# DYNAMIC RESPONSE OF A REINFORCED CONCRETE COLUMN MODEL

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## SUMMARY

A numerical model has been developed for predicting the behaviour of members subjected to axial load and alternate bending. Comparison of model response with available experimental results shows that the model satisfactorily reproduces the important characteristics of the degrading strength/ /stiffness hysteretic behaviour of r.c. members.

Analysis of the way the model responds to cyclic deformation reveals the need for a better insight into the problem of defining the allowable ductility for the member whose idealization is far from being elastoplastic.

The seismic response of an r.c. column, the restoring force of which is represented by the proposed model, is presented. An extensive investigation is in progress to construct the inelastic 'degrading' spectra - similar to those for elastic SDOF system - for different mechanical and geometric parameters of the model; an example of these synthetic diagrams based on the results obtained so far is presented.

## INTRODUCTION

For many simple but frequently used structures the seismic response is closely related to the behaviour of the columns. Since the ability of structures to resist violent seismic loads dependes largerly on the possibility of their withstanding large deformations beyond the elastic range without collapsing, it is important to have available an accurate representation of the inelastic behaviour of the elements subjected to axial load and cyclic bending.

Furthermore even now in the design, account is taken implicitly of the strength reserves of the members beyond the yield level through the use of reduced spectra and explicit reference is made thereto in the recent proposal to introduce limit-state design in the seismic verification of structures too.

This explains the large number of experiments dedicated to the study of the behaviour of r.c. members. It has shown that this behaviour depends closely on the load path; its main characteristics are a continuous variation of stiffness and a decrease in strength, corresponding to a certain amount of deterioration.

In this contest models have been developed which describe the behaviour of r.c. members subjected to repeated loads. In some of these the force-displacement relationship is modelled through analytical laws and deterioration is controlled by certain parameters which are empirically related to the physical causes of degradation; in others the f-$\delta$ relationship is the result of the analysis of a section in which the time evolution of each layer is followed, on the basis of laws which appropriately describe the behaviour of steel and concrete under cyclic strains.

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The model proposed here belongs to the latter category. Through a qualitative analysis and a comparison with experimental results it has been shown that the model is able to reproduce the main features of the hysteretic behaviour of r.c. members.

The model is used to describe the restoring force of an r.c. column; thus it has been possible to utilize this numerical tool for a sufficiently extensive, yet economical, investigation of a simple but realistic deteriorating structure under seismic action.

## MECHANICAL MODEL

A model has been developed which will represent the behaviour of a r.c. beam element subjected to compression and alternating bending.

Since attention is focused only on critical regions, a beam element of limited dimension is considered, which has constant generalized stresses (M,N) and strains ($u/l, \varphi/l$) as indicated in Fig. 1a. According to these assumptions the force-displacement relationship of the element can be obtained from the analysis of the behaviour of a single section.

For the purpose of numerical analysis the beam section is discretized into concrete and steel layers; as shown in Fig. 1b. The numerical procedure follows an incremental formulation based on the displacement method. As for the constitutive relationships of the materials, sufficiently accurate laws are assumed.

The uniaxial stress-strain relationship for concrete is described in Figs. 2 and 3. The main features can be summarized as follows:

- the envelope curve is that suggested in ref.1 .
  Three branches define the envelope curve: a parabolic ascending branch, a linear falling branch and a horizontal branch,which last indicates a residual stress which the confined concrete can offer also for high values of strain. For unconfined concrete the curve is interrupted either at the strain $\varepsilon_{br}$ (a) or at the intersection of the falling linear branch with the zero stress axis (b).
- the cyclic behaviour is described by two distinct unloading and reloading stiffness moduli, which are both a decreasing function of the maximum plastic strain.

The uniaxial stress-strain relationship for the reinforcing steel is described in Fig. 4 where the analytical law is reported; it furnishes an accurate representation of the reversal curves through the function R. A set of additional rules is introduced for the application of the hysteresis law when a general strain history is given, in order to avoid the necessity of storing the parameters of all the reversal curves. Three fundamental curves which represent particular bounds during cyclic behaviour have been selected: 'skeleton curve' (a), 'upper curve' (b) and 'lower curve' (c). More detail can be found in [5].

## MODEL BEHAVIOUR DESCRIPTION

Though the proposed model is essentially concerned with behaviour under repeated loads, an initial qualitative analysis is performed on the basis of the response to an increasing assigned deformation. From this test -which is certainly very simple- the first load curve is obtained, on which it is readily possible to observe the influence of the different parameters that define the model; the shape of that curve becomes a distinguishing feature of the model and thus also permits prediction of the hysteretic behaviour. Fig. 5

shows the first load curves for the variations in value of the axial load (a), the slope of the second branch of the confined concrete $\sigma$-$\varepsilon$ relationship (b) and of its ultimate strain value $\varepsilon_{bu}$ (c). As the values of these parameters increase the behaviour of the model becomes more degrading; in this test the deterioration is evidenced by a larger or smaller decrease of restoring moment with the increase of the imposed rotation as a result of collapse in some concrete layers.

On the basis of the numerical tests carried out so far it ensues that the most important parameters influencing model behaviour are: the slope of falling branch of the envelope curve of concrete, the value of the average compressive stess, the 'nominal' value of the ultimate strain of concrete. Depending on the values of these parameters the hysteretic behaviour of the model may or may not be stable. An example is given in Fig. 8 which illustrates both the M-$\varphi$ and the u-$\varphi$ diagram for a more complete description of the response; on the u-$\varphi$ diagram the deterioration of the member is revealed by a continuous increase in its axial deformability.

As regards the hysteretic behaviour of the model, a comparison with experimental tests is shown in Figs. 6 and 7. Four different cases are considered with different values of the maximum imposed deformation, axial load and stirrup spacing. Good agreement comes out both from a qualitative and a quantitative point of view. It can be noticed that the main aspects of the experimentally observed behaviour are reproduced: a) the reduction of stiffness with increasing amplitude of inelastic deformation; b) the reduction of strength at fixed amplitude of deformation; c) the loop's shape and the predicted numbers of cycles (in these tests it is assumed $\varepsilon_{br}$ = 0.020).

Observation of the model response to a steady increase in the imposed rotation and to a cyclic history (Fig. 5 and 8) reveals that for r.c.-members it is strongly difficult to establish an ultimate deformation (ductility) in order to define collapse, because the response is closely dependent on the load history.

The first load curve of a sufficiently strong member is shown in Fig.9a. Numerical tests are run with imposed cyclic rotations having different maximum values; eight cycles are performed without arriving at collapse in the case of $\varphi_{max}$ equal to $\varphi_1$ and $\varphi_2$, but with a marked reduction in maximum force, while four and two cycles are possible prior to collapse when $\varphi_{max}$ is equal to $\varphi_3$ and $\varphi_4$. This signifies, for example, that the ductility values corresponding to $\varphi_3$ and $\varphi_4$ cannot be considered as possible values, even if they are attached on the first load curve and the corresponding reduction in restoring moment does not seen to be very great.

## SEISMIC RESPONSE OF AN R.C. COLUMN MODEL

The proposed model has been used to describe the restoring force-displacement relationship f(x) for an r.c. column; this relationship is derived from the M-$\varphi$ law considering the column deformation as concentrated at the column end (Fig. $1_c$). The equation of motion can be written:

$$\ddot{x} + 2\nu\omega_0\dot{x} + f(x)/m - \beta g/H = \ddot{x}_G \qquad (1)$$

where $\nu$ is the fraction of linear viscous damping, $\omega_0$ the 'nominal' circular frequency of the structural model, f(x) the restoring force, $\beta$ a coefficient which accounts for geometric effects. In order to test the dynamic model two recorded accelerograms have been considered: Forgaria-Cornino 15.9.76, NS (Friuli) (Exc. 1) and Taft 21.07.52, N 69 W (Exc. 2).

The response of some of these first tests is presented here. In Fig. 10 three structural cases are considered in which column height and depth and the percentage of reinforcing steel, are held constant, while the mass (and consequently the average compressive stress $\sigma$) and the slope $E_d/E_i$ are changed. The typical response is described through the time history of the displacement and the cyclic diagram of the restoring force. The two responses (a) and (b) are substantially different and in particular collapse occurs in the latter while in the former stable hysteretic loops are obtained. In case (c) a high degree of deterioration is reached even though collapse does not occur.

In Fig. 11 the numerical response of the structural model under exc. 2 is presented. The 2nd order effect is considered, or not considered, respectively in case (d) and (c). After an almost identical initial behaviour case (d) is shown to diverge rapidly, up to collapse.

This example shows that the combination of phisical deterioration and geometric effect is very damaging. It is also interesting to note that this effect does not emerge clearly from the hysteretic diagram (Fig. 11a,b) where for the same imposed deformation all that is apparent is a greater decrease of the internal force.

In order to obtain synthetic diagrams of more general application the equation of motion can be modified in the following manner

$$\ddot{z} + 2\nu\omega_0\dot{z} + \omega_0^2\, f(zx_y)/f_y - \beta\omega_0^2 z = \omega_0^2\, a(t)/\lambda \qquad (2)$$

where $x_y$ is the yield displacement of the column, $z = x/x_y$ is a dimensionless quantity, and $\lambda$ is the design coefficient defined as the ratio between the $f_y$ and the force $ma_{max}$ related to the peak acceleration $a_{max}$ of the actual excitation.

Equation (2) is solved by step-by-step numerical integration for various values of the natural frequency of the structure; the ductility request of the column is furnished directly by the maximum value of z. The results of the calculation are illustrated in Fig. 12 which also shows the first load curve of the model adopted. The qualitative trend is fairly like that for the analogous diagrams of the elastoplastic model, though it is more irregular here; it is evident that the elastoplastic model does not give a cautions result, especially for the low $\lambda$ values in the low-period range.

This prompts two comments: a) at all design coefficient values, the curves for this column are limited to a $\mu$ value of between 12 and 14, and the systems for which the ductility request exceeded this value, arrived at collapse before the end of the test with the $\mu$ values indicated in the figure by means at a dotted line; b) for a system of given period T, with small variations of $\lambda$ there are big variations of $\mu$.

On the basis of the results referred to earlier it has been possible to plot the Fig. 13, diagrams which are analogous to the elastic or, rather, the elastoplastic spectra. For a model of given hysteretic characteristics, each curve illustrates the relationship $\lambda$-T for a maximum ductility value $\mu$.

A more extensive investigation is now under way in which a greater number of accelerograms is being considered to average out the effect of the excitation shape and variations of the most important parameters that influence the behaviour of the model are being examined; in the synthetic diagrams of the results, second order effects will be also taken into account.

## CONCLUSIONS

A model representing the dynamic behaviour of a simple r.c. structural member is presented; the behaviour is described by analysis of the static re sponse under assigned deformation (steadily increasing and cyclic) and by analysis of the dynamic response to seismic excitation. Much more work must be done but some concluding remarks can be made on the basis of the results obtained so far. The proposed model is capable of accurately representing the behaviour of r.c. elements, while at the same time beeing relatively simple to use. This simplicity stems not least from the limited number of easily controlled parameters, having a clear physical meaning, on which model behaviour depends. Thus the model constitutes a useful investigative tool to furnish a better insight into the meaning of maximum allowable ductility of r.c. members, whose restoring force-displacement relationship is markedly history-dependent.

Moreover from the seismic response of a column, based on this model, it is possible to construct inelastic spectra -of the type presented here- which take into account the phenomenon of physical deterioration of r.c. structures, with the combined deterioration of the second order geometric effect.

This analysis -static and dynamic- should help achieve a better understanding of the behaviour of deteriorating structures and a more correct evalutation of their seismic reliability.

ACKNOWLEDGEMENT

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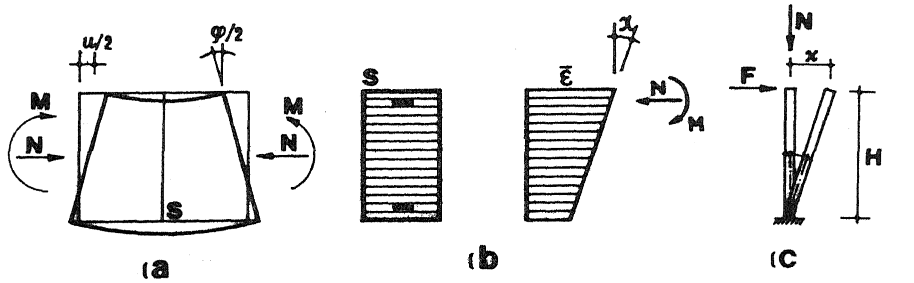
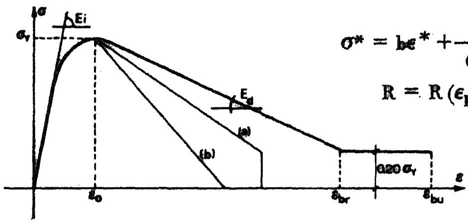



Fig. 1



$$\sigma^* = b\epsilon^* + \frac{(1-b)\,\epsilon^*}{(1+\epsilon^{*R})^{1/R}}$$

$$R = R(\epsilon_{p_{max}})$$

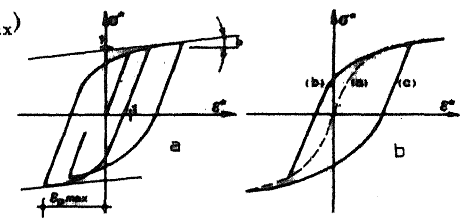
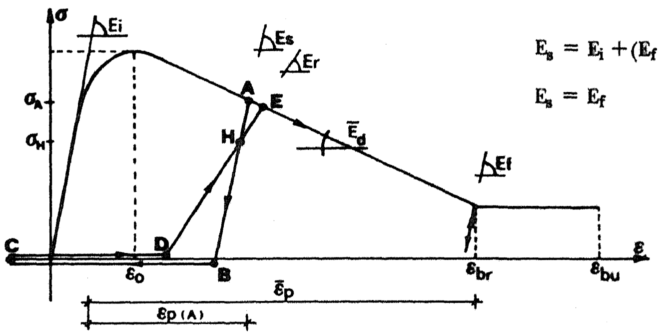



Fig. 2

Fig.4




$$E_s = E_i + (E_f - E_i)\frac{\epsilon_p}{\bar{\epsilon}_p} \qquad \frac{\epsilon_D}{2} \leq \epsilon_p \leq \bar{\epsilon}_p$$

$$E_s = E_f \qquad \epsilon_p > \bar{\epsilon}_p$$

$$\sigma_H = \beta\,\sigma_A$$

$$E_r = \alpha(\epsilon_p)\,E_s$$

$$\alpha(\epsilon_p) = \alpha_i + (\alpha_f - \alpha_i)\frac{\epsilon_p}{\bar{\epsilon}_p}$$

Fig.3

$n = N/bh\,\sigma_Y$

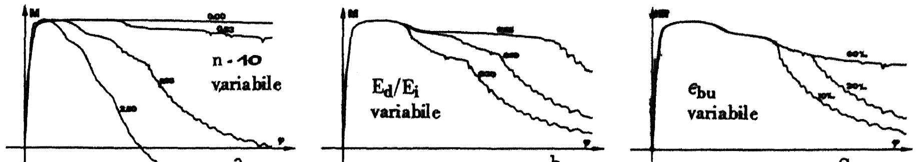



Fig. 5

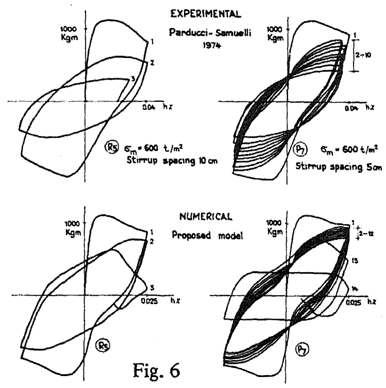



Fig. 6

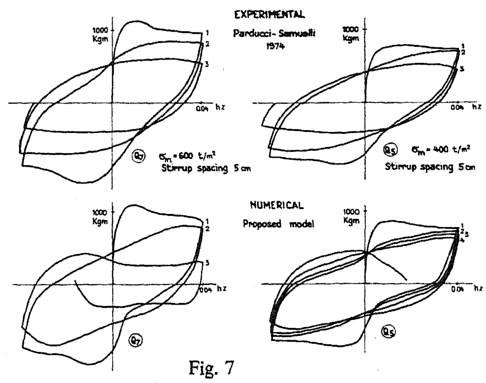



Fig. 7

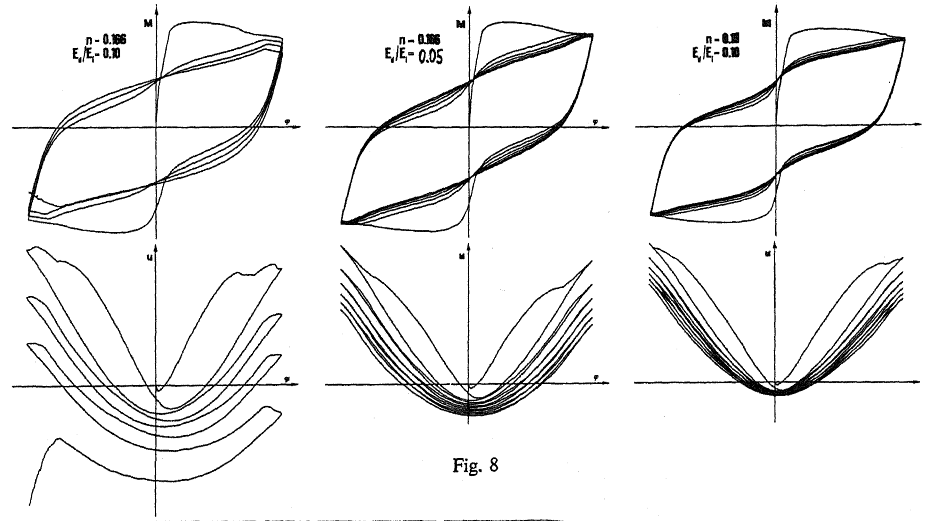



Fig. 8

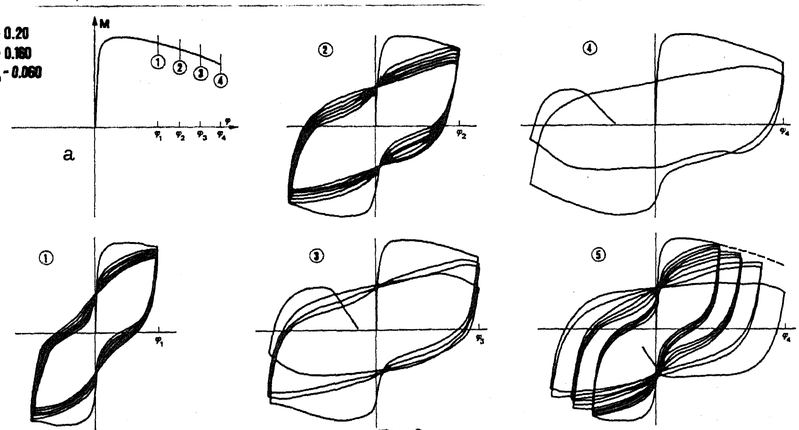



Fig. 9

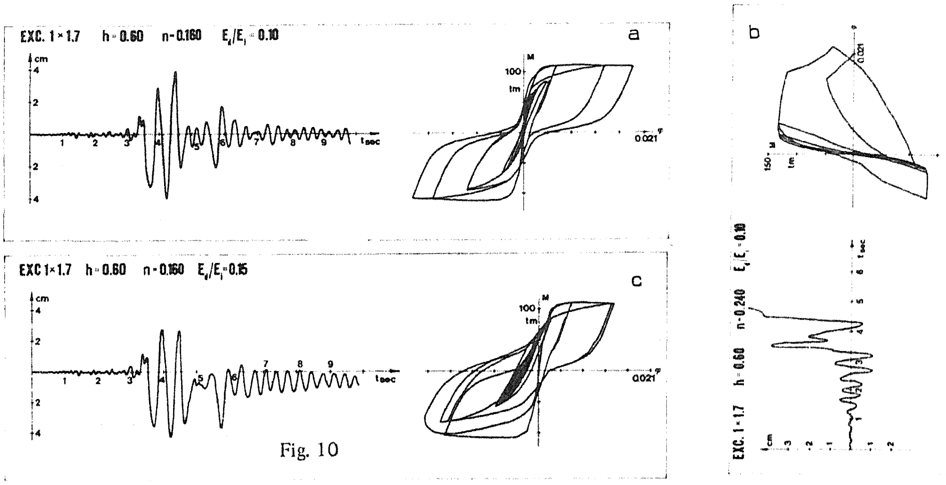



Fig. 10

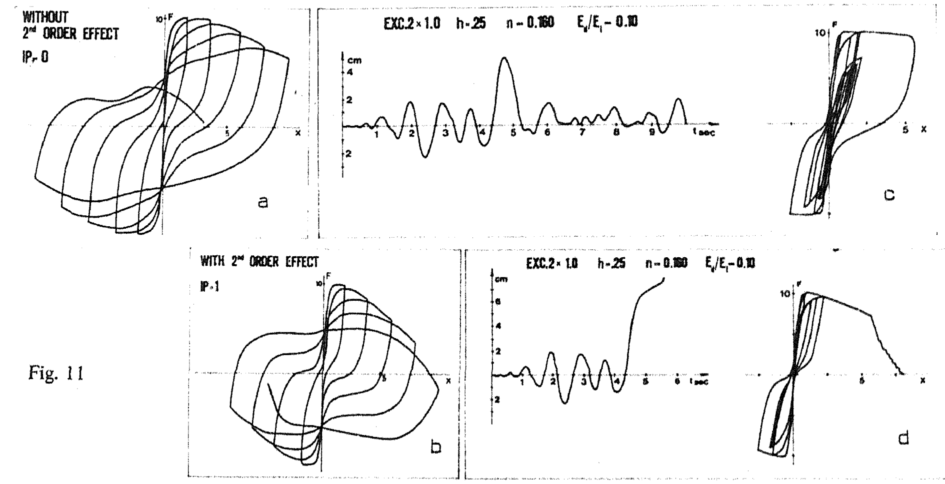



Fig. 11

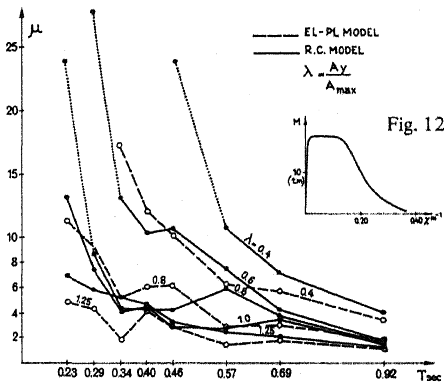



Fig. 12

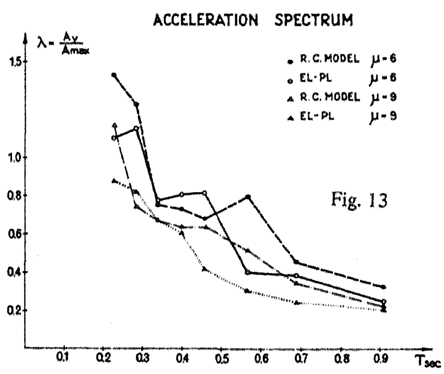



Fig. 13