INELASTIC BEHAVIOR OF WIDE-FLANGE BEAM-COLUMNS UNDER CONSTANT VERTICAL AND TWO-DIMENSIONAL ALTERNATING HORIZONTAL LOADS

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Summary Wide-flange steel beam-columns are tested under constant vertical and two-dimensional monotonic or cyclic horizontal loads. Experimental behavior is investigated and compared with results of elasto-plastic and rigid-plastic analyses. It is shown that results of elasto-plastic analysis agree with experimental values nearly up to the maximum load. The conventional strength formula is shown to be too conservative.

INTRODUCTION

In conjunction with the seismic design, a number of investigations have been performed to make clear the cyclic behavior of structural members and frames subjected to constant vertical and alternating horizontal loads. Most of them are limited to the investigation on the in-plane behavior of structures. However, the real behavior of structures under earthquake excitation is presumably affected by the horizontal displacements in two directions and torsional deformation. Few researches have been done in this regard. It is needed to investigate the three-dimensional inelastic behavior, and to grasp new problems which do not appear in the plane frames. The paper presents the results of experimental and theoretical studies on the behavior of wide-flange steel beam-columns under constant vertical and two-dimensional monotonic or alternating horizontal loads.

TEST

Test Specimen and Loading Apparatus Test specimen is a cantilever column of a rolled H-shaped cross section H-100x100x6x8 as shown in Fig.1. The material is mild steel(SS41, Japanese Industrial Standards), having average yield stress equal to $2.87t/cm^2$, and average ultimate strength equal to $4.27t/cm^2$, both obtained from tensile tests of 3 pieces.

Loading apparatus shown in Fig.2 is designed to satisfy the boundary conditions that the following displacements and rotations at the top of the specimen must be free; relative sway displacements in two directions, flexural rotation in two directions and torsional rotation. First, constant vertical load P is applied on the specimen ① by an Amsler type testing machine ②, and thus the top of the specimen is not allowed to move horizontally. Relative horizontal sway due to the application of horizontal load H by a double acting hydraulic jack ③ actually occurs at the bottom of the specimen, with the movement of a loading frame ④, which is allowed by two-directional rollers ⑤ and ⑥. Because of rollers ⑦, the original position and direction of the hydraulic jack are kept unchanged, regardless of the movement of the loading frame. Three-directional rotation at the top of the specimen becomes free by a two-directional hinge ⑧ and a radial bearing ⑨, both built in a loading plate ⑩. Torsional rotation of the specimen is not restricted by the testing machine because of a thrust bearing ⑪. Supporting frame ⑫ is composed of straight bars with universal joints at

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ends and couplers which connect bars without friction in the longitudinal direction. This supporting frame is connected to the loading plate ⑩ and the loading frame ④, and has such a mechanism that the loading plate is kept horizontally, but can move downward freely following the shortening of the specimen.

Test Program   Tests are performed under the combination of two parameters; vertical load ratio $n=P/P_y$(=0.1 or 0.3) and the direction of the horizontal load θ (=0°, 30°, 60°, or 90°), where P_y is the yield load. The loading program employed for the horizontal load is as follows: Specimens in Series I are subjected to basically monotonic horizontal load, one cycle of reversed loading applied at the large amplitude of plastic deflection. Specimens in Series II are tested under cyclic loading, where the amplitude of displacement $\bar{u}$ (see Fig.1) is increased by 1% of the column height ℓ in a step-wise manner every four cycles of loading completed, until the specimen becomes unable to sustain the vertical load.

THEORETICAL ANALYSIS

Two types of theoretical analysis are performed to obtain the load-deflection curves of cantilever beam-columns under monotonic biaxial bending; elasto-plastic analysis taking the spread of plastic zone into account, and rigid-plastic analysis based on a plastic hinge method.

Elasto-Plastic Analysis   Governing differential equations of equilibrium of elasto-plastic beam-columns under biaxial bending can be written as follows[1]:

$$\begin{aligned} &N'=0 \\ &(M_y+M_x\phi)''-(Nu')'=0 \\ &(M_x-M_y\phi)''+(Nv')'=0 \\ &M_w''-T_s'-(K\phi')'+M_xu''+M_yv''=0 \end{aligned} \qquad (1)$$

where N denotes axial force, M_x and M_y bending moments about x- and y-axes, respectively, T_s torsional moment due to St.Venant's torsion, M_w bi-moment, K stress resultant due to inclination of fibers of the twisted column, u and v displacements of centroid in x- and y-directions, respectively, ϕ angle of twist, and prime denotes the differentiation with respect to z taken along the longitudinal axis of the beam-column.

Equation (1) is solved by the numerical integration under the prescribed boundary conditions, based on the bi-linear stress-strain relation shown in Fig.3. In the analysis the following assumptions are made: 1) Beam-column is free from initial imperfections, such as residual stress and crookedness. 2) Effects of strain reversal and shear deformation are negligible. 3) Yielding is governed by normal stresses only. 4) Constant normal stress is distributed over a subdivided segment of the cross section. 5) Warping function is unchanged, regardless of the spread of the plastic zone. 6) St.Venant's torsion constant is reduced in proportion to the ratio of the area remaining elastic to the total area of the cross section.

As it can be seen, the stress resultants in Eq.(1) depend on the strain distribution which must satisfy the strain-displacement relations, and thus the numerical computation is a non-linear, iterative task.

Rigid-Plastic Analysis In addition to the elasto-plastic analysis, rigid plastic analysis is performed to investigate the state of collapse mechanism, by assuming a generalized plastic hinge forming at the base of the beam-column. The load-deflection relation is numerically obtained from the plastic condition and associated flow rule, the former being formulated by Heymann[2] for a wide-flange section of rigid-perfectly plastic material.

RESULTS AND DISCUSSIONS

Monotonic Behavior Figures 4 and 5 show load-deflection relations under the monotonic loading. In Fig.4, experimental results are shown by solid lines with circles, and results of the elasto-plastic analysis and the rigid-plastic mechanism curves are shown by solid and dash-dotted lines, respectively. Definitions of the displacements u and v are given in Fig.1.

Figures 4(c) and 4(d) show the results under n equal to 0.1, and in the rest of Fig.4, the value of n is kept constant to be 0.3 and the value of θ changes from 0° to 90°. Note that Figs.4(a) and 4(b) are for the uniaxial bending cases. All experimental curves are compared in Fig.5. It is generally concluded from the experimental results that the maximum load carrying capacity decreases as the value of n or θ becomes large. In all cases, H-u curve becomes approximately linear after the maximum load attained, with the great increase in the displacement u which corresponds to the weak-axis bending. Similar behavior is observed in H-v curves for the uniaxial bending and for the biaxial bending with n equal to 0.1. However, in H-v curves of specimens subjected to biaxial bending with n equal to 0.3, it seemes that there exists a limiting value which the displacement v cannot exceed. This phenomenon clearly appears on the mechanism curves. In fact, the specimen becomes unable to sustain the vertical load during the test, in the vicinity of the limiting point. As to the theoretical results, experimental behavior can be predicted to some degree by the elasto-plastic analysis nearly up to the maximum load, and the mechanism curves estimate the general tendency after the maxmum load attained.

Strain distributions at the states of the maximum load and the unloading are shown in Fig.6, where dash-dotted line drawn connecting zero strain points of two flanges approximates the neutral axis. Among specimens subjected to biaxial bending, the neutral axis hardly changes the position or rotates between two loading states in case of n equal to 0.1, while it moves and rotates in specimens of n equal to 0.3. Particularly, in case of θ equal to 60° the neutral axis almost coincides with the y-axis at the state of unloading, and it seems as if the specimen is subjected to pure weak-axis bending. This is the reason why H-v curve shows a limit in the value of v mentioned above.

Cyclic Behavior Cyclic load-deflection curves of Series II are shown by solid lines in Fig.7, together with the results of Series I by solid lines with circles, where definitions of $\bar{u}$ and $\bar{v}$ are given in Fig.1. The effects of the vertical load ratio n and the angle θ are quite apparent. As to the cyclic behavior under the biaxial bending, H-$\bar{v}$ loops of the specimen of n equal to 0.1 are stable and symmetric during the test. On the other hand, in specimens of n equal to 0.3, H-$\bar{v}$ loops are approximately symmetric about the origin when the displacement amplitude of $\bar{u}/\ell$ is 1%, but they start to drift away from the origin in one direction, when the amplitude of $\bar{u}/\ell$ becomes 2%. When the amplitude of $\bar{u}/\ell$ is increased to 3%, the displacement $\bar{v}$ increased rapidly in the first stage of loading, and the vertical load could not be sustained. Similar characteristics are observed in case of θ equal to 60°, but they seem to appear rather slowly, compared with the case of θ equal to 30°. In the specimen of θ equal to 0°, the situation becomes quite different, the displacement $\bar{v}$ keeping on increasing after the combined local and lateral bucklings occur. It may be said that H-$\bar{v}$ loops become more stable as the angle θ becomes large.

Relations between the number of loading cycle and the strain at the centroid of the cross section are shown in Fig.8. It seems that H-$\bar{v}$ loops start to drift away when the strains at the centroid increase remarkably.

Strength formula Figure 9 and Table 1 show the experimental and theoretical results of the maximum horizontal loads compared with the values computed by a strength formula given in Ref.[3],

$$\frac{N}{N_{cr}} + \frac{C_x M_x}{(1-\frac{N}{N_{ex}})M_{cr}} + \frac{C_y M_y}{(1-\frac{N}{N_{ey}})M_{py}} = 1.0 \qquad (2)$$

in which N denotes axial load, Ncr critical load, Ne Euler buckling load, M applied moment, Mcr lateral buckling moment, Mp full plastic moment, C factor computed in relation to moment gradient, and subscripts x and y indicate quantities computed about x- and y-axes, respectively. The values of Ncr and Mcr are computed from the design formula [3], taking the buckling length equal to 2ℓ. The ratio of the experimental maximum load to the stregth computed by the formula above ranges from 1.5 to 2.2. It seems in Table 1 that the conservative discrepancy in the case of bi-axial bending is more pronounced with the increase in the vertical load.

CONCLUSIONS

(1) For beam-columns subjected to monotonic biaxial bending, it seemes that the displacement v is limited to a certain value, different from in case of uniaxial bending. This phenomenon also appears on the mechanism curves, which estimate the general tendency in the experimental behavior. To some degree, the results of the elasto-plastic analysis agree with experimental values up to the maximum load.

(2) As to the cyclic behavior, the effect of the vertical load ratio n is quite apparent. In specimens of n equal to 0.3. H-$\bar{v}$ loops are approximately symmetric about the origin in the region where the displacement amplitude of $\bar{u}/\ell$ is small, and they start to drift away from the original point in one direction, when it becomes 2%. There is no drift observed during the test in the specimen of n equal to 0.1. Such behavior must be taken into consideration when the dynamic analysis of space frames is performed.

(3) The conventional strength formula used in the plastic design of steel structures is conservative, and needed to be improved.

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[2] Heymann, J. : PLASTIC DESIGN OF FRAMES, Cambridge University Press, 1971, Vol.2, pp. 56.
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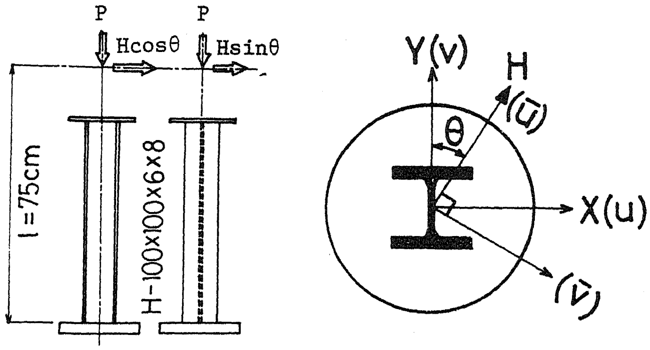


Fig.1 Specimen and Coordinates

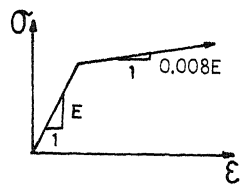


Fig.3 Stress-Strain Relation

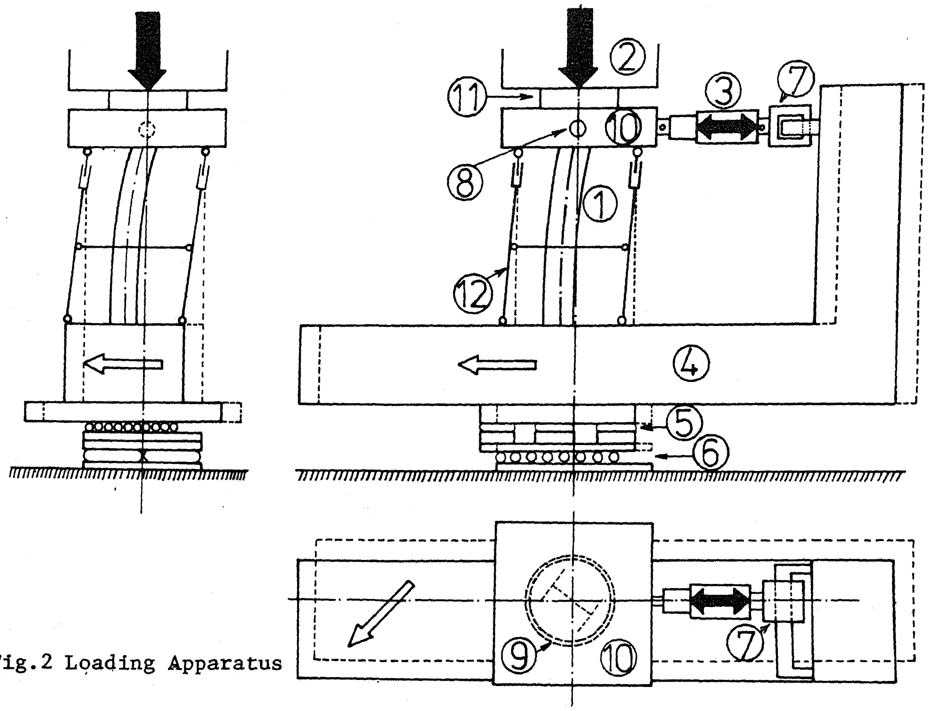


Fig.2 Loading Apparatus

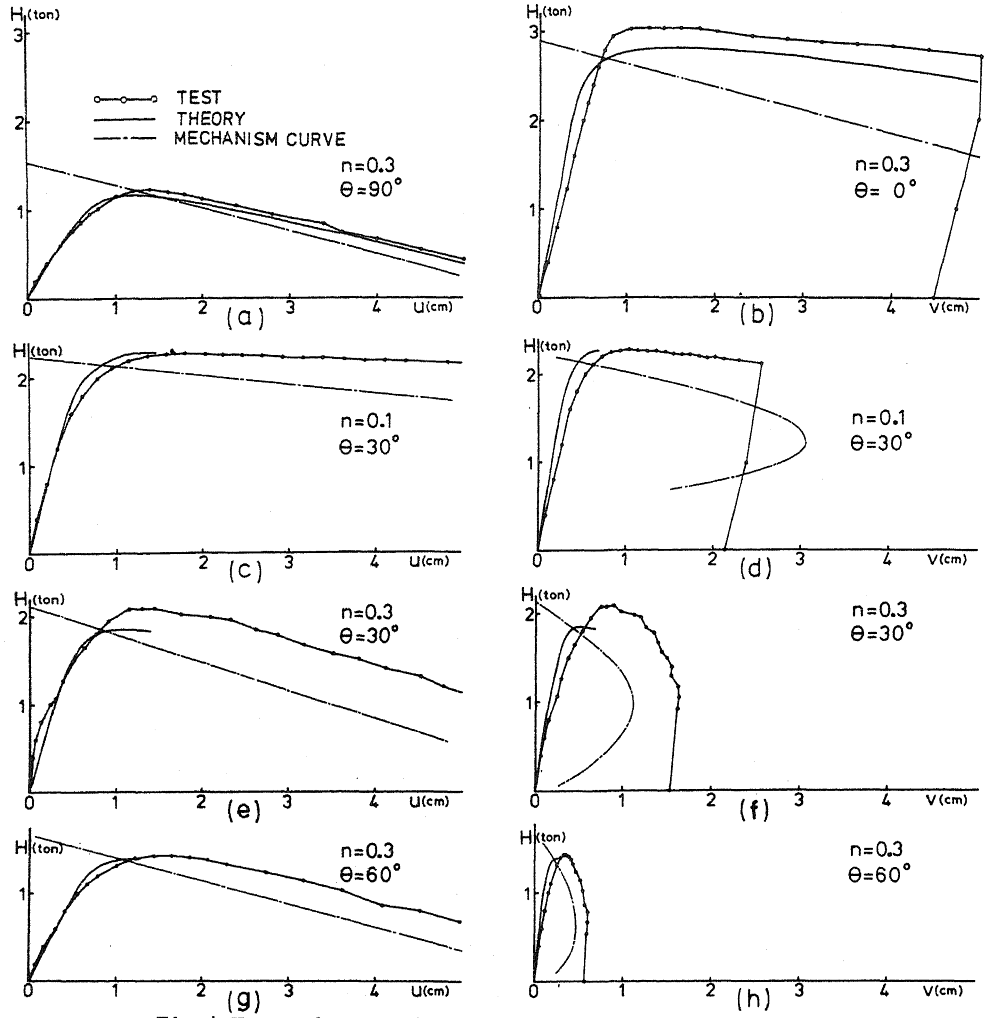


Fig.4 H-u and H-v relations under Monotonic Loading

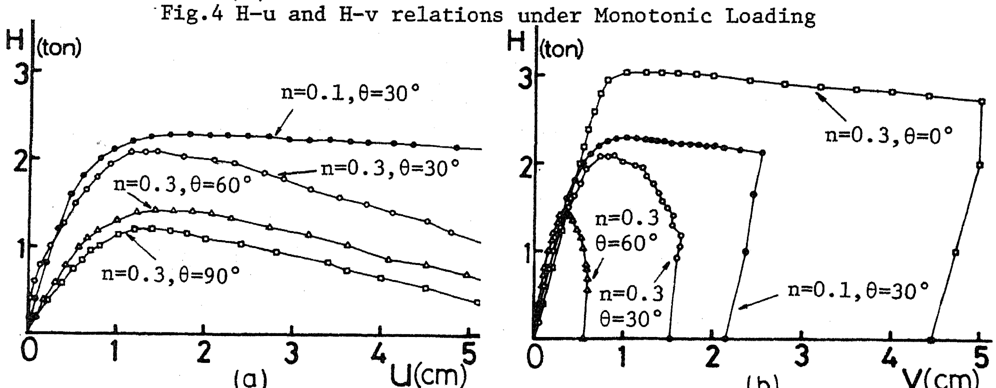


Fig.5 H-u and H-v Relations

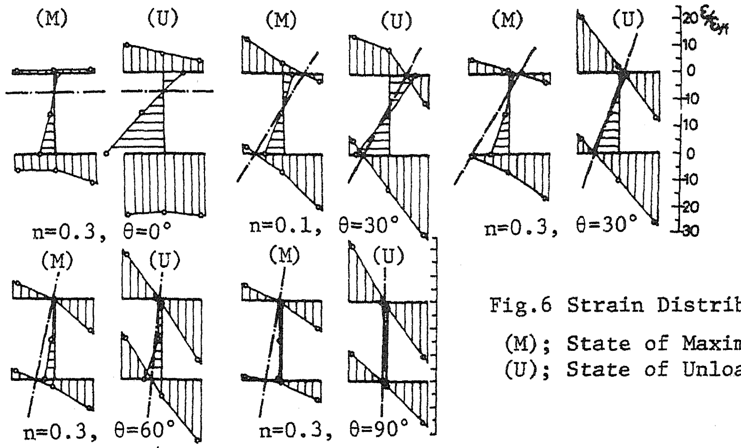


Fig.6 Strain Distribution

(M); State of Maximum Loading

(U); State of Unloading

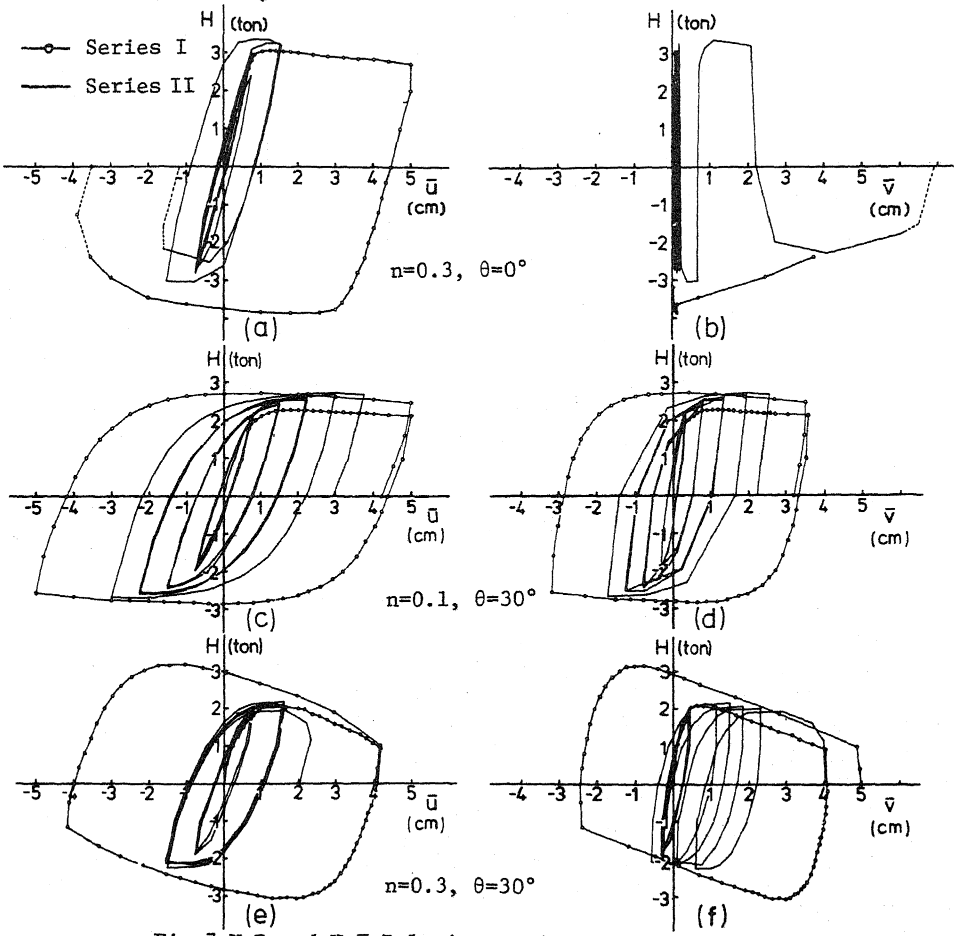
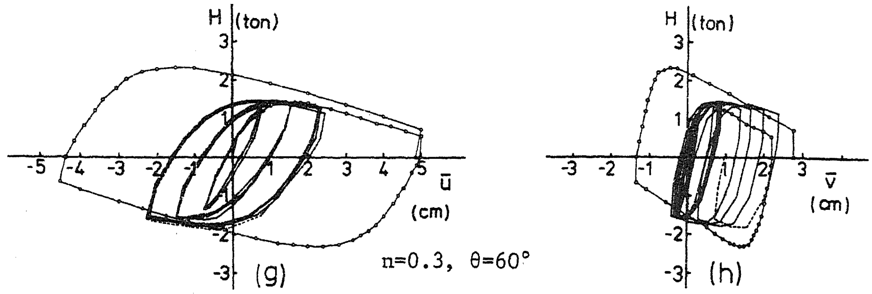


Fig.7 H-ū and H-v̄ Relations under Cyclic Loading



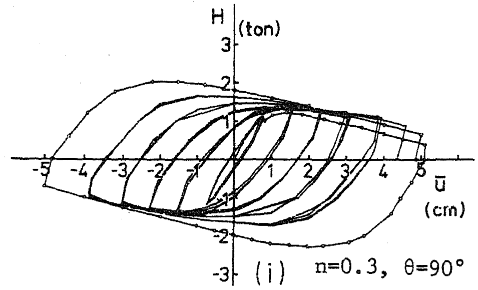


Fig.7 H-ū and H-v̄ Relations under Cyclic Loading (continued)

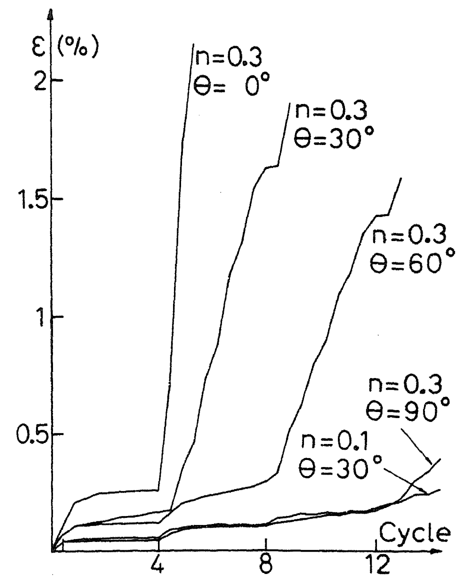


Fig.8 Strain-Cycle Relations between Strain and Number of Loading Cycles

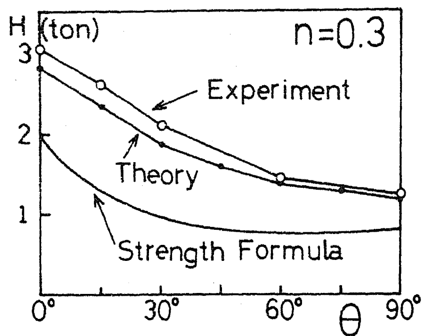


Fig.9 H-θ Relations

Table 1 Comparison of Maximum Loads (Unit:ton)

n	θ	(1)	(2)	(3)	(1)/(2)	(2)/(3)	(1)/(3)
0.1	30°	2.28	2.28	1.46	1.00	1.56	1.56
	60°	-	1.72	1.18	-	1.46	-
0.3	0°	3.04	2.81	2.00	1.08	1.41	1.52
	15°	2.61	2.33	1.24	1.12	1.88	1.99
	30°	2.09	1.85	0.94	1.13	1.96	2.20
	45°	-	1.57	0.81	-	1.93	-
	60°	1.43	1.39	0.75	1.03	1.85	1.89
	75°	-	1.29	0.75	-	1.72	-
	90°	1.23	1.16	0.80	1.06	1.45	1.53
0.6	30°	-	1.09	0.30	-	3.60	-
	60°	-	0.72	0.23	-	3.11	-

(1) Experiment,
(2) Elasto-Plastic Analysis, and
(3) Strength Formula