HYSTERETIC ENERGY SPECTRA IN SEISMIC DESIGN

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SUMMARY

A design method that uses spectra of required energy capacity is proposed. Spectra derived from single degree of freedom models for the required total energy capacity (total energy dissipated in the structure) and for the proportion of energy dissipated in hysteretic and viscous damping mechanisms are presented. The use of such spectra and the distribution of dissipated energy within the structure is extended to multi degree of freedom systems.

INTRODUCTION

The object of seismic design is to provide the capacity to accept without undue distress, the energy that will be imparted to the structure by the earthquake. It is expected that structures will behave inelastically in major earthquakes, and therefore a large percentage of the energy must be dissipated hysteretically. The problem is to anticipate the amount and distribution through the structure of this hysteretic energy, and to detail the structure appropriately, so that it can be absorbed without unacceptable distress.

In current North American practice, this is achieved as follows:

It is assumed that the total deflection of the structure will be approximately the same as that of a similar structure which remains elastic. Then knowing the ductility (deflection at limit of acceptable distress/deflection at limit of elastic behaviour) inherent in the chosen structural system, the force level up to which the structure must remain elastic can be deduced. The structure is then designed to be capable of resisting this force, and detailed to provide the assumed ductility. If the assumption with respect to deflection at the beginning of this paragraph is correct, and if hysteretic energy is dissipated uniformly throughout the structure, this procedure implies that the appropriate capacity to absorb energy in the inelastic range will be available.

The present work is directed towards developing a method of achieving this goal more directly, so that the true distribution of energy-dissipative mechanisms can be accounted for.

Studies have been carried out on single and multi-degree of freedom

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models for various ground motions, damping values, hysteretic models, periods, and strength levels, to determine the amount and distribution of energy dissipated by viscous and hysteretic damping. A design procedure has been proposed and is being tested and developed.

SINGLE DEGREE OF FREEDOM SYSTEMS

The time-variation of the following forms of energy were computed:

Energy dissipated by hysteresis in the members.
Energy dissipated by viscous damping.
Strain Energy.
Kinetic Energy.
Total energy imparted to the structure (the sum of the above).

The influence on the energy values of the following parameters was determined:

Ground motion: Pacoima Dam S16W 1971; Parkville N65E 1956; (both near-source impulsive earthquakes). Taft N69W 1952; El Centro NS 1940; (both more distant earthquakes).

Hysteretic properties (see Fig. 1): Elasto-plastic; Ramberg-Osgood (1)* with A = 0.1, R = 7 and 9; Tri-linear degrading stiffness (2) with

$$P_{BY}/P_{BC} = 3 , \quad K_2/K_1 = 0.5 .$$

(The last of these is typical of reinforced concrete frames, while the others represent steel structures.)

Viscous damping: 0, 1, and 5% of critical damping.

Yield Strength Ratio: this quantity is defined as

$$YSR = \frac{\text{Horizontal Accel}^{\underline{n}} \text{ to produce yield}}{\text{Maximum peak ground acceleration}}$$

It was varied from 0.1 to 10. Values implied by the base shear of the National Building Code of Canada, for example, range from about 0.3 to 1.5. A typical 10 storey frame corresponds to about 0.5.

Briefly, one may classify the response in three broad groups on the basis of Yield Strength Ratio (2):

1. Strong systems (YSR = 1 to 10). These would include realistic buildings, which would, in practice, tend to have short periods characteristic of shear wall systems.
   Response is predominantly elastic; ductility demand is in the order of 1 to 3 when the period is less than 0.5 secs, but there are no major yield excursions. The equal displacement criterion is good, provided that there is viscous damping at 1% of critical or greater, and the period is greater than 0.5 secs. The energy dissipated by the hysteresis in the members is very sensitive to the period and frequency content of the earthquake.
2. Moderate systems (YSR = 0.3 to 1). These would also include realistic buildings, with longer periods characteristic of frame structures.
   Major yield excursions occur, with ductility demands up to 10

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*Numbers in parentheses refer to references.

for periods over 1 sec. (Higher for less realistic structures with short periods.) The equal displacement criterion is good for members with non-degrading stiffnesses, but does not hold when the stiffness degrades unless the viscous damping is high. The hysteretic energy dissipated is less strongly period dependent.

3. Weak systems (YSR < 0.3). Such systems would seldom be encountered in practice; they would tend to have very long periods.

DESIGN METHOD

At the point in the design process when seismic resistance is considered, the overall stiffness, and thus the period, of the structure have generally been established. The Yield Strength Ratio, since it depends mainly on the type of structural system, would also be known within a narrow range. It is intended, then, that the designer should enter an energy spectrum with the appropriate period, YSR, and viscous damping values, and determine the total Required Hysteretic Energy Capacity for the applicable hysteretic characteristics.

The amount of energy to be dissipated in the major yield excursion can then be estimated, and the structure detailed accordingly, or, if this is not possible, strengthened to reduce the Required Hysteretic Energy Capacity.

With regard to the last step, it has been found that the number of excursions into the yield range depends upon the type of earthquake and the Yield Strength Ratio. For impulsive earthquakes, and for strong systems in more distant earthquakes, there are generally 2 to 3 such excursions, with the largest dissipating up to 50% of the total hysteretic energy. For moderate systems in more distant earthquakes there are 3 to 5 yield excursions, with the largest dissipating up to 30% of the hysteretic energy. Although the duration of an earthquake is significant in this respect, the major activity is generally concentrated in the early part of the record. Fig. 2, typical of many such results, shows the energy dissipation in a degrading-stiffness system with YSR = 1, in the Pacoima S16E earthquake, an impulsive-type event.

The remainder of this paper will be devoted to a discussion of the quantities necessary for such a design procedure: the total energy spectrum, the energy distribution, and the relevant behaviour of multi-degree of freedom systems.

DESIGN SPECTRUM FOR TOTAL ENERGY

Eighteen sets of data were obtained: various combinations of the 3 hysteresis models, 3 values of viscous damping, and 4 earthquake records. In each set, 6 values of the YSR were used, with 10 values of the period, for 60 analyses per set.

Fig. 3 shows spectra of the total dissipated energy. For each YSR the curve represents a linear or bilinear fit to the mean plus one standard deviation of the total dissipated energy at each period value. The one exception to this procedure occurred for the strong system case where there was a wide spread in the response; a curve through the upper end of the response range closely matched that for moderate systems with YSR 1.0. For moderate systems the response formed a reasonably narrow band for all hysteretic models, damping values and ground motions. For weak systems the total dissipated energy proved to be essentially independent of viscous damping and earthquake type.

An attempt had been made to predict the total energy dissipated by means of the "elastic spectral kinetic energy" as proposed by Housner (1)

$$\text{ESKE} = \frac{1}{2} S_u^2 \text{ per unit of mass}$$

where S_u is the velocity response spectra of the ground motion. It was found that this method gave a reasonable upper bound for strong systems with longer periods but was unconservative for all other cases.

ENERGY DISSIPATED BY HYSTERETIC DAMPING

The proportion of the total energy dissipated by hysteretic damping was found to depend upon the hysteretic model, YSR, and the amount of viscous damping, but to be sensibly independent of the earthquake type, as can be seen in Fig. 4. Fig. 5 shows the percentage of energy dissipated by hysteretic damping for particular YSR's and viscous damping ratios. These curves are mean value plots of the response for the different earthquakes.

For each strength classification and hysteretic model (in effect, for each type of structural system) it is thus possible to calculate the energy dissipated by hysteretic damping using the spectra of total dissipated energy and distribution of energy.

TOTAL ENERGY DISSIPATED IN MULTI-DEGREE OF FREEDOM SYSTEMS

Six structures were studied, as shown in Fig. 6. Five 3 storey frames represented tapered and uniform stiffness and strength, soft first storeys, and different YSR values. A ten storey frame of uniform strength and stiffness completed the set. Combinations of mass and stiffness-proportional viscous damping of different magnitudes were used, with the four earthquake excitations. The values of the relevant parameters are detailed together with the results in Table 1 for elasto-plastic models and in Table II for stiffness-degrading models. Agreement with values predicted on the basis of the spectra constructed from single degree of freedom results was generally good; in nearly every case the spectrum values would be slightly conservative. In the case of strong elasto-plastic systems, the total energy is very sensitive to variations in the YSR and the interpolation from the previous results was inaccurate. However, when a single degree of freedom model with these specific parameters was studied, agreement was good. These direct comparisons, in which interpolation was avoided by running a specific S.D.F. system, are marked with an asterisk in Table 1.

It appears, therefore, that the total energy dissipated by MDF systems can be determined from the spectra derived from SDF systems, which implies that the first mode governs the energy dissipation.

DISTRIBUTION OF DISSIPATED ENERGY BETWEEN MECHANISMS: MDF SYSTEMS

The distribution of the total energy between hysteretic and viscous damping is shown in Tables III and IV (for elasto-plastic and degrading-stiffness systems respectively), together with the values predicted by means of the spectra of Fig. 5. Agreement is seen to be good except for strong elasto-plastic systems, where comparison with specific SDF systems with the particular values of the design parameters was again good.

DISTRIBUTION OF HYSTERETIC ENERGY WITHIN STRUCTURE

In the elasto-plastic systems, it was found that essentially all the hysteretic energy was dissipated in the lowest storey when the stiffness and strength were uniform, and, of course, when the bottom storey was deliberately softened. In structure 1, with stiffness and strength sharply tapered, 75% of the hysteretic energy was absorbed in the bottom storey. In the degrading stiffness models, the tapered and strong uniform buildings showed two thirds of the energy going to the bottom storey, while in the weaker uniform and soft storey buildings nearly all of it was concentrated in the bottom storey. These results are shown in Tables V and VI. In the 10 storey structure 6, up to 75% of the hysteretic energy was again lost in the first storey. (see Table VII and VIII.)

CONCLUSIONS

The total energy to be absorbed by a structure can be reasonably predicted on the basis of the Yield Strength Ratio (YSR) and period; the fraction of this energy that is dissipated hysteretically can, in turn, be predicted by means of spectra that depend upon the YSR, the viscous damping ratio, and the hysteretic model. Such spectra may be constructed from the response of single degree of freedom systems. Results so far suggest that it may be possible to predict in advance the approximate distribution of hysteretic energy dissipation through the structure; if this does indeed prove to be practicable, the basis for a rational design method exists. It will be possible to predict the amount of hysteretic energy that must be accepted by each location in the structure, and, possibly, the size of the hysteresis loop for which the detailing must provide.

REFERENCES

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TABLE V

RESULTS FOR ELASTO-PLASTIC MDF 3 STOREY STRUCTURE

STRUCTURE	PERIOD (SEC)	YSR	TOTAL ENERGY (INCH KIPS)	ENERGY DISSIPATED IN HYSTERESIS (%)	DISTRIBUTION 1	2	3
1	.43	.8	3715	65.5	48.9	13.9	2.6
2	.33	1.4	682	34.0	24.0	0.0	0.0
3	.33	.45	1413	90.0	83.0	7.0	0.0
4	.47	.68	2356	73.0	73.0	0.0	0.0
5	.57	.45	2152	77.0	77.0	0.0	0.0

TABLE VI

RESULTS FOR DEGRADING-STIFFNESS MDF 3-STOREY BUILDINGS

STRUCTURE	PERIOD (SEC)	YSR	TOTAL ENERGY (INCH KIPS)	ENERGY DISSIPATED IN HYSTERESIS (%)	DISTRIBUTION 1	2	3
1	0.43	0.8	6744.3	86.4	50.8	29.4	6.2
2	0.33	1.4	1308.5	76.6	51.9	22.3	2.4
3	0.33	0.45	1946.8	73.4	71.0	1.9	0.5
4	0.47	0.68	2709.6	60.0	67.0	3.1	0.0
5	0.57	0.45	1908.5	69.1	68.7	0.4	0.0

TABLE VII

10 STOREY ELASTO-PLASTIC 3% MASS DAMPING

EL CENTRO N.S.			PACOIMA S16E		
STOREY	ENERGY DISSIPATED BY HYSTERESIS	ENERGY DISSIPATED BY VISCOUS DAMPING	STOREY	ENERGY DISSIPATED BY HYSTERESIS	ENERGY DISSIPATED BY VISCOUS DAMPING
10	0.0	5.1	10	0.0	5.5
9	1.0	4.9	9	0.1	5.5
8	2.4	4.5	8	0.3	5.4
7	3.9	4.0	7	0.4	5.4
6	5.9	3.4	6	0.6	5.2
5	7.2	2.8	5	1.2	5.2
4	8.4	2.2	4	1.6	4.9
3	9.5	1.6	3	1.6	4.7
2	14.2	1.1	2	3.8	4.5
1	17.2	0.6	1	39.8	4.2
TOTAL	69.7	30.2	TOTAL	49.5	50.5

TABLE VIII

10 STOREY DEGRADING STIFFNESS 3% MASS DAMPING

EL CENTRO N.S.			PACOIMA S16E		
STOREY	ENERGY DISSIPATED BY HYSTERESIS	ENERGY DISSIPATED BY VISCOUS DAMPING	STOREY	ENERGY DISSIPATED BY HYSTERESIS	ENERGY DISSIPATED BY VISCOUS DAMPING
10	0.0	6.6	10	0.0	4.3
9	0.0	5.8	9	0.2	4.1
8	0.0	5.0	8	0.9	4.0
7	0.0	4.3	7	1.5	3.8
6	1.5	3.6	6	3.1	3.8
5	3.7	2.9	5	3.6	3.6
4	6.2	2.5	4	2.6	3.3
3	10.7	1.9	3	3.1	3.1
2	9.3	1.5	2	2.6	2.9
1	33.1	1.1	1	46.6	2.7
TOTAL	64.6	35.4	TOTAL	64.1	35.9

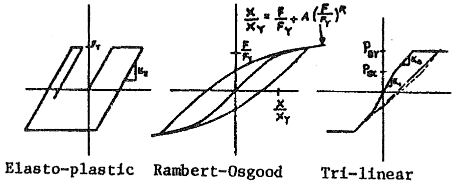


Figure 1 - Hysteretic models

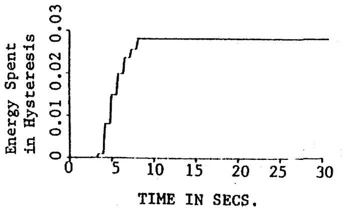


Figure 2 - Hysteretic Energy/ unit mass (secs^2) v. time YSR = 1, Pacoima Earthquake

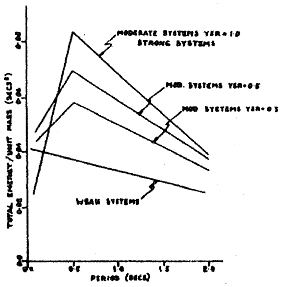
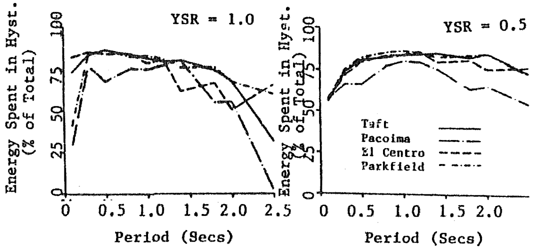


Figure 3 - Hysteretic Energy v. Period



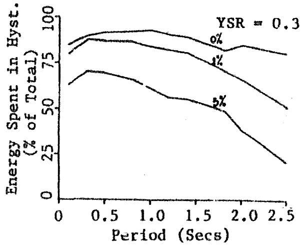


Figure 5 - Hyst. Energy v. Period for different YSR

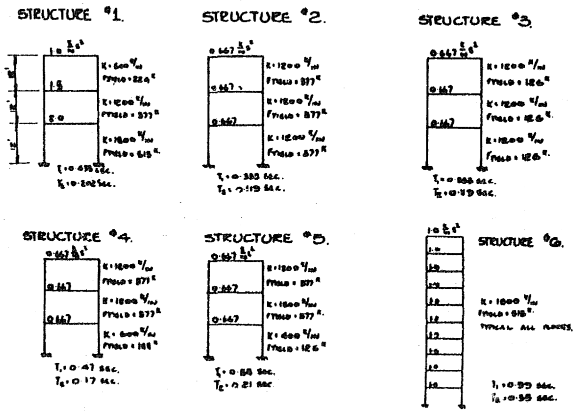


Figure 6 - Test Structures