SEISMIC RESPONSE OF HYSTERETIC DEGRADING STRUCTURES

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SUMMARY

A hysteretic restoring force model based on nonlinear differential equation and an equivalent linearization solution procedure for inelastic, degrading or nondegrading multi-degree-of-freedom systems are presented. Covariance matrix for the response variables, maximum response and energy dissipation statistics are obtained. The accuracy of this method is verified against Monte-Carlo simulation. Models for multi-story frames based on a shear beam, soft beam-strong column, and discrete hysteretic hinge concept are studied. The results indicate that the proposed method gives a realistic representation of the inelastic structural behavior and a computationally efficient solution procedure.

INTRODUCTION

In the study of safety of structures under earthquakes, the response in the inelastic range has to be considered. Empirical method based on time-history analysis has been most widely used. However, in order to account for the statistical variation of excitation and structural resistance, the determination of the statistics and probability of the inelastic response is required. Although such information may be obtained by repeated time-history analysis, the process could be very costly if accurate estimate is required. Therefore, there is a need for analytical procedure for the random vibration of multi-degree-of-freedom, inelastic structures. Such analysis procedures should take into account the yielding, hysteretic behavior of the structure, and the possibility of structural deterioration. Herein, such a mathematical model is presented in which the inelastic behavior of structures is represented by a set of nonlinear differential equations. The resulting equations of motion is solved approximately using the method of equivalent linearization, without recourse to the Krylov-Boboliubov approximation. The covariance matrix of the responses obtained agrees more closely with solutions obtained from Monte Carlo simulation than do other available approximate solutions. Further, the model is capable of estimating the energy dissipation statistics at critical locations on the structure which provide useful information for assessment of structural damage and failure.

HYSTERETIC ELEMENT AND LINEARIZATION

A general hysteretic restoring force model which can exhibit hardening or softening behavior, with a wide range of cyclic energy dissipation properties is given by (4,8,3), for the i-th element in the structure,

$$\dot{z}_i = \{A_i\dot{u}_i - \nu_i[\beta_i|\dot{u}_i||z_i|^{n_i-1} z_i + \gamma_i\dot{u}_i|z_i|^{n_i}]\}/\eta_i \qquad (1)$$

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where z_i is the hysteretic restoring force, and u_i is the relative displacement. The rate of energy dissipation by the hysteresis is proportional to

$$\dot{\varepsilon} = z_i \dot{u}_i \tag{2}$$

Noting that the hysteretic energy dissipation is an indicator of the response severity, deterioration may be obtained by making the parameters A_i, ν_i or η_i functions of ε_i, e.g.

$$A_i = A_{oi} - \delta_{Ai}\varepsilon_i$$

$$\eta_i = \eta_{oi} + \delta_{\eta i}\varepsilon_i \tag{3}$$

$$\nu_i = \nu_{oi} + \delta_{\nu i}\varepsilon_i$$

where δ_{Ai}, $\delta_{\eta i}$ and $\delta_{\nu i}$ are positive constants and A_{oi}, η_{oi} and ν_{oi} are initial values. When implemented as the restoring force element in models of mechanical systems Eq. 1 is a principal source of nonlinearity. To obtain r.m.s. response under stocahstic excitation, e.g. white noise or filtered white noise, Eq. 1 is linearized (3),

$$\dot{z}_i = C_{ei}\dot{u}_i + K_{ei}z_i \tag{4}$$

It has been shown that, if the parameters A_i, η_i and ν_i are assumed to vary slowly (or constant for nondeteriorating systems), a closed form linearization is possible, which minimizes the mean square equation differences. For details of the linearization, refer to (3). The resulting expressions for C_{ei} and K_{ei} are functions of the zero time lag covariance matrix S(o) which is given by the well known matrix equation (8,3).

$$\dot{\underset{\sim}{S}} + \underset{\sim}{G}\,\underset{\sim}{S} + \underset{\sim}{S}\,\underset{\sim}{G} = \underset{\sim}{B} \tag{5}$$

in which $\underset{\sim}{G}$ and $\underset{\sim}{B}$ are matrices of structural system and ground excitation parameters. Stationary solution ($\dot{\underset{\sim}{S}} = 0$) is obtained iteratively by solving the Liapunov matrix equation, while transient solution requires numerical integration of Eq. 5 with an updated system matrix G computed via Eq. 4 at each step.

POWER SPECTRAL DENSITY FUNCTION COMPUTATION

For stationary response analysis, the power spectral density functions for the linearized but non-selfadjoint system are obtained using modal decomposition, which is equivalent to a partial fraction expansion. If the input is a filtered white noise, one obtains, for example, for the displacement response Y

$$W_{YY}(\omega) = \sum_{j=1}\sum_{k=1} \frac{1}{(\omega-\sigma_j)}\frac{1}{(\omega+\sigma_k^*)}\;\underset{\sim}{\phi}_j\;\underset{\sim}{\psi}_j^T W_{\xi\xi}\underset{\sim}{\psi}_k^*\;\underset{\sim}{\phi}_k^* \tag{6}$$

where σ_j, ϕ_j and ψ_j are the jth eigenvalue, and the corresponding right and left eigenvectors respectively of the structural-ground system, * indicates complex conjugate, and $W_{\xi\xi}$ is the (constant) input power spectral density function. In this instance, the $\underset{\sim}{B}$ matrix simplifies to $2\pi W_{\xi\xi}$. Knowledge of the power spectral density function also allows approximate computation of first passage time probability (6). Several first passage approximations for models with the hysteresis of Eq. 1 are discussed elsewhere (2,3). It has been shown (3) that the response power spectral density function of the linearized system has the essential characteristics expected of the inelastic systems.

MODELS FOR YIELDING SYSTEMS

The simplest multidegree of freedom model using Eq. 1 is the shear beam model, which results for an n degree of freedom systems in n equations of the form

$$\ddot{u}_i - (1-\delta_{ii}) \frac{q_{i-1}}{m_{i-1}} + \frac{q_i}{m_i} [1+(1-\delta_{1i}) \frac{m_i}{m_{i-1}}] - [1-\delta_{in}] \frac{q_{i+1}}{m_{i+1}} (\frac{m_{i+1}}{m_i}) = -\delta_{i1}\ddot{\xi}_B \qquad i = 1,n \tag{7}$$

where δ_{ij} are Kronecker deltas, $\ddot{\xi}_B$ is the base motion, q_i is the total restoring force, given by

$$q_i = c_i\dot{u}_i + \alpha_i K_i u_k + (1-\alpha_i) K_i z_i \tag{8}$$

and u_i is the relative displacement between floors i and i+1. In Eq. 8 the hysteresis is given by Eq. 1. Zero time lag covariance matrix of the responses computed for this type of system have been reported previously (1,3) and agree more closely with the results of Monte Carlo simulation than other available approximate methods. First passage time approximations computed using the covariance matrix and power spectral density matrices have also been reported (2). Agreement between simulations and first passage approximations based on a method for Gaussian processes is somewhat less satisfactory, particularly as yielding and the consequent non-Gaussian behavior of the system become more important.

The shear beam model of course cannot account for the interaction between stories of multistory structures as has been printed out by Takizawa (5). He proposed, instead, a single degree of freedom system model whose initial stiffness reproduces the preyield natural frequency of the systems, and whose yield level corresponds to the expected yield mechanism of the structure. Herein, a variation of Takizawa's model, with columns assumed rigid, except at the fixed base, is considered as an alternative to the shear beam model. The system hysteresis is modeled by Eq. 1 and the linearization procedure described herein is used for the response analysis.

A second, more general, alternative is also studied. The structure is idealized as an assemblage of linearly elastic beam (column) elements, connected at the joints by discrete inelastic hinges, whose hysteretic behavior is described by Eq. 1 as shown in Fig. 1. Clearly, three types of generalized displacements ($\underset{\sim}{u}$) and associated generalized forces ($\underset{\sim}{U}$) must be considered, the translations and shears ($\underset{\sim}{u}_T$ and $\underset{\sim}{U}_T$), the elastic element end rotations relative to the hysteretic elements ($\underset{\sim}{u}_R$ and $\underset{\sim}{U}_R$) and the hysteretic element rotations and moments ($\bar{\underset{\sim}{u}}$ and $\bar{\underset{\sim}{U}}$). The last pair corresponds to u and z in Eq. 1. The elastic end forces are described in terms of the vectors $\underset{\sim}{u}^T = \{\underset{\sim}{u}_T^T, \underset{\sim}{u}_R^T\}$, and $\bar{\underset{\sim}{u}}$, viz.

$$\underset{\sim}{U} = K_e \underset{\sim}{u} - \underset{\sim}{K}_H \bar{\underset{\sim}{u}} \tag{9}$$

The linearized hysteretic restoring forces are written in matrix form, following Eq. 4

$$\dot{\bar{\underset{\sim}{U}}} = \underset{\sim}{K}_e \bar{\underset{\sim}{U}} + \underset{\sim}{C}_e \dot{\bar{\underset{\sim}{u}}} \ , \tag{10}$$

The hysteretic moments are equated to the elastic element end moments,

$$\underset{\sim}{U}_R = \underset{\sim}{K}_p \bar{\underset{\sim}{u}} + \underset{\sim}{K}_y \bar{\underset{\sim}{U}} \tag{11}$$

where the first term on the r.m.s. of Eq. 11 is introduced to allow a (linear) post yield stiffness, and the translational equations of motion are wirtten

$$\ddot{\underset{\sim}{u}}_T + \underset{\sim}{M}^{-1} \underset{\sim}{C}_T \dot{\underset{\sim}{u}}_T + M^{-1} \underset{\sim}{C}_{ST} \underset{\sim}{U}_T = -\underset{\sim}{i} \ddot{\xi}_B \tag{12}$$

Since the system is non-self adjoint, a complete formulation for the covariance matrix of the response can be obtained by a transformation of variables resulting in a first order system of the equations of motion

$$\dot{\underset{\sim}{V}} + \underset{\sim}{G} \underset{\sim}{V} = -\hat{\underset{\sim}{i}} \ddot{\xi}_B \tag{13}$$

where

$$\underset{\sim}{V}^T = \{\underset{\sim}{u}_T^T \ \dot{\underset{\sim}{u}}_T^T \ \bar{\underset{\sim}{u}}^T\} \tag{14}$$

which may be solved for the covariance matrix terms $E[\underset{\sim}{v}(t) \ \underset{\sim}{v}^T(t)]$ as previously indicated (Eq. 5). The additional response terms needed to compute the updated $\underset{\sim}{G}$ matrix, which is required for iteration or numerical integration, are obtained from the transformation

$$\hat{\underset{\sim}{V}} = \underset{\sim}{T}_1 \underset{\sim}{V} \tag{15}$$

where

$$\hat{\underset{\sim}{V}}^T = \{\dot{\bar{\underset{\sim}{u}}}^T \ \bar{\underset{\sim}{U}}^T\} \tag{16}$$

Then

$$E[\hat{\underset{\sim}{V}} \ \hat{\underset{\sim}{V}}^T] = \underset{\sim}{T}_1 \ E[\underset{\sim}{V} \ \underset{\sim}{V}^T] \ \underset{\sim}{T}_1^T \tag{17}$$

The cross terms in the left hand side of Eq. 17 also provide the energy dissipation rates at the individual discrete hinge elements. By similar transformations, other terms of the covariance response are obtained. For details, see Baber and Wen (3).

NUMERICAL STUDIES

Two structures, shown in Fig. 2, were analyzed by the different modeling techniques. Structure A is a strong girder structure for which the shear beam idealization may be satisfactory, while Structure B is a strong column structure typical of those designed in seismic regions. The designs are based on an 80 psf live load and a 100 psf dead load. One half of the live load was taken as contributing to the effective mass, and both frames were considered to be interior with 30 feet spaces between frames.

Frame A was analyzed using the discrete hinge (Fig. 3) and shear beam models under stationary excitation modeled by Kanai filtered white noise with r.m.s. absolute base accelerations from 1.4 to 22.4 ft/sec^2. The resulting r.m.s. story displacements as functions of excitation level are shown in Fig. 5 and the energy dissipation rates are shown in Fig. 6. The discrete hinge model predicts somewhat larger r.m.s. displacements than the shear beam model, a consequence of the characteristic of the discrete hinge model being closer to that of an elastoplastic system but the trends are quite similar. The total energy dissipation computed by the two models are quite close. The energy dissipation rates provide significant information about where serious damage may have occurred. The dotted curves, ij, in Fig. 6 are the energy dissipation rates of the individual discrete hinges at joint j of member i (see Fig. 3).

Likewise, Frame B was analyzed by the discrete hinge model (Fig. 4), the shear beam model and a SDOF model. The r.m.s. displacements and energy dissipation rate predictions are shown in Fig. 8 and 7, respecitvely. As expected, the behavior of the discrete hinge model more closely resembles that of the SDOF model. Once again, significant information regarding the distribution of yielding is given by the energy dissipation rates. In this case, yielding of the girders is clearly a more important source of energy dissipation.

CONCLUSIONS

A hysteretic element model which is quite versatile, and compatible with stochastic equivalent linearization analysis is presented. Various modeling concepts for multistory structures are studied and numerical studies of two simple structures using the modeling concepts are discussed. The studies indicate that the proposed method gives a realistic representation of the inelastic structural behavior and a computationally efficient solution procedure. The discrete hinge model provides significantly more insight into the response of hysteretic structures to stochastically described base excitations than do simpler SDOF or shear beam models. Extension to more complex structures is straightforward, although numerical computations will become more extensive. Extension to consider midmember hinges, axial load effects and nonzero mean excitation are also possible.

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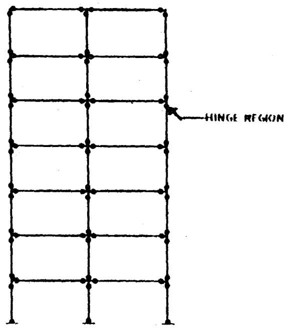

Fig. 1 Frame with Discrete Yield Regions

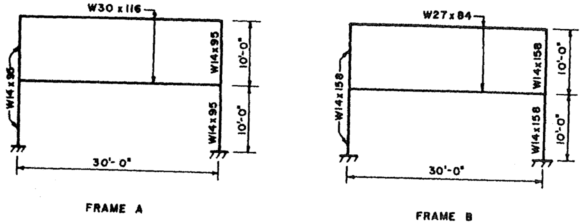

Fig. 2 Plane Frames for Equivalent Linearization Analysis

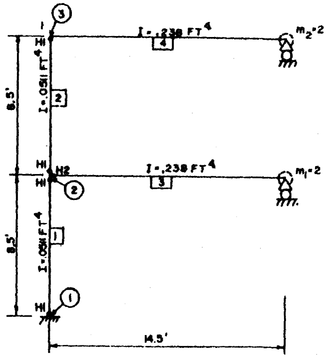

Fig. 3 Discrete Hinge Model for Frame A

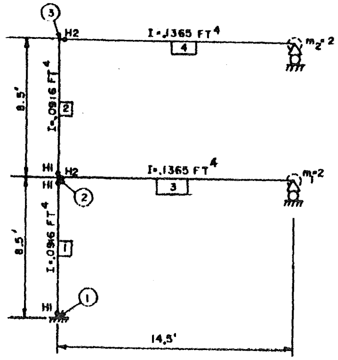

Fig. 4 Discrete Hinge Model for Frame B

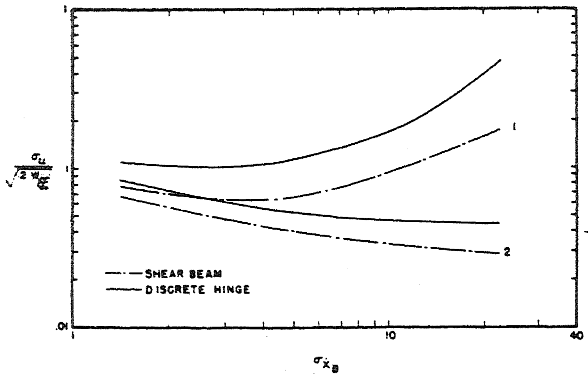

Fig. 5 R.M.S. Displacement
Frame A

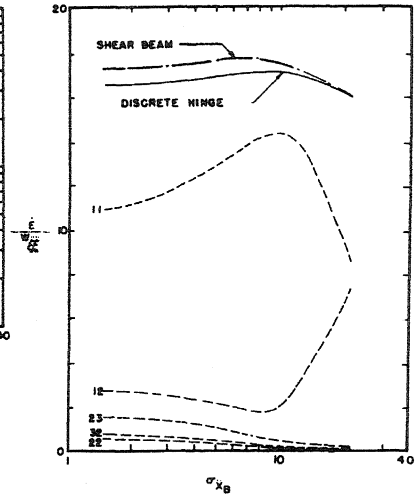

Fig. 6 Energy Dissipation
Rates - Frame A

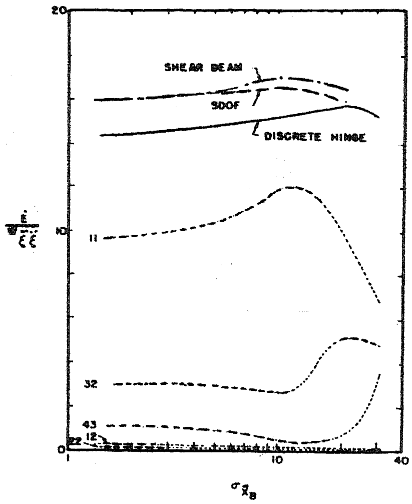

Fig. 7 Energy Dissipation
Rates - Frame B

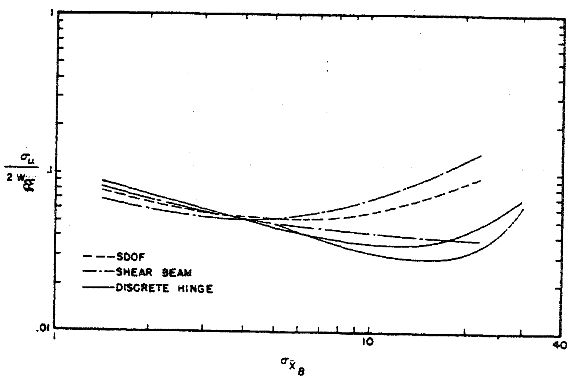

Fig. 8 R.M.S. Displacement
Frame B