3-DIMENSIONAL LATERAL AND LONGITUDINAL SEISMIC STABILITY OF EARTH AND ROCK-FILL DAMS

by

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SUMMARY

The paper describes two analytical procedures to evaluate the permanent sliding deformations and the tensile longitudinal strains that may occur in earth or rockfill dams due to earthquake-induced lateral and longitudinal vibrations. The procedures are based on a 3-Dimensional dynamic theory which: (1) models the dam as a prism with a wedge-shaped crosss-section bounded by a rectangular canyon and (2) accounts for a realistic variation of soil stiffness within the dam, thus offering a significant improvement over the currently used homogeneous shear-slice model. Results are presented in the form of natural frequencies, modal acceleration and strain shapes, average induced accelerations and resulting permanent deformations of potentially sliding masses due to recorded earthquake motions.

INTRODUCTION

It is widely recognized that criteria of seismic performance of embankment dams during earthquakes should involve deformations rather than factors of safety against shear strength exceedance. Thus, to ensure the integrity of earth dams built with compacted clays or dense/dry sands and of rockfill dams during and after strong earthquakes the designer must limit:

1. the permanent relative displacements of potential sliding masses on the upstream or down-stream slope due to the lateral vibrations of the dam; and
2. the tensile strains in the abutment-dam or outlet works-dam interfaces due to longitudinal vibrations of the dam.

The permanent displacement of a potential sliding mass due to the "design" earthquake is currently assessed by a simplified procedure [5,8]

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that involves essentially three separate steps: (a) Determination of the yield acceleration, k_y, that would develop a factor of safety equal to 1 against shear strength failure of the sliding mass; (b) Determination of the time history of the average induced acceleration, k(t), on the sliding mass; and (c) Estimation of the plastic relative displacements that take place whenever k exceeds k_y. Dynamic response analyses that are needed in step b are performed using two simplified models that approximate the actual dynamic behavior to varying degrees; the 's h e a r' model and the 'p l a n e - s t r a i n' model. The former treats the dam as a symmetrical homogeneous shear beam with variable cross-section (triangular or truncated-wedge) and has been very popular with earthquake engineers because it is very simple and satisfactorily predicts horizontal displacements and accelerations [1,2,4,7]. The plane-strain model treats the dam as a 2-Dimensional continuum and uses a plane-strain finite-element (or finite-difference) discretization to obtain the response. Despite its theoretical superiority, it has enjoyed only limited application (at least in preliminary design calculations) becuase of cost-and-time requirements of the numerical solution. Truly 3-Dimensional models, although in principle feasible, are almost never used in practice due to the complexity of the actual boundary conditions and the projibitive expense of a dynamic 3-Dimensional finite element analysis.

The tensile strains in the abutment-dam or outlet works-dam interfaces can be estimated by studying the longitudinal vibrations of the dam. To this end, the author [3] has recently developed a simplified theory of longitudinal vibrations that accounts for both shear and dilatational deformations and models the dam as a homogeneous prism with a wedge-shaped cross-section, bounded by two vertical (abutments) and one horizontal (river-bed) planes.

Most of the above-mentioned theories consider the dam as a homogeneous medium. However, it is well known that even within a uniform mass the shear modulus of soil increases approximately as the square-root of the effective confining pressure, σ_o'. In earth dams the average σ_o' across a horizontal plane appears to increase with the distance from the crest in such a way that the average shear modulus at a depth z can be expressed as:

$$G(z) = G_m \left(\frac{z}{H}\right)^{2/3} \qquad (1)$$

in which: G_m = the modulus at the base of the dam, i.e., at z=H. This form of variation of soil stiffness in earth and rockfill dams has been directly confirmed from in-situ measurements of S-wave velocities in several dams [6] and the author has recently presented a theory of lateral vibrations of earth and rockfill dams modeled as 1-D Shear beams whose modulus increases with depth in a form described by Eq. 1 [2].

This paper extends the analysis of Ref. 2 to account in a simple way for the geometry of the canyon. The objective of the paper is to present results pertaining to the evaluation of seismic stability (during lateral and longitudinal vibrations) of embankment dams modeled as simplified 3-D

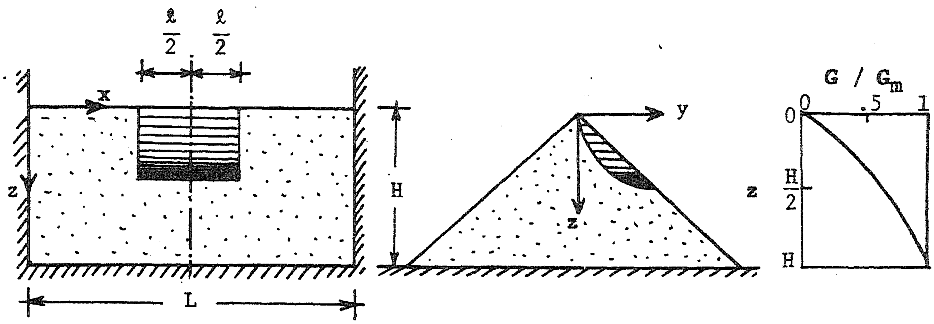


FIGURE 1

structures built in a rectangural conyon and having stiffness that increases with the 2/3-power of the distance from the crest.

LATERAL STABILITY

Average Acceleration Induced on Cylindrical Potential Sliding Mass

Let x, y and z be the orthogonal coordinates of any point in an earth dam occupying a rectangular canyon, as shown in Fig. 1. The linear governing differential equations describing the relative-to-the-ground motion during earthquake shaking that consists exclusively of vertically pro-propagating S-waves is

$$\rho\ddot{v} - \frac{1}{z}\frac{\partial}{\partial z}\left(G\,z\frac{\partial v}{\partial z}\right) - \frac{\partial}{\partial x}\left(G\,\frac{\partial v}{\partial w}\right) = \rho\ddot{v}_g \qquad (2)$$

in which $\ddot{v}_g(t)$ = the y component of the ground acceleration. Solution of Eq. 2 that satisfies the boundary conditions (zero relative displacements between dam and surrounding canyon, zero shear stresses at the crest) can be most conviently obtained through modal superposition. The latter calls for determination of natural frequencies and modal shapes. For the particular case of a dam having a shear modulus (G) increasing with depth according to Eq. 1, a constant Poisson's ratio (ν) and a constant soil density (ρ), Eq. 3 closely approximates the true natural frequencies of the dam:

$$\omega_{nr} \simeq \frac{\overline{C}}{H}\left[\left(\frac{7\pi}{9}n\right)^2 + \left(\frac{H}{L}\pi\, r\right)^2\right]^{1/2} \qquad (3)$$

$$n,r = 1,2,3,\ldots$$

in which: $\overline{C}$ = average S-wave velocity within the dam=$[\overline{G}/\rho]^{1/2}$; H and L are the height and length of the dam (Fig. 1). Fig. 2 portrays the variation of the normalized frequency $\omega_{11}H/\overline{C}$ versus the length-to-height ratio, L/H.

The corresponding modal shapes for the acceleration and shear-strain are approximated by

$$\ddot{v}_{nr} \approx \frac{\omega_{nr}^2}{z^{2/3}} \sin\left[n\pi[1-(z/H)^{2/3}]\right] \sin(r\pi x/L) \qquad (4)$$

and

$$\gamma_{zy,nr} \approx \frac{2}{3}\frac{H^{2/3}}{z^{5/3}} \left\{ \sin\left[n\pi[1-(z/H)^{2/3}]\right] + n\pi\left(\frac{z}{H}\right)^{2/3} \cos\left[n\pi[1-(z-H)^{2/3}]\right] \right\} \cdot \sin(r\pi x/L) \qquad (5)$$

Fig. 3 displays in a vertical cross-section the first two normalized acceleration and shear-strain modal shapes (i.e. Eq. 4 and 5 for constant x= L/2). Notice the sharp amplification of accelerations near the crest and the relatively uniform distribution with depth of first-mode shear-strains.

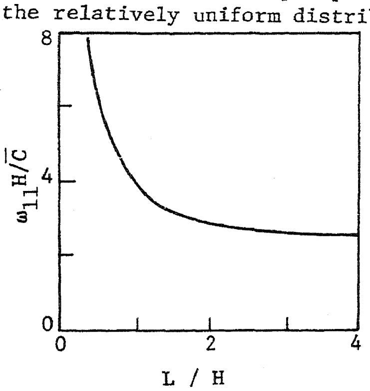


FIGURE 2

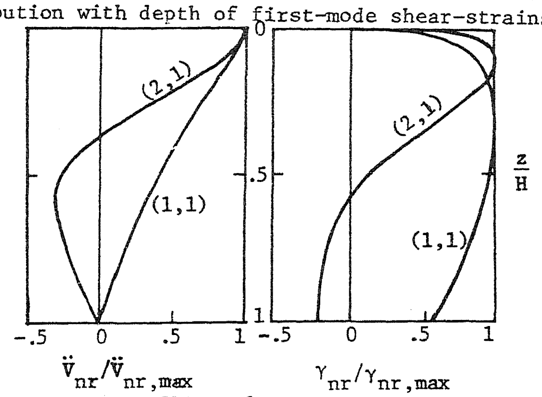


FIGURE 3

Using the principle of modal superposition, the expression for the absolute acceleration in any level z<H as function of time is given by:

$$\ddot{v}(x,z;t) = \sum_{n=1,2}^{\infty} \sum_{n=1,3}^{\infty} \Gamma_{nr}\, \ddot{v}_{nr}\, D_{nr}(t) \qquad (6)$$

in which: Γ=the participation factor of the n,r mode in the overall motion of the dam and is given by

$$\Gamma_{nr} = \frac{\int_0^H \int_0^L \ddot{V}_{nr} \cdot z \cdot dx \cdot dz}{\int_0^H \int_0^L \ddot{V}_{nr}^2 \cdot z \cdot dz \cdot dz} = \frac{8}{\pi^2 nr} \qquad (7)$$

$D_{nr}(t)$=the displacement time hisotry of a single-degree-of-freedom oscillator having the natural frequency, ω_{nr}, and the critical damping ratio, β_{nr}, of the n,r vibrational mode and subjected to the ground motion, $\ddot{v}_g$. $D_{nr}(t)$ is obtained by numerical evaluation of the appropriate Duhamel integral (e.g., see Ref. 7, p 168).

In order to estimate the sliding deformations of a cylindrical mass, shown in Fig. 1, we must first compute the average induced acceleration (function of time). To this end, the cylindrical surface is calculated as the sum of infinitesimal forces, dF(t), acting on horizontal elementary slices of volume $\lambda z \cdot dx \cdot dz$:

$$F(t) = \int_{z=0}^{z} \int_{(L-\ell)/2}^{(L+\ell)/2} \rho\lambda z \; dx \; dz \; \ddot{v}(x,z;t) \qquad (8)$$

the average induced acceleration is then obtained from

$$k(t) = \frac{F(t)}{M} = \frac{F(t)}{\frac{1}{2}\rho\lambda z^2 \ell} \qquad (9)$$

Plane-Strain Approximation

Two slightly conservative simplifications are commonly made to determine the average induced acceleration. First the thickness ℓ of the cylindrical sliding mass is considered infinitesimaly; in such a case, F is the inertia force on a thin triangular slice located at the central section of the dam. Second, only modes with r=1 contribute significantly to the inertia force of this sliding slice. The average induced acceleration is then obtained from the simpler relation

$$k(t) \equiv K(z,x=\frac{L}{2};t) \simeq \frac{16}{\pi^2 \rho z} \sum_{n=1,2}^{\infty} \phi_{n1}(z) \; D_{n1}(t) \qquad (10)$$

where : $\phi_{n1}(z)=G\gamma_{zy,n1}/n$ and $\gamma_{zy,n1}$ is obtained from Eq. 5 for r=1, x=L/2. Fig. 4a shows the distribution with depth of sliding mass of the peak values of k for a dam subjected to the El Centro 1940 recorded accelerogram (NS component), while Fig. 4b shows the distribution of peak shear strains within the dam. Notice the fast attenuation of k_m away from the crest and the nearly uniform distribution of γ_m. Strain-compatible equivalent linear analyses can thus be performed by using the "effective" average peak strain from the presented theory (average $\gamma_{eff} \approx 0.65$ x average γ_m) to read the new equivalent-linear shear modulus and damping ratios from published G/G_o and β versus γ curves and repeat the analysis until the initial and final shear strains coincide.

Determination of Yield Acceleration

Yield acceleration of a potential sliding mass is defined as the critical acceleration that produces an inertia force which brings the stresses along the slip surface into equilibrium with the available cyclic strength of the soil. All the classical methods of limiting equilibrium analysis (Fellenius, Bishop, Morgenstern-Price, etc.) can be used, after some modification, to determine the yield acceleration. A recently proposed method

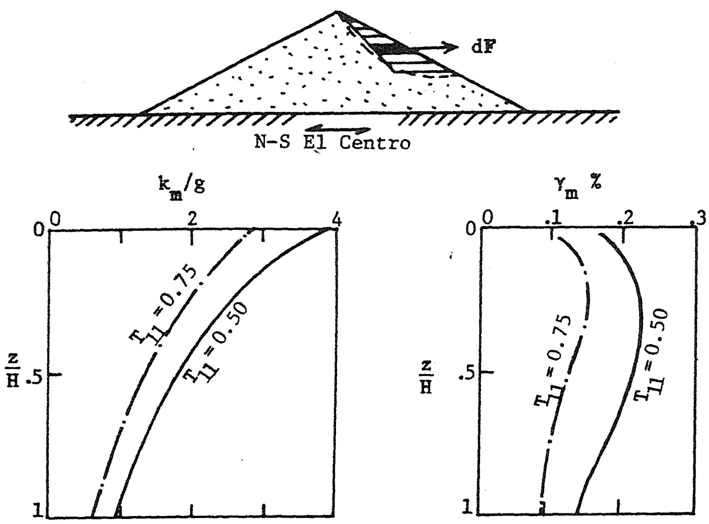


FIGURE 4

by Sarma [9] seems to be better suited for this purpose, since k_y is directly obtained in explicit form, while the shear strength on internal shear surfaces is taken into account. A crucial step in determining k_y is the estimation of the cyclic shear strength. For non-liquefiable soils, Makdisi & Seed [5] recommend that no reduction in the static strength is necessary and that, therefore, the available cyclic shear strength be estimated on the basis of the Mohr-Coulomb failure criterion. This author believes that the conservatism introduced by Eq. 10 in the "forcing" function would in most cases compensate for the higher "resistance" resulting from the above proposition of Ref. 5. But whenever the peak cyclic strain (estimated on the basis of Eq. 5) exceeds 1/2 of the static failure strain (obtained from laboratory testing), the cyclic shear strength may be significantly smaller than the static strength and cyclic triaxial or simple shear tests must be conducted to determine it.

<u>Calculation Of Permanent Sliding Deformations</u>

The slippage (δ) that occurs between a 'failing' mass and the main body of the dam every time the induced average acceleration exceeds the yield acceleration may be evaluated by double integration of the differential equation

$$\ddot{\delta} = \frac{\cos(\beta-\phi')}{\cos \phi'} (k-k_y) \qquad (11)$$

in which: ϕ'=the friction angle of the soil determined from a $\overline{CK_oU}$ triaxial compression test; and β=the average inclination of the sliding plane with respect to the horigontal, which is usually taken as 0.

By solving Eq. 11 with a large number of recorded accelerograms in place of k(t) and several values of k_y for each record, Sarma [8] and Makdisi et al. [5] have come up with 'design' curves relating, in a dimensionless fashion, the maximum permanent sliding deformation, δ_m, with the ratio k_y/k_m of the yield over the peak average induced acceleration. Fig. 5 presents a set of such

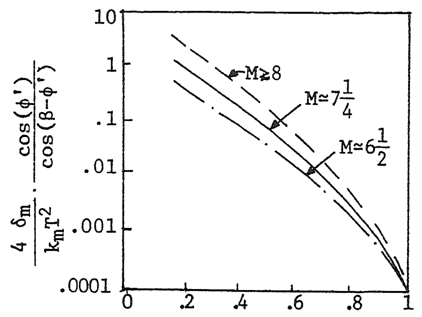


FIGURE 5

curves corresponding to three different earthquake magnitudes. They were obtained by combining the curves of Ref. [5] and Ref. [8]. It is obvious that such curves are particularly useful for design purposes, especially when the 'design' earthquake is described through a response spectrum. In that case, one directly estimate the maximum values, k_m, of the induced average accelerations by taking, e.g. the square root of the sum of the squares of the maximum accelerations of the first three or four modes.

PERFORMANCE DURING LONGITUDINAL VIBRATIONS

During earthquake shaking that consists exclusively of vertically propagating S-waves, the longitudinal displacements of the dam relative to the surrounding canyon satisfy the governing differential equation

$$\rho\ddot{u} - \frac{1}{z}\frac{\partial}{\partial z}\left(Gz\frac{\partial u}{\partial z}\right) - \frac{\partial}{\partial x}\left(E\frac{\partial u}{\partial x}\right) = \rho\ddot{u}_g \qquad (12)$$

in which $\ddot{u}_g(t)$=the x component of the ground acceleration. Eq. 12 accounts for both shear and dilation deformations [Ref. 3] and can be conveniently solved through modal superposition. For a dam with G varying according to Eq. 1 and a constant Poisson's ratio, ν, the natural frequencies and the modal shapes of the absolute acceleration and the axial strain are approximated respectively [Ref. 3] by

$$\omega_{nr} \simeq \frac{\bar{C}}{H}\left[\left(\frac{7\pi}{9}n\right)^2 + 2(1+\nu)\left(\frac{H}{L}\pi r\right)^2\right]^{1/2} \qquad (13)$$

$$\ddot{u}_{nr} \simeq \frac{\omega_{nr}^2}{z^{2/3}}\sin\left[n\pi[1-(z/H)^{2/3}]\right]\sin(r\pi x/L) \qquad (14)$$

$$\varepsilon_{x,nr} \simeq \frac{r\pi}{L}\frac{1}{z^{2/3}}\sin\left[n\pi[1-(z/H)^{2/3}]\right]\cos(r\pi x/L) \qquad (15)$$

$$n, r = 1, 2, 3, \ldots$$

Fig. 6 portrays the variation of the normalized circular frequency $\omega_{11}H/\bar{C}$ versus the length-to-height ratio, L/H, for two values of Poisson's ratio, ν=0.3 and ν=0.5. Fig. 7 depicts in a horizontal cross-section the first four modal shapes of the axial strain; it is clear that maximum tensile strains develop at the abutment-dam interfaces.

Using the principle of modal superposition, the expression for the axial strain as function of time is given by

FIGURE 6

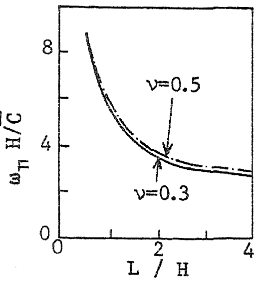
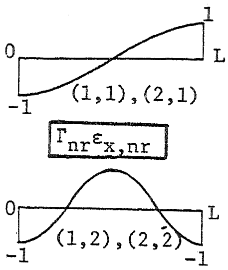


FIGURE 7



$$\varepsilon_x(x,z;t) = \sum_{n=1,2}^{\infty} \sum_{r=1,2}^{\infty} \Gamma_{nr}\ \varepsilon_{x,nr}\ D_{nr}(t) \qquad (16)$$

where: Γ_{nr} is given by Eq. 7, i.e. it is identical with Γ_{nr} of the lateral vibrations; and $D_{nr}(t)$=the displacement time history of a single-degree-of freedom oscillator having the natural frequency, ω_{nr}, and the critical damping ratio, β_{nr}, of the n,r vibrational mode and subjected to the ground motion, u_g. Eq. 16 can be used to estimate the maximum tensile strains that develop near the crest of the dam at the abutment-dam interface and also at the contact surfaces of the dam with outlet works, such as side spillways, sluiceways and buried pipes. Evaluation of the anticipated strains in the light of past experience with performance of similar dams during earthquakes will serve as a guide in the design.

CONCLUDING REMARKS

The methods described in this paper to estimate permanent sliding deformations due to lateral vibrations and tensile strains due to longitudinal vibrations provide a rational approach to design embankment dams against earthquakes. The dynamic model on which the analyses are based offers a significant improvement over the currently used homogeneous shear slice model [1,4,5] because it accounts for a realistic variation of soil stiffness within the dam (Eq. 1), while considering the end effects of a simplified canyon geometry. Nevertheless, the nature of approximations involved in all steps of the analysis requires that the two methods be used with caution and good judgement. Additional research is also needed to establish quantitative performance criteria (maximum allowable deformations & tensile strains) for various types of dams. This can be achieved through estensive evaluation of recorded performance of dams during and after strong earthquakes.

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