MEASUREMENTS AND CALCULATIONS OF NATURAL FREQUENCIES AND COUPLED BENDING-TORSION MODES OF HIGHRISE BUILDINGS

by

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SUMMARY

Measurements of natural frequencies and of the appertaining bending-torsion modes for two highrise buildings are presented here. The first building is a reinforced concrete structure of a quadrate plan and eccentric kernel, height 55 m. The second one is a steel skeleton building with concrete floor plates and kernels; it consists of three single towers which are eccentrically coupled by footbridges at every story, heights from 91 m to 143 m. The results of vibration measurements are compared with results of calculations based on a mechanical model which allows to map the mechanical properties of a building into a one-dimensional continuum.

1. INTRODUCTION

A basic requirement in order to dimension buildings against earthquake loads is the existence of an adequate mechanical model thereof which is to handle easily enough in practical operation and which is exact enough to determine frequencies and modes of the building as requested for the above-mentioned purpose. It was the authors' main topic to develop such a model and to test it not only with respect to its frugality and convenience at calculation, but also with respect to its usefulness and efficiency in the representation of real buildings. It was the basic idea to treat the problem analytically as far as possible in order to keep a general view at any phase of calculation.

2. MECHANICAL MODEL AND CALCULATION PROCEDURE

The mapping of the mechanical properties of a building into a one-dimensional continuum allows a convenient mathematical description and also the simple derivation and application of approximate methods, for instance an energy method. Consider a single multistory frame with beams and stanchions and distribute the mass and the stiffness of the beams continually over the height of a story, by halves up- and downward. Together with the bending stiffness of the stanchions we get the equation of motion of such a continuum which is equal to that of a string with bending stiffness, where the string force consists of the weight of the frame as a compressive force and of a tensile force caused by the bending stiffness of the beams, whereby the latter one predominates.

The continual model of the whole building is composed of such single frames coupled by the floor plates, assumed as rigid in their plane and also continually distributed over the height. Their perpendicular arrangement in the plan is most convenient for calculation and corresponds with

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most instances in practice. Let the origin of a coordinate system x (height), y,z be in the center of gravity at x = 0. The normal position of the gravity-line - which connects the gravity centers of the model and which has neither to be straight nor perpendicular - may be given by a(x) and b(x). The horizontal displacements of the gravity centers are v(x,t) and w(x,t) in y- and z-direction respectively - taken from the gravity center at x = 0 -, $\varphi(x,t)$ is the rotation round the x-axis, t the time. For details see [1], [2].

Then we get a system of three coupled partial differential equations of 4th order. In matrix form this system has the following form:

$$M_{ij}\ddot{u}_j + (B_{ij}u_j'')'' + (S_{ij}u_j')' = p_i(x,t) \qquad (1)$$

summed over j = 1,2,3 ' = d/dx

With the boundary conditions (L = height of building):

$$\begin{aligned} u_i(0,t) &= 0 \qquad & (B_{ij}u_j''(L,t))' + S_{ij}u_j'(L,t) &= 0 \\ u_i'(0,t) &= 0 \qquad & B_{ij}u_j''(L,t) &= 0 \end{aligned} \qquad (2)$$

summed over j = 1,2,3

The displacements and the coefficients - in general variable in x - are defined as follows:

Displacement vector:

$$u_i(x,t) = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \underbrace{\begin{Bmatrix} v - b \\ w - a \\ \varphi \end{Bmatrix}}_{=\bar{u}_i} + \underbrace{\begin{Bmatrix} a \\ -b \\ 0 \end{Bmatrix}}_{=c_i}\varphi = \bar{u}_k(\delta_{ik} + c_i\delta_{3k}) \qquad (3)$$

summed over k = 1,2,3

The displacements u_i may be interpreted as the displacements of these points which lie on the x-axis in the normal position.

Mass matrix:

$$M_{ij}(x) = \begin{Bmatrix} \mu & 0 & -a\mu \\ 0 & \mu & b\mu \\ -a\mu & b\mu & (a^2 + b^2)\mu + \Theta \end{Bmatrix} \qquad (4)$$

Matrices of bending-stiffnesses and string-stiffnesses:

$$B_{ij}(x) = \begin{Bmatrix} EI_y & 0 & -EI_{y\varphi} \\ 0 & EI_z & EI_{z\varphi} \\ -EI_{y\varphi} & EI_{z\varphi} & EI_\varphi \end{Bmatrix} \quad (5) \qquad S_{ij}(x) = \begin{Bmatrix} -A_y & 0 & A_{y\varphi} \\ 0 & -A_z & -A_{z\varphi} \\ A_{y\varphi} & -A_{z\varphi} & -A_\varphi \end{Bmatrix} \quad (6)$$

The elements of the matrices are composed by the parameters of the several frames: μ is the mass of the building per height unit, Θ the moment of inertia of a building layer with respect to its gravity center. The elements of Eq. 5 denote the bending-stiffnesses of columns and kernels, those of Eq. 6 are composed of the bending-stiffnesses of beams and floor plates.

The solution of the boundary value problem - in general with variable coefficients - is provided by means of a finite-difference method [3]. The method reduces the eigenvalue problem of the system of differential equations to an eigenvalue problem of matrices with 50 to 200 unknowns their number depending on the desired accuracy. With the aid of basic matrices which are independent of a special problem it is possible to build up the final matrices of the eigenvalue problem by the computer in a comparatively easy manner.

4. VIBRATION MEASUREMENTS

To perform the measurements seismometers (system Willmore) were used. They were placed at several points in plan and elevation of the building, at every point in two perpendicular horizontal directions, in order to be able to determine both, natural frequencies and modes of the building. The signals of the seismometers were recorded simultaneously by a tape recorder and analysed by a spectrum analyzer. By means of the power spectral density of the signals modes and natural frequencies were gained. Excitation of the building by micro-tremor or wind is sufficient.

5. RESULTS OF MEASUREMENTS AND CALCULATIONS

Measurements and calculations for two buildings were performed. Building A is a reinforced concrete structure, see Fig.1, which lodges the Material Test Institute of Stuttgart University; building B, see Fig.3, are the three towers of the "Deutsche Welle" radio station at Cologne.

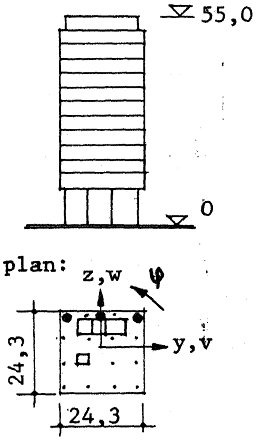


Fig.1:Building A
•measuring points

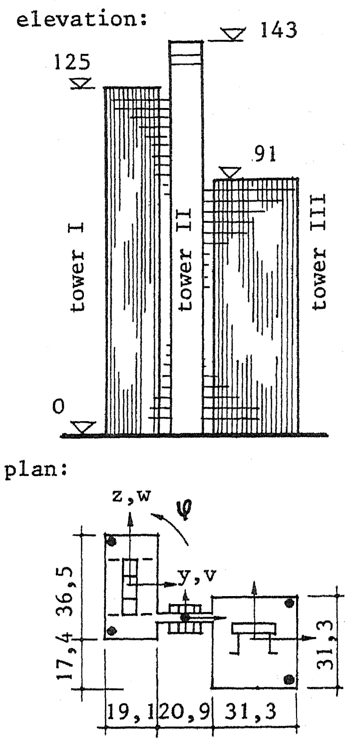
The two main towers have concrete kernels with a connected steel skeleton construction and special steel-concrete floor plates [4]. An elevator tower is a concrete structure. All three towers are coupled eccentrically by means of footbridges. At building A at 26 points, distributed over 12 stories, measurements were taken; at building B every 5th story, altogether at 28 points. The following frequencies (Hz) were determined:

Building A :	1.	2.	3.	4.	5.
measured :	0,52	0,73	1,06	2,39	2,57
calculated :	0,54	0,69	1,20	1,93	2,90

Building B :

	1.	2.	3.	4.	5.	6.	7.	8.	9.
m:	0,20	0,35	0,50	0,55	0,71	0,78	1,05	1,33	1,70
c:	0,18	0,37	0,51	0,53	0,67	0,74	1,06	1,35	1,85

One example of the modes is shown for each building: In Fig.2 for building A, in Fig.4 for building B. Numerical values of the mass- and stiffness-coefficients cannot be given here for reasons of space, but it should be mentioned that Young's modulus of the concrete was taken as $3{,}5\ 10^{10}\ N/m^2$. There is a good correspondence between measurements and calculations. But nevertheless more refinements of the measuring method and systematical variations of the parameters are necessary to get the feel of the whole problem.



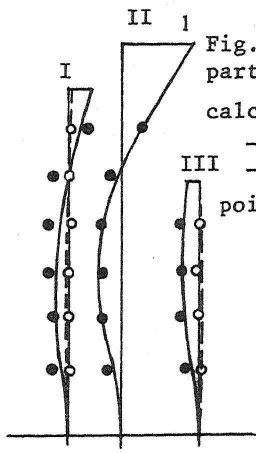


Fig.4: Building B, part of 9th mode.
calculated:
--- φ
___ v
points measured:
o φ • v

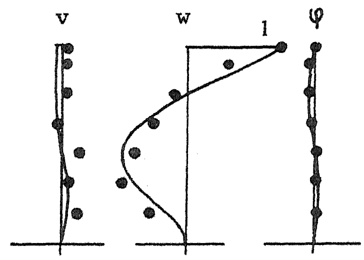


Fig.3: Building B
• measuring points

Fig.2: Building A, 5th mode, calculated values, • points measured

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