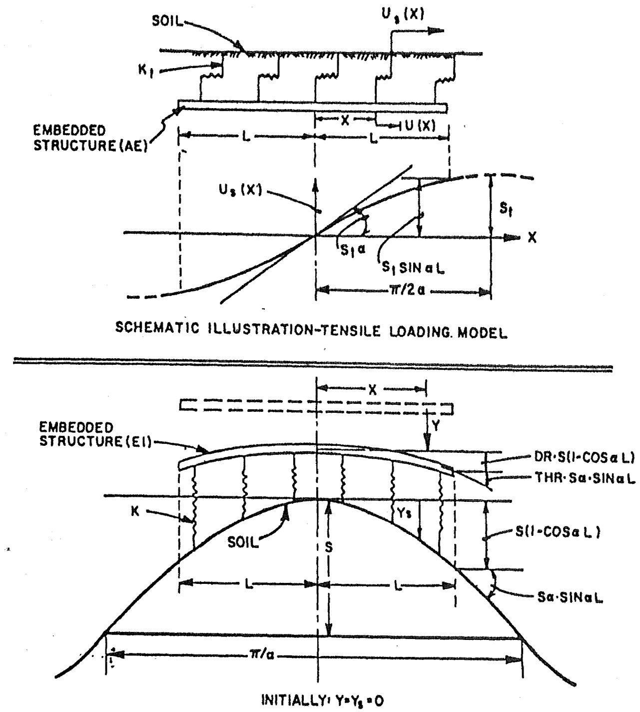


FIG. 1 TRANSVERSE STRESSES, TENSILE AND BENDING LOADING



SCHEMATIC ILLUSTRATION-BENDING MODEL

FIGURE 2 TENSILE ANr ɜENDING MODELS

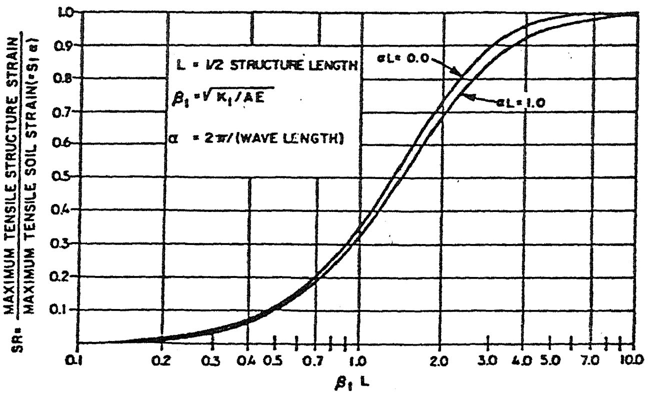


FIG. 3 MAXIMUM TENSILE STRAIN

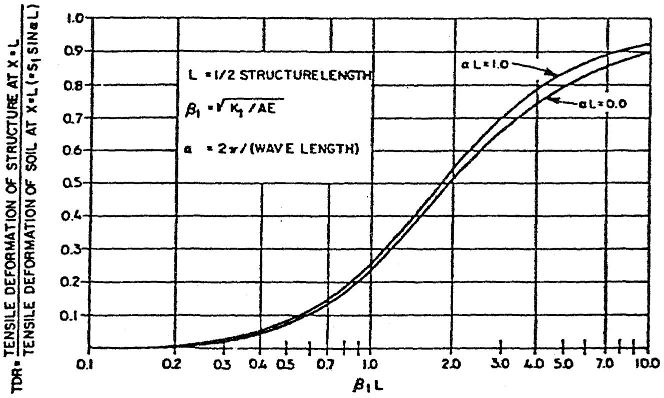


FIG. 4 MAXIMUM TENSILE DEFORMATION

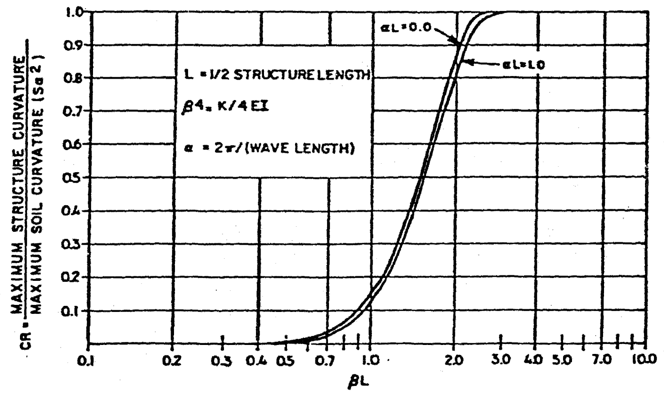


FIG 5 MAXIMUM CURVATURE DUE TO BENDING

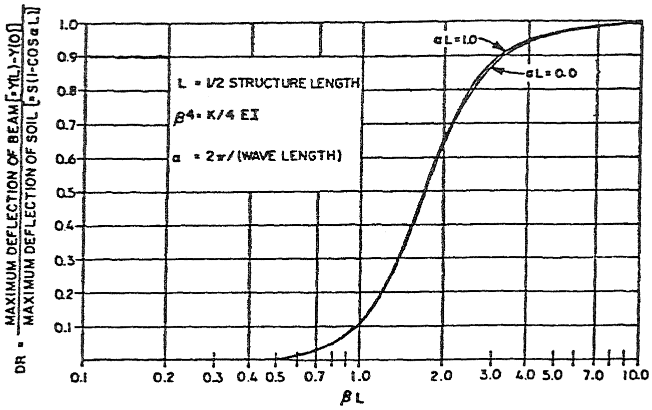


FIG. 6 MAXIMUM RELATIVE DEFLECTION DUE TO BENDING

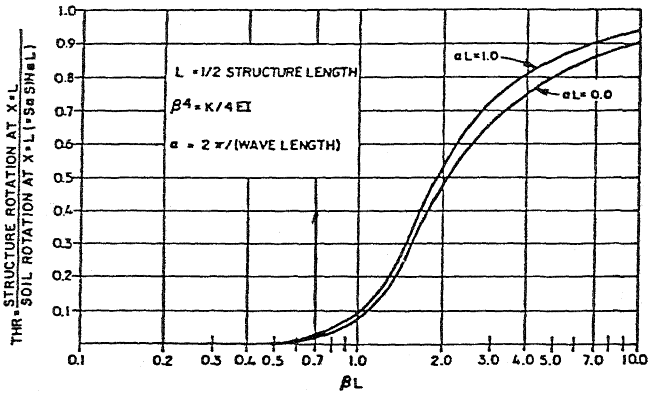


FIG. 7 MAXIMUM ROTATION DUE TO BENDING

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