# A NAIVE MODEL FOR NONLINEAR RESPONSE OF REINFORCED CONCRETE BUILDINGS

By

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## SUMMARY

A simple and economical model is introduced for the calculation of the nonlinear displacement-response histories of multi-story structures subjected to strong earthquakes. A structure is idealized as a mass connected to a rigid bar that in turn is connected to the ground by a hinge and a rotational spring. The calculated responses are compared with measured experimental results from dynamic testing of eight small-scale ten-story model structures. Satisfactory correlation between the analytical and experimental results has been observed.

## INTRODUCTION

Structures designed according to current engineering practice in the U.S. are expected to develop nonlinear deformations when subjected to strong ground motions. Although nonlinear analysis of structures is a complicated and lengthy process, with the help of sophisticated digital computers successful analytical models have been developed for this purpose [7, 9]. Because of the involved data preparation procedures and, at times, due to lack of confidence in complicated programs (which cannot be checked easily) these models have not been utilized by the engineer in practice who needs a simple model which can be easily used for several possible alternative designs.

This paper introduces a simple nonlinear model (called the Q-Model) to calculate the seismic displacement-response histories of multi-story reinforced concrete structures. Measured response histories of eight small-scale ten-story structures are used to evaluate the results of the model.

## DESCRIPTION OF THE MODEL

The idea of representing a mutli-degree-of-freedom system by a "single-degree" system with some generalized mass, stiffness, and damping has been used for elastic structural systems. The extension of such idealization for inelastic problems has been viewed with some caution because of the changing stiffness properties and, therefore, dynamic properties of inelastic systems.

Current engineering practice encourages the designer to proportion the columns of a structure such that they experience only limited yielding during the design earthquake. Experimental results from testing of reinforced concrete structures designed according to this criterion indicate that the deflected shape will tend to remain essentially unchanged as nonlinear deformations are developed [1,3,4,5]. Furthermore, displacement responses have been shown to be dominated by the first mode. Therefore, a multi-story structure with the above properties can be reduced to a single-degree system with some source of hysteretic energy dissipation.

Equivalent Mass. The Q-Model is shown in Fig. 1. The governing dynamic differential equation can be described as [2].

$$\alpha_m M_t \ddot{x} + \alpha_\ell K x = -\alpha_\ell M_t \ddot{y} \qquad (1)$$

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where

$M_t$ = total mass of the MDOF system

$K$ = stiffness of the MDOF system (overall stiffness defined in terms of a particular lateral force and a particular horizontal displacement)

$x$ = lateral displacement of the mass of the SDOF oscillator with respect to its base

$\alpha_\ell = \left(\sum_{r=1}^{j} M_r \phi_r\right) / M_t$

$\alpha_m = \left(\sum_{r=1}^{j} M_r {\phi^2}_r\right) / M_t$

$r$ = numeral identifying level in MDOF system

$j$ = total number of levels in MDOF system

$M_r$ = mass at level r

$\phi_r$ = ratio of assumed displacement at level r to that at level j.

$\ddot{y}$ = acceleration of the base.

To simplify the equation, both sides are divided by $\alpha_\ell$, and a damping force is added.

$$M_e \ddot{x} + C\dot{x} + Kx = -M_t\ddot{y} \qquad (2)$$

in which $M_e = (\alpha_m/\alpha_\ell)\, M_t$, equivalent mass; and C = viscous damping coefficent.

Stiffness Properties. To define the stiffness characteristics, assumptions are made about the primary force-deformation relationship and stiffness variations for unloading and load-reversal stages. The primary curve is directly related to the stiffness of the multi-story structure and is obtained from a static analysis of the structure for a set of monotonically increasing lateral forces applied at floor levels (Fig. 1). The lateral force at a given level is proportional to the mass and height at that level. The primary curve is then approximated by a bilinear curve. One possible set of rules for such approximation is given in Reference 11.

The assumptions about stiffness variations upon unloading and subsequent loadings are included in a simple hysteresis model described by four rules. Appendix A in Reference 11 describes the details of the hysteresis model.

Corresponding to each point of the primary curve there is a lateral deflected shape for the multi-story structure. The shape corresponding to the beak point of the idealized binary curve is assumed to represent the vibration shape of the structure. The height of the mass in the Q-Model is assumed to be

$$L_e = \frac{\sum_{r=1}^{J} M_r\, \phi_r\, h_r}{\sum_{r=1}^{j} M_r\, \phi_r}$$

in which $h_r$ = the height of level r from base.

Solution Technique. With an arbitrary damping factor of 2%. Equation 1 was integrated using Newmark's $\beta$-method [6]. The value of $\beta$ was taken as 0.25.

## MODEL STRUCTURES

Eight small-scale ten-story reinforced concrete model structures were analyzed using the Q-Model. Four of these (MF1, MF2, H1, and H2) consisted of only two frames. Each of the other four (FW1, FW2, FW3, and FW4) comprised two frames as well as a central shear wall. The structures were subjected to simulated earthquakes at the University of Illinois at Urbana.

The input motion was applied to the structures in horizontal direction and parallel to the strong axis of each building. Structures MF1, MF2, H1, H2, FW1, and FW4 were subjected to a simulated north-south component of El Centro, 1940. The input motion for the other two was modeled after a north-east component of Taft 1952. All but one of the structures (H2) were subjected to three motions with increasing intensity from one run to the other. The first run for each case corresponded to the "design earthquake" and was strong enough to cause nonlinear response. Structure H2 was subjected to seven motions, the third one representing the design earthquake. In all tests, the time axis of input acceleration was compressed by a factor of 2.5 to obtain realistic proportions between the frequencies of structures and base accelerations. Therefore, six seconds of simulated earthquake corresponds to fifteen seconds of the actual records. The complete information regarding the casting and testing of these structures has been provided in References 1, 3, 4, and 5.

The calculated characteristics of the structures, as used for the Q-Model, are listed in Table 1. The assumed deflected shape for each structure is normalized with respect to the top level displacement and is presented in Table 2.

## ANALYTICAL AND EXPERIMENTAL RESULTS

Calculated (solid line) and measured (broken line) top-level displacement histories for the "design runs" are presented in Fig. 2 to 5. The maximum displacements are plotted in Fig. 6. Note that all levels of each structure were in phase and the maximum displacement at different levels occurred at the same time.

The evaluation of calculated response histories needs to be comprehensive to cover all different aspects included in the response. The customary method of judging an analytical model based on the calculated maxima, although it may be justified for models not operating with the "time" dimension, is clearly inadequate and can represent only one factor. Other factors such as frequency content and waveform are important and need to be considered.

It can be seen in the figures that the frequencies of calculated and measured responses were quite close for most instances in all structures. Also, the waveforms were generally similar and in parts excellent correlation was observed between the experimental and analytical results (structures H2, MF1, and MF2). The time of maximum response was calculated very close to the measured one for all structures except H2 and FW3. In terms of the maximum amplitude, it is evident in Fig. 2 to 5 that the Q-Model was successful for all but structures FW3 and FW4. For all cases the calculated deflected shapes were close to those measured (Fig. 6).

Based on the above observations, and considering the fact that computer cost for each analysis was only three percent of a corresponding "multi-degree" analysis, the overall performance of the Q-Model is regarded as satisfactory in simulating displacement response.

## CONCLUSIONS

Due to nondeterministic nature of earthquake motions satisfactory performance of a structure against a particular earthquake record does not necessarily guarantee its adequacy to resist forces caused by other earthquakes. A structure needs to be checked against several earthquakes and modified if necessary. The available "multi-degree" analytical models, because of their complicated theory and application procedures, do not provide the practicing engineer with the tool needed for design evaluation.

The Q-Model uses a simple concept (presentation of a MDOF system by a SDOF model) accompanied by low cost in terms of engineer and computer time. And yet, the results from the Q-Model (lateral floor displacements) are adequate to view the overall behavior of a structure for a given motion and judge if the performance is likely to be satisfactory.

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The CDC 175 computing system at the Digital Computer Laboratories of the University of Illinois was used for the analytical work.

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TABLE 1 CALCULATED PARAMETERS FOR DIFFERENT STRUCTURES

| Structure | Equivalent Mass (ton) | Equivalent Height (m) | $(\frac{M}{M^*}) \times 10^2$ at break point | $S_1$ | $S_2$ | Idealized Initial Frequency rad/sec. |
|---|---|---|---|---|---|---|
| H1 & H2 | 3.69 | 1.58 | 25 | 48 | 9 | 17 |
| MF1 | 3.68 | 1.59 | 29 | 64 | 8 | 20 |
| MF2 | 3.60 | 1.59 | 29 | 64 | 8 | 20 |
| FW1 & FW4 | 3.36 | 1.64 | 33 | 113 | 29 | 27 |
| FW2 & FW3 | 3.36 | 1.63 | 38 | 93 | 12 | 25 |

M* = sum of the products of story weights and corresponding heights from base.

$S_1$ = slope of idealized primary curve for the equivalent system. (See Fig. 1)

$S_2$ = slope of "post-yielding" branch of the idealized primary curve for the equivalent system. (See Fig. 1)

TABLE 2 ASSUMED DEFORMED SHAPES FOR STRUCTURES ANALYZED

| Level | H1 & H2 | MF1 | MF2 | FW1 & FW4 | FW2 & FW3 |
|---|---|---|---|---|---|
| 10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 9 | 0.98 | 0.97 | 0.97 | 0.93 | 0.92 |
| 8 | 0.95 | 0.92 | 0.92 | 0.85 | 0.83 |
| 7 | 0.88 | 0.86 | 0.87 | 0.75 | 0.74 |
| 6 | 0.79 | 0.79 | 0.79 | 0.64 | 0.63 |
| 5 | 0.66 | 0.69 | 0.70 | 0.51 | 0.51 |
| 4 | 0.52 | 0.57 | 0.59 | 0.37 | 0.39 |
| 3 | 0.37 | 0.43 | 0.46 | 0.24 | 0.26 |
| 2 | 0.22 | 0.27 | 0.29 | 0.12 | 0.15 |
| 1 | 0.08 | 0.13 | 0.13 | 0.03 | 0.05 |

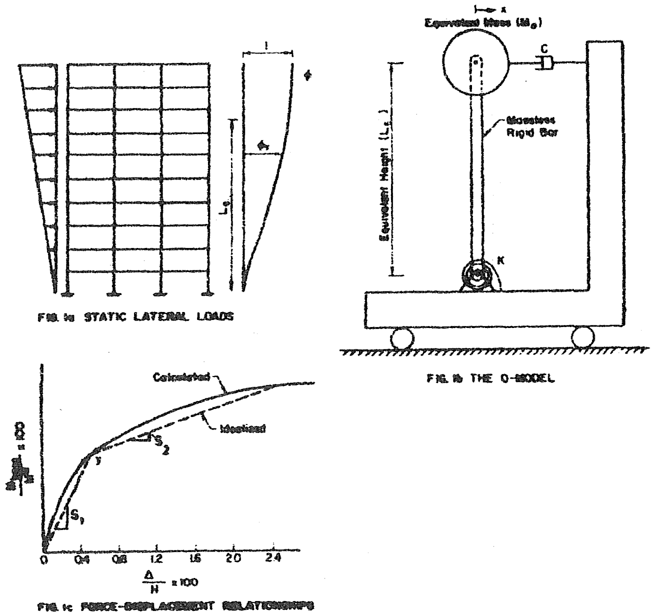

Fig. 1 Multi-Story Structure and the O-Model

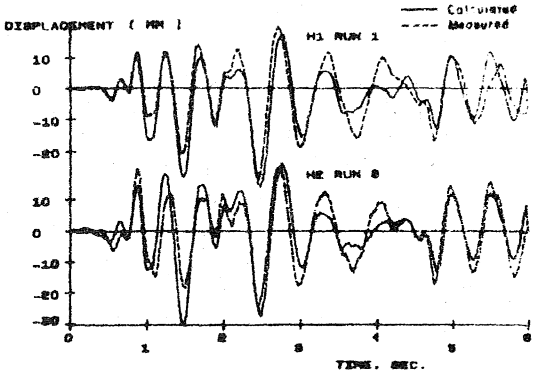

Fig. 2 Top-Level Response for H1 and H2

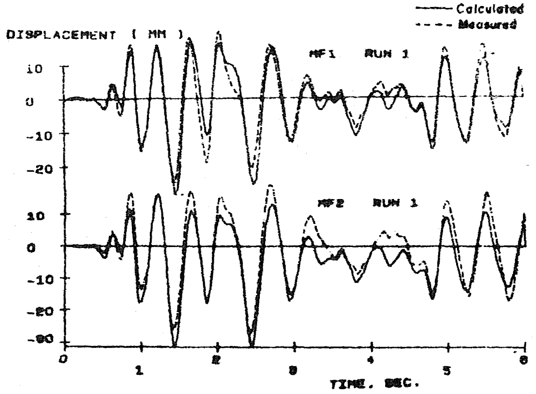

Fig. 3 Top-Level Response for MF1 and MF2

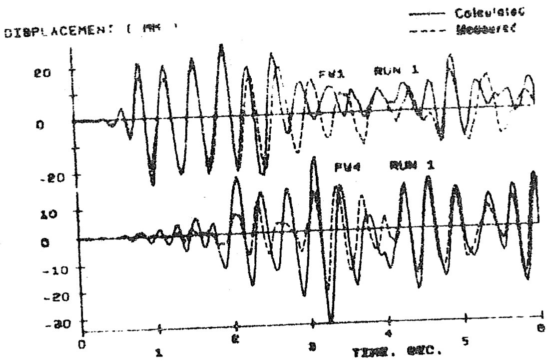

Fig. 4 Top-Level Response for FW1 and FW4

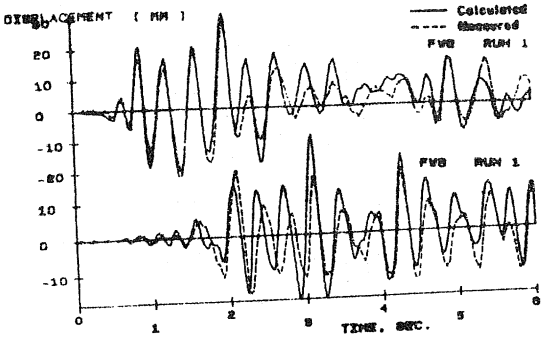

Fig. 5 Top-Level Response for FW2 and FW3

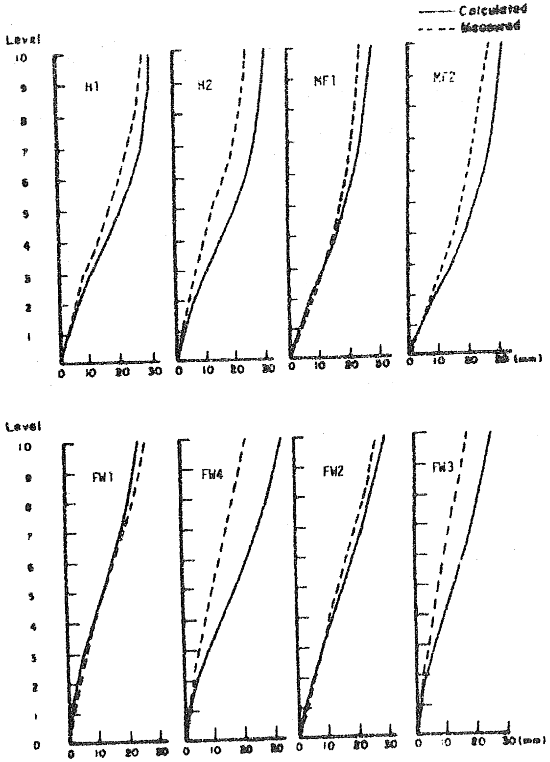

Fig. 6 Maximum Floor Displacements