

## MECHANISM OF CONFINEMENT IN TIED COLUMNS

Shamim A. Sheikh<sup>I</sup> and S.M. Uzumeri<sup>II</sup>

### SUMMARY

A brief summary of the experimental program related to the determination of stress vs. strain curve of concrete confined by rectangular ties is presented in this paper. The development of an analytical model to predict the behaviour of confined concrete is reported.

A comparison is made between the experimental results and the results obtained from using the proposed models for the tests conducted during this study and those reported in the literature. The model was also used to predict the behaviour of a column under axial load and at the same time subjected to cyclic moment. The comparisons of the experimental and analytical results show that the model predicts the behaviour of concrete elements fairly accurately. The confinement requirements of the ACI Code are also examined.

### EXPERIMENTAL WORK

Twenty-four 12" (305 mm) square, 6'-5" (1960 mm) high reinforced concrete columns were tested under monotonic axial compression. Columns were cast vertically. The core dimension (centre to centre of the perimeter tie) was 10.5" (267 mm) square in all the specimens. The concrete strength at the time of testing varied between 4.5 ksi (31 MPa) and 6.0 ksi (41 MPa). The variables examined were: tie arrangement resulting from the distribution of the longitudinal steel around the core perimeter (Fig. 1), tie spacing, amount of lateral and longitudinal steel and strength of the lateral steel.

The details of the experimental program are reported elsewhere (1,2) and a brief summary of conclusions is given below.

1. Confinement by rectangular ties increases the strength (up to 70% in the tests reported) as well as ductility of concrete.
2. Well distributed longitudinal steel around the core-perimeter - and the resulting tie configuration supporting each and every longitudinal bar - enhances the strength and ductility of the confined core.
3. While maintaining the same volumetric ratio of the lateral steel, reduced tie spacing results in higher concrete strength and ductility.
4. Increase in the amount and strength of lateral steel results in less than proportional increase in strength and ductility of the confined concrete.

---

<sup>I</sup> Design Engineer - Civil, Ontario Hydro, Toronto, Ontario, Canada.

<sup>II</sup> Professor, Department of Civil Engineering, University of Toronto, Toronto, Ontario, Canada.

5. Within the range used in the tests, the amount of longitudinal reinforcement appears to have little effect on the behaviour of the confined concrete.

#### ANALYTICAL WORK

The analytical models proposed by previous researchers (3-7) did not satisfactorily predict the results of the tests conducted in this study (Fig. 2). A major reason for this failure is that the distribution of the longitudinal steel and the resulting tie configuration is not considered as a variable in any of these models.

A model was developed to determine the increase in the strength of the confined concrete as indicated by the term  $K_s$  which is the ratio of confined strength to the unconfined strength of concrete. The application of the model to previous tests as well as Toronto tests showed good correlation between the calculated and the experimental values of  $K_s$  (Fig. 3). A stress vs. strain curve for confined concrete, using the value of  $K_s$ , is proposed as shown in Figure 4.

The increase in strength of confined concrete is calculated on the basis of 'effectively confined' concrete area. The 'effectively confined' concrete area is less than the core area enclosed by the centreline of the perimeter tie. This reduced area is a function of the tie configuration and tie spacing. Figure 1 shows the confined and unconfined (shaded) areas of concrete at the tie level and Figure 5 shows the 'effectively confined' area of concrete at the critical section (hatched).

#### Determination of Effectively Confined Core Area ( $A_{ec}$ )

It is assumed that the 'effectively confined' concrete arches vertically as well as horizontally in the form of a second degree curve. The area of the 'effectively confined' concrete is determined as follows:

If  $b$  and  $h$  are the dimensions of core as measured from the centrelines of the perimeter tie,

$$\begin{aligned} \text{Area of core} &= A_{co} = bh \\ \text{Area of effectively confined concrete} &= \lambda A_{co} = \lambda bh \\ \text{at tie level} & \end{aligned}$$

$$\text{where } \lambda = 1 - \frac{\sum_{i=1}^n c_i^2}{\alpha A_{co}}$$

$c_i$  is the distance between the longitudinal bars

$n$  is the number of arcs of unconfined concrete (Fig. 1)

and  $\alpha$  is a constant.

Area of effectively confined concrete at the critical section is then defined as

$$A_{ec} = \lambda (b - 2y_m)(h - 2y_m)$$

where  $y_m = 0.25s \tan \theta$

$s$  is the spacing of ties

and  $\theta$  is the angle between the initial tangent to the curve and the vertical (or direction of longitudinal bars).

Effect of Confining Pressure:

The additional load carrying capacity of concrete due to the effect of confining pressure on the 'effectively confined' concrete is calculated as:

$$P_{add} = \frac{(\rho_s f'_s)^\gamma}{\beta} A_{ce}$$

where  $\beta$  and  $\gamma$  are constants

$\rho_s$  is the ratio of the volume of total lateral steel to the volume of core

and  $f'_s$  is the stress in the lateral steel.

Determination of  $K_s$ :

Since  $K_s$  is the ratio of confined strength to unconfined strength of concrete, it can be evaluated from:

$$K_s = \frac{P_{occ} + P_{add}}{P_{occ}}$$

where  $P_{occ} =$  unconfined strength of concrete core  $= 0.85 f'_c A_{co}$

and  $f'_c$  is the cylinder strength of concrete.

$$K_s = 1 + \frac{1}{P_{occ}} \left\{ \left( 1 - \frac{\sum_{i=1}^n c_i^2}{\alpha A_{co}} \right) \left( 1 - \frac{0.5s}{b} \tan \theta \right) \left( 1 - \frac{0.5s}{h} \tan \theta \right) bh \right\} \frac{(\rho_s f'_s)^\gamma}{\beta}$$

The constants  $\alpha$ ,  $\theta$ ,  $\beta$  and  $\gamma$  were determined from a regression analysis using the  $K_s$  values obtained from the Toronto tests. For the best correlation (Fig. 3), the values of the constants are:  $\alpha = 5.5$ ,  $\theta = 45^\circ$ ,  $\gamma = 0.5$ . For  $P_{occ}$  in kN,  $f'_s$  in MPa and the linear dimensions in mm,  $\beta = 140$  (If the units are kips, ksi and inches respectively,  $\beta = 0.366$ ).

The final  $K_s$  equation for square section thus becomes:

$$K_s = 1 + \frac{b^2}{140 P_{occ}} \left\{ \left( 1 - \frac{nc^2}{5.5 b^2} \right) \left( 1 - \frac{0.5s}{b} \right)^2 \right\} \sqrt{\rho_s f'_s}$$

Determination of Strain Parameters:

In order to completely define the relation shown in Figure 4, the following equations are suggested to calculate the strain values:

$$\epsilon_{s1} = 80 K_s f'_c \times 10^{-6} \quad [\text{adopted from Soliman \& Yu (7)}]$$

$$\frac{\epsilon_{s2}}{\epsilon_o} = 1 + \frac{248}{c} \left(1 - 5\left(\frac{s}{b}\right)^2\right) \frac{\rho_s f'_s}{\sqrt{F'_c}} \quad [\text{adopted from Sargin (6)}]$$

$$\epsilon_{s85} = 0.225 \rho_s \sqrt{b/s} + \epsilon_{s2} \quad [\text{adopted from Kent \& Park (4)}]$$

where  $f'_c$  and  $f'_s$  are in MPa and linear dimensions are in mm.,  $\epsilon_o$  is the strain corresponding to maximum stress in plain concrete.

#### Application of the Proposed Model:

The proposed equations were applied to determine the behaviour of all the columns tested in this study (1) and Figure 6 shows comparison between the experimental and analytical curves for two of the specimens, 2C1-16 and 2C6-18.

Using the confinement criterion of making up for the strength loss due to the spalling of the cover concrete (8), the proposed model was used to determine the tie requirements for a column 22" x 22" (560 x 560 mm) in section. The results are shown in Table I for various tie configurations. It should be noted that buckling of bars is not considered in calculating the tie spacing. The ACI Code (8) requirements, based on the same criterion, are:

$$\rho_s (\text{min}) = 2.04\%, \quad s (\text{max}) = 4.0 \text{ in.}, \quad \text{and tie bar diameter } (d_h) = 3/8".$$

According to the proposed model, it is almost impossible in practice to maintain axial load carrying capacity of the column with only four corner bars after the cover is spalled off. The Code (8) does not differentiate between different tie configurations, and the effect of spacing is also not properly recognized. While in some cases the code requirements are conservative, they appear to be unsafe in cases of columns with only four corner bars. There are doubts as to the validity of the ACI criteria itself, namely that the function of confinement steel is to make up for the loss of axial load carrying capacity of the column due to the loss of cover. Since all columns in a building subjected to earthquakes are acted upon by the combination of axial, bending and shear forces, the confinement criteria should relate to achieving a satisfactory moment-curvature relationship and not to the replacement of the lost cover capacity, as if the column was under pure axial compression.

The proposed stress vs. strain curve can be utilized to predict the moment-curvature relationship of a column subjected to bending and axial compression. Figure 7 shows the experimental moment-average curvature hysteresis loops for a column as reported by Gill, Park and Priestley (9). Also shown on the same figure are the monotonic moment-curvature curves predicted by the proposed model as well as the previous models (4,10).

#### CONCLUSIONS

A brief summary of conclusions from the experimental program is given under Experimental Work.

An analytical model has been developed to predict the behaviour of confined concrete under axial compression. A stress-strain curve of confined concrete under axial compression is proposed.

Application of the proposed model shows that the model predicts the behaviour of confined concrete elements under the actions of axial as well as combined axial and flexural loading in a satisfactory manner.

The confinement requirements of the ACI Code (8) are examined in the light of the experimental and analytical work reported here. It is observed that the steel requirements of the Code do not satisfy the criteria on which the Code requirements are based. It is suggested that the confinement criteria should be related to achieving a satisfactory moment-curvature relationship and not to the replacement of the lost cover capacity in compression. An example of the application of the model to this effect gives promising results.

#### ACKNOWLEDGEMENTS

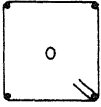

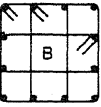
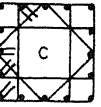
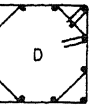
Research reported was supported by a Natural Sciences and Engineering Research Council of Canada Grant (NSERC A4273), and by the University of Toronto. Experimental work was carried out in the Structures Laboratory of the Department of Civil Engineering of the University of Toronto.

#### REFERENCES

1. Sheikh, Shamim A., "Effectiveness of Rectangular Ties as Confinement Steel in Reinforced Concrete Columns", Ph.D. Thesis, Department of Civil Engineering, University of Toronto, Toronto, Canada, June, 1978, 256 pp.
2. Sheikh, Shamim A., and Uzumeri, S.M., "Strength and Ductility of Tied Concrete Columns", to be published in Journal of the Structural Division, ASCE, May, 1980.
3. Chan, W.W.L., "The Ultimate Strength and Deformation of Plastic Hinges in Reinforced Concrete Frameworks", Magazine of Concrete Research, Vol. 7, No. 21, Nov. 1955, pp. 121-132.
4. Kent, D.C., and Park, R., "Flexural Members with Confined Concrete", Journal of the Structural Division, ASCE, Vol. 97, ST7, July, 1971, pp. 1969-1990.
5. Roy, H.E.H., and Sozen, M.A., "Ductility of Concrete", Proceedings of the International Symposium on the Flexural Mechanics of Reinforced Concrete, ASCE - ACI, Miami, November, 1964, pp. 213-224.
6. Sargin, M., "Stress-Strain Relationships for Concrete and the Analysis of Structural Concrete Sections", Study No. 4, Solid Mechanics Division, University of Waterloo, Waterloo, Canada, 1971, 167 pp.

7. Soliman, M.T.M., and Yu, C.W., "The Flexural Stress-Strain Relationship of Concrete Confined by Rectangular Transverse Reinforcement", Magazine of Concrete Research, Vol. 19, No. 61, Dec., 1967, pp. 223-238.
8. American Concrete Institute, "Building Code Requirements for Reinforced Concrete", ACI 318-77, 1977, Detroit, Michigan, 78 pp.
9. Gill, W.D., Park, R., and Priestley, M.J.N., "Ductility of Rectangular Reinforced Concrete Columns With Axial Loads", Research Report 79-1, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, Feb., 1979, 136 pp.
10. Vallenas, J., Bertero, V.V., and Popov, E.P., "Concrete Confined by Rectangular Hoops and Subjected to Axial Loads", Earthquake Engineering Research Centre, Collete of Engineering, University of California, Berkeley, California, Report No. UCB/EERC-77/13, August, 1977, 114 pp.

TABLE I: Tie Steel Requirements for Various Configurations Using the Proposed Model

Configuration					
Longitudinal Steel	4-2 in. $\phi$	8 #11	12 #9	16 #8	12 #9
% of Gross Area	2.60	2.58	2.48	2.61	2.48
Tie diameter (in.)	0.5	0.5	0.5	0.5	0.5
Spacing (in.)*	0.65	4.25	6.25	8.00	5.25
Volumetric ratio of total tie steel (%)	6.25	1.73	1.62	1.74	1.43
$f'_c = 4 \text{ ksi}$ $f'_s = f_y = 60 \text{ ksi}$ Column Size = 22 in. square Core size to centre line of perimeter tie = 18.5 in. square *Buckling of longitudinal bars may require smaller spacing.					

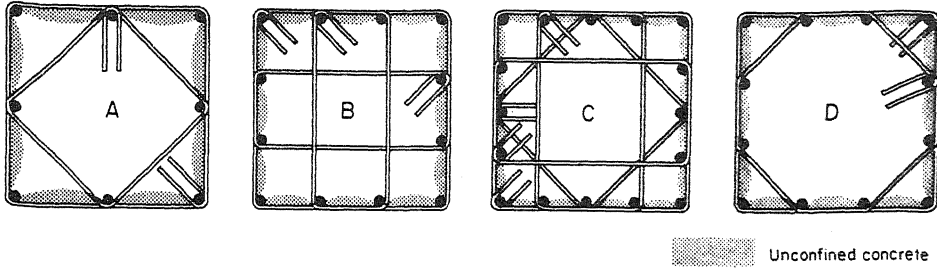


Fig. 1. Tie Configurations

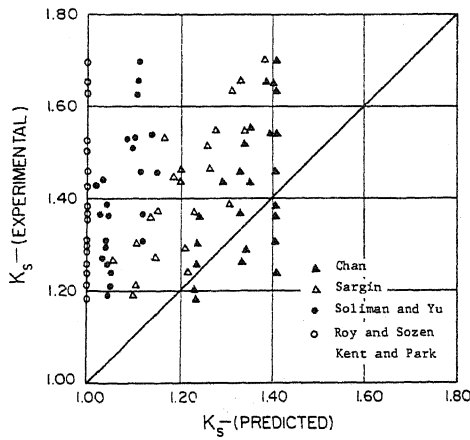


Fig. 2. Prediction of Toronto results by previous models

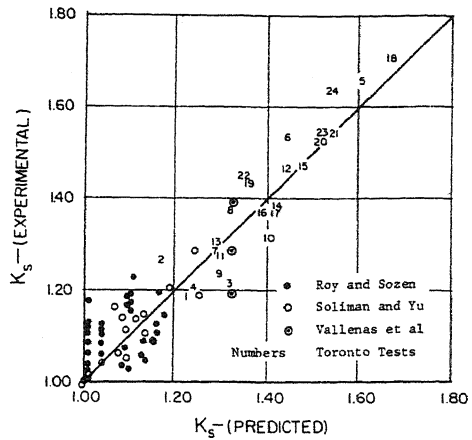


Fig. 3. Prediction of previous and Toronto results by the proposed model

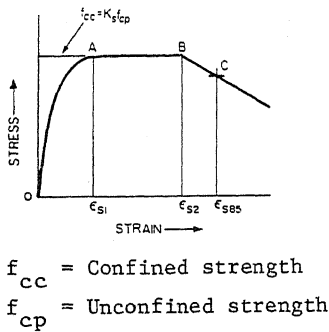


Fig. 4. Proposed Stress-Strain Curve

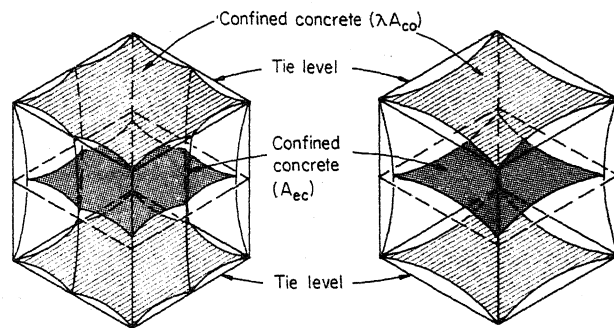


Fig. 5. Modelling of Effectively Confined Concrete Area

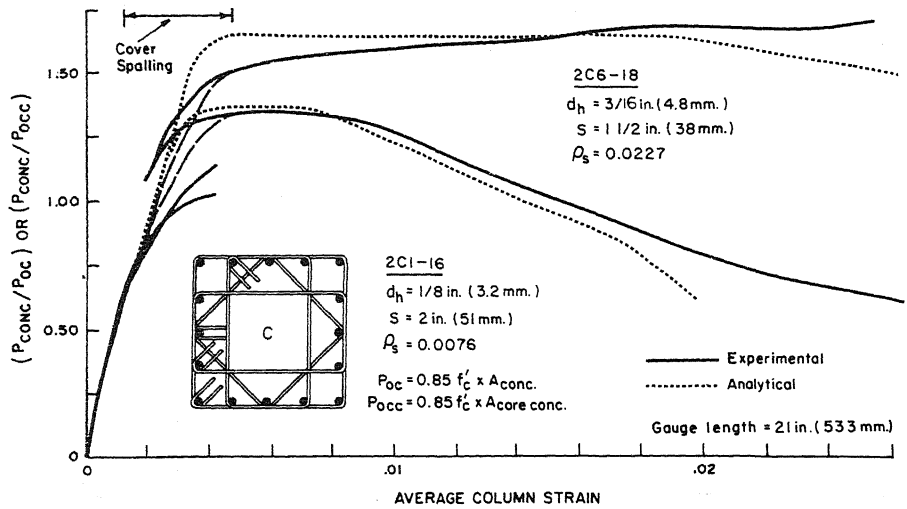


Fig. 6. Comparison of experimental and analytical stress-strain curves

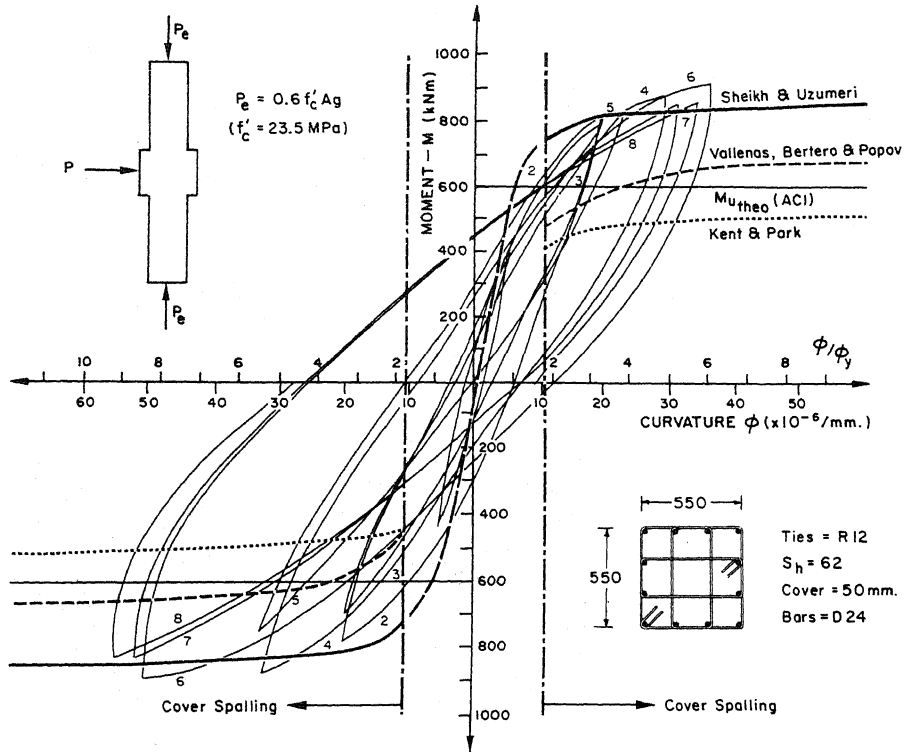


Fig. 7. Comparison of experimental (Specimen 4, by Gill et al (9)) and analytical moment curvature relationships