DAMPING ESTIMATION FROM FULL-SCALE CYCLIC TESTING OF A FIVE-STORY STEEL FRAME

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SUMMARY

In this paper a method is presented to identify both stiffness and damping parameters of building structures from observed acceleration measurements at each of the stories undergoing harmonic excitation. The method is based on a modified Newton-Raphson scheme which minimizes the weighted sum of errors in both the acceleration amplitude and phase angle relative to the exciter on the roof. The technique is applied to a five-story steel frame in which the frequency range of excitation encompassed only the first two resonant frequencies in each of the three movements associated with the orthogonal lateral displacements and torsion about the center of geometry of the building.

INTRODUCTION

The observed dynamic behavior of a structure rarely concords with that predicted by an available mathematical model, no matter how refined it may be. The objective of this paper is to present a method of systematically varying the parameters of this model in an effort to match more closely the observed behavior. In particular sinusoidal acceleration measurements at all the floors are taken for various inertial loads on the top story in both the lateral directions as well as in torsion at different frequencies of excitation.

Limitations precluded tests at high frequencies and thus tests encompassed only the two lowest natural frequencies in each of the three responses observed of this five-story rigid steel frame. Four hundred records digitized at .01 s intervals over 10 s time intervals were considered. Both the response amplitude and the phase angle relative to the exciter are treated as data. Further information on the building, apparatus, and method may be found in Ref. 1.

BASIC EQUATIONS OF DYNAMICS

The dynamic behavior is assumed linear and the symmetric mass [M], stiffness, [K], and viscous damping [C] matrices are assumed to have constant coefficients. For sinusoidal excitation and response, once the transient phenomena have disappeared there results

$$\{X\} = [H]\,\{F\} \tag{1}$$

in which {F} corresponds to the force at each level, {X} corresponds to the displacement and [H] is the matrix of frequency response equal to the

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inverse of the impedance matrix, [Z]

$$[Z] = [K] + i\ 2\pi f\ [C] - 4\pi^2 f^2\ [M] \tag{2}$$

f is the frequency of displacement in hertz and $i = \sqrt{-1}$ is the imaginary symbol. Thus, {F} and {X} are in general complex vectors. {F} may be supposed real if all loads are in phase, but with damping, {X} is, in general, complex even in this situation.

BUILDING MODEL

Only the very simple shear model for building behavior is considered in the example presented though the development is applicable to a more complete representation, provided of course that a sufficient amount of data is available. The mass matrix is assumed diagonal, the so-called lumped mass model, and the stiffness and damping matrices satisfy the tria-diagonal relationship. This simple model is assumed to be valid in each of the two lateral directions as well as in torsion.

INITIAL ESTIMATES

Though excitation at the top story only precludes that the displacements measure be a normal mode, it will resemble one if damping is small and excitation is near a resonant frequency. Based on this observation and the simple model chosen a system of linear equations results for the unknown stiffness parameters

$$\begin{bmatrix} [B_1] \\ \hline [B_2] \end{bmatrix} \{k\} = \begin{Bmatrix} \Omega_1^{\ 2}\ [M]\ \{\phi_1\} \\ \hline \Omega_2^{\ 2}\ [M]\ \{\phi_2\} \end{Bmatrix} \tag{3}$$

in which {k} is the vector of unknowns, $\{\phi_1\}$ and $\{\phi_2\}$ are the measured normal mode corresponding to the measured resonant frequencies Ω_1 and Ω_2 respectively. A similar linear relation results for the damping constants as shown in Ref. 2 with the addition of modal damping ratios obtained by means of the standard half-power bandwidth.

MODIFIED NEWTON-RAPHSON

Both the acceleration amplitude and phase angle are expanded about estimates to the parameters

$$\{y\} = \{g\} + \sum_{j=1}^{k} \left\{\frac{\partial g}{\partial \theta_1}\right\} \delta\theta_j + \{e\} \tag{4}$$

$$\{\theta^o\} = \{\theta\} + \{\delta\theta\} \tag{5}$$

in which $\{\theta^o\}$ is the initial estimate to the vector, $\{\delta\theta\}$ is the correction, and $\{\theta\}$ is the vector of estimates at the current iteration. {y} are the measured quantities associated with the {g} and {e} incorporates both measurement errors and those due to the first order Taylor series expansion.

These two sets of equations are then weighted appropriately to form a weighted sum of errors given by the relationship

$$S(\delta\theta) = \{e\}' [W_D] \{e\} + \{\delta\theta\}' [W_P] \{\delta\theta\} \qquad (6)$$

in which ' represents the transpose and $[W_D]$ and $[W_P]$ represent the respective weights (usually diagonal) of the data and estimates to the parameters respectively.

Optimization of this with respect to $\{\delta\theta\}$ yields a set of linear equations in the corrections to the parameters

$$[A] \{\delta\theta\} = \{b\} \qquad (7)$$

for which the elements may be found in Ref. 1. Furthermore, the variance-covariance matrix of the final estimates is the inverse of the matrix [A].

SENSITIVITY

The differentiation of the measurement quantities with respect to the parameters, needed in Eqn. 7, is easily obtained from the differentiation of the mass, stiffness, and damping matrices. For a complex x having a particular real, a, and imaginary portion b, and an amplitude, d and phase angle, ψ,

$$x = a + ib \qquad (8)$$

$$d = \sqrt{a^2 + b^2} \qquad (9)$$

$$\psi = \tan^{-1} (b/a) \qquad (10)$$

there results for θ a parameter

$$\frac{\partial x}{\partial \theta} = \frac{1}{d} R \left(x \frac{\partial x}{\partial \theta}\right) \qquad (11)$$

$$\frac{\partial \psi}{\partial \theta} = \frac{1}{d^2} I \left(x \frac{\partial x}{\partial \theta}\right) \qquad (12)$$

in which

$$\left\{\frac{\partial x}{\partial \theta}\right\} = - [H] \left[\frac{\partial Z}{\partial \theta}\right] [H] \{F\} \qquad (13)$$

and

$$\left[\frac{\partial Z}{\partial \theta}\right] = \left[\frac{\partial K}{\partial \theta}\right] + i\ 2\pi f \left[\frac{\partial C}{\partial \theta}\right] - 4\pi^2 f^2 \left[\frac{\partial M}{\partial \theta}\right] \qquad (14)$$

RESULTS

Two different models of damping were used in arriving at best sets. The first has already been discussed and corresponds to five unknown parameters of damping together with the five stiffness parameters. The masses or mass moment of inertias in the case of torsion were considered as known. In the second model, model damping ratios were used in the first two modes only in each direction which leads to dyadic damping formula of Ref. 3

$$[C] = 2[M] [\phi] [\zeta] [\Omega] [\phi]' [M] \qquad (15)$$

in which the higher modal damping ratios are assumed as zero, and $[\Omega]$ is the diagonal matrix of undamped natural frequencies associated with the normal modes $[\phi]$.

Table 1
Percentage of sum of errors

Direction	7 parameters No. Iter.	Amp.	Phase	10 parameters No. Iter.	Amp.	Phase
East-West	5	37	82	6	8	9
North-South	7	33	52	7	9	18
Torsion	9	19	16	7	9	14

The results of the identification of parameters are given in Table 1 and are expressed as a percentage of the errors squared of both the amplitude and phase angle measurements with respect to those calculated with the initial estimates. Also given are the number of iterations to converge at the results shown. Although it would appear that the model having 10 parameters is much better in terms of convergence, it should be emphasized that the initial damping parameters gave transfer functions which generally had very low damping and thus had large initial errors near resonance. Furthermore, some of the interstory damping constants were negative at convergence for this model.

CONCLUSION

A method is developed to estimate the dynamic parameters of a building based on sinusoidal measurements of the acceleration amplitude and corresponding phase angle with the excitation force. The results are as good as the model chosen and an estimate of the variance-covariance matrix is also given. More complete models of the stiffness, damping, or mass could be attempted, but caution is suggested when the measurements are done over a limited frequency range. This method could also be used to estimate three-dimensional models of buildings provided data in all three degrees of freedom at each level was available.

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