

# SEISMIC BEHAVIOR OF INELASTIC MEMBERS OF BRACED FRAME STRUCTURE

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## SUMMARY

A method of nonlinear earthquake response analysis of braced frame structures having elasto-plastic joints at the middle of X-type braces and at both ends of frame members is proposed in order to clarify the effects of the braces on the aseismic safety of local joints as well as the whole structure. From the results of the root mean square responses of 7-story, single-bay braced frame structures due to random excitation, an optimum distribution of dynamic characteristics of the structural members is presented. The results of elasto-plastic local response analyses of such a model subjected to artificial earthquakes, considering the post-buckling behavior of braces, make clear the effectiveness of adequate aseismic elements to optimize the safety distribution of structural members.

## INTRODUCTION

It is well known that building structures with effective aseismic elements such as shear walls or braces are able to resist strong earthquakes. Elasto-plastic earthquake response analysis of structures with aseismic elements is very important for the aseismic design of the structures and the structural members in seismically affected regions. Investigations on the nonlinear behavior of the braced frame structures have been carried out, but the effects on the aseismic safety of the structural members due to post-buckling behavior of the braces has not yet been sufficiently clarified.

In this paper, a formulation of hysteretic characteristics of the X-type braces, supported by two hinges, which has an elasto-plastic joint at the middle of the bar, is summarized and typical hysteretic behavior of the brace, computed here, is compared with an experimental result. Then, a method of the earthquake response analysis of the braced frame structures with elasto-plastic joints is formulated, extending author's earlier method<sup>1,2)</sup> to the method containing post-buckling behavior of the braces. Elastic response characteristics of braced frame structures such as optimum distribution of the dynamic characteristics of the structural members and the effect of the braces on the local ductility ratio responses, are shown by applying the above-mentioned method to the stationally random response analysis. From the results of the nonlinear response analysis of such structures due to filtered artificial earthquakes, the aseismic safety of column members when the braces behave inelastically, is mainly discussed.

## ANALYTICAL REPRESENTATION

### OF RESTORING FORCE CHARACTERISTICS OF A BRACE

The relation between the bending moment and the rotation of a member under the axial force is generally given by the formulae of the slope deflection method considering the axial force. As shown in "Fig.1", deviding a brace into two parts and introducing the relative rotation of the elasto-

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plastic joint, the axial deformation of a pin-ended brace,  $u_b$ , can be represented<sup>3)</sup> by the three kinds of deformations in the nondimensional incremental form; the axial deformation of elastic member,  $u_m$ , the flexural deformation,  $u_f$ , and the elasto-plastic deformation of the joint,  $u_j$ .

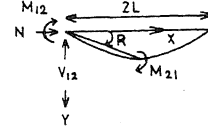


Fig.1 Physical Model of Brace

$$\dot{u}_b = \frac{\dot{U}_b}{L\Delta} = -2(\dot{u}_m + \dot{u}_f + \dot{u}_j) \quad (1)$$

$$\dot{u}_f = \frac{\dot{U}_f}{L\Delta} = -\frac{\Delta}{2} \left[ r \left( \frac{Z^2}{\sinh^2 Z} + \frac{Z \cosh Z}{\sinh Z} \right) \dot{r} + r^2 \left( \frac{Z^2}{2 \sinh^2 Z} + \frac{Z \cosh Z}{2 \sinh Z} - \frac{Z^3 \cosh Z}{\sinh^3 Z} \right) \dot{Z} \right] \quad (2)$$

$$\dot{u}_m = \frac{\dot{U}_m}{L\Delta} = \frac{L}{2\Delta L} \cdot \frac{H}{L} \cdot \dot{n}_b \quad (3)$$

$$\dot{u}_j = \frac{\dot{U}_j}{L\Delta} = \begin{cases} \frac{1}{2} \cdot \frac{H}{L} \cdot \dot{n}_b & \text{for elastic range} \\ \phi(\beta_b, n_b, \dot{\beta}_b, \dot{n}_b, \dot{\theta}) & \text{for plastic range} \end{cases} \quad (4)$$

in which

$$\dot{\beta}_b = \frac{\dot{M}}{M_y} = \frac{\Delta}{8} \cdot \lambda^2 \cdot \frac{2H}{L} (n_b \cdot \dot{r} + r \cdot \dot{n}_b) \quad (5)$$

$$\dot{\theta} = \frac{\dot{\Theta}}{\Delta} = - \left[ Z \coth Z \cdot \dot{r} + r \left( Z \coth Z - \frac{Z^2}{\sinh^2 Z} \right) \dot{Z} \right] \quad (6)$$

and

$$\dot{n}_b = \frac{\dot{N}_b}{N_y} \quad (7)$$

are increments of bending moment, relative rotation and axial force of the joint,  $L$ ,  $\Delta L$  and  $H$  are half lengths of the member and joint, and the depth of the member,  $\lambda$  is the slenderness ratio,  $\Delta$ ,  $\Delta_n$  are the elastic limit relative rotation and the elastic limit axial deformation of the joint,  $Z^2 = \sigma_y \cdot n_b \cdot \lambda^2 / 4E$ , and the function  $\phi(\beta_b, n_b, \dot{\beta}_b, \dot{n}_b, \dot{\theta})$  is determined by the yield function and the flow rule.

$$\begin{Bmatrix} \dot{\beta}_b \\ \dot{n}_b \end{Bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \begin{Bmatrix} \dot{\theta} \\ \dot{u}_j \end{Bmatrix} \quad (8)$$

$$\rho_{11} = 1 - \frac{\left( \frac{\partial \phi}{\partial \beta_b} \right)^2}{\left( \frac{\partial \phi}{\partial \beta_b} \right)^2 + \left( \frac{\partial \phi}{\partial n_b} \right)^2} \quad \rho_{12} = \rho_{21} = - \frac{\left( \frac{\partial \phi}{\partial \beta_b} \right) \left( \frac{\partial \phi}{\partial n_b} \right)}{\left( \frac{\partial \phi}{\partial \beta_b} \right)^2 + \left( \frac{\partial \phi}{\partial n_b} \right)^2} \quad \rho_{22} = 1 - \frac{\left( \frac{\partial \phi}{\partial n_b} \right)^2}{\left( \frac{\partial \phi}{\partial \beta_b} \right)^2 + \left( \frac{\partial \phi}{\partial n_b} \right)^2}$$

$$\dot{u}_j = \frac{\dot{U}_j}{\Delta_n} = 2 \frac{L}{H} \dot{u}_j$$

The incremental deformation of the elastic brace without flexural deformation is temporarily computed and then, the elasto-plastic relations among axial force  $n_b$ , bending moment  $\beta_b$ , relative rotation  $\theta$ , slope  $r$ , and deformations  $u_f$ ,  $u_j$ ,  $u_m$  are exactly obtained corresponding to each branch of hysteretic curves of the brace. A typical example of the restoring force characteristics of X-type braces, when  $2\Delta L/L=1/10$ ,  $\lambda=100$  and yield function  $|\beta_b| + n_b^2 = 1$ , shows the similar hysteresis with an experimental result of H-shaped brace<sup>4)</sup> as shown in "Fig.2".

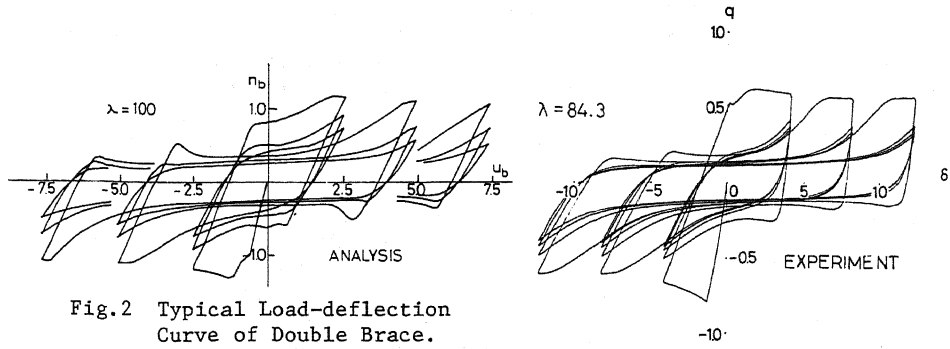


Fig.2 Typical Load-deflection Curve of Double Brace.

### EARTHQUAKE RESPONSE ANALYSIS OF A BRACED FRAME STRUCTURE

The above-mentioned X-type braces are introduced into the frame structure in order to obtain the aseismic design data of braced frame structures. The nondimensional fundamental equation of motion is written as :

$$([a]\frac{d^2}{dt^2} + [b]\frac{d}{dt} + [n_g])\{\eta\} + [c]\{\beta\} + [c_b]\{n_b\} = -[a]\{I\}\alpha \cdot \alpha(\tau) \quad (9)$$

where,  $\{\eta\} = \{X\} / \bar{L}\bar{\Delta}$  is horizontal displacement,  $\{\beta\} = \{M\} / \bar{B}$  is bending moment of member joint,  $\{n_b\} = \{N_b\} \bar{H} / \bar{B}$  is axial force of brace newly defined here, which is calculated from the relative displacement of each brace as mentioned above, and other notations are given in the reference <sup>3)</sup>.

Structural model considered here are 7-story, single-bay frame structures with and without pin-ended X type braces in every story, as shown in "Fig.3". The dynamic characteristics of the model structures are represented by the following nondimensional parameters: the strength ratio of girder to column assembled at one connection  $\beta$ , the strength ratio of yield moment to yield axial force multiplied by the depth of the member  $\mu = M_y / N_y H$ , the area ratio of brace to column at the base story  $\xi = A_b / A$  and the stiffness and strength distributions of columns and braces defined by

$$k_{ji} = b_{ji} = 1 - \lambda_j \cdot \left(\frac{i-1}{N-1}\right)^{\nu_j} \quad (10)$$

where, suffix  $j$  means the column members,  $j=c$ , or the brace members,  $j=b$  and  $i$  represents  $i$ -th story.

The following nondimensional responses are mainly selected for pointing out the qualitative characteristics of the responses of the braced structures :

$$\{\gamma_b\} = [\delta_b]^{-1} \{n_b\} \quad (11)$$

$$\{\gamma_j\} = [\delta_j]^{-1} \{\theta_j\} \quad (12)$$

$$\{\eta\}_r = [J] \frac{\{X\}}{L\Delta} \quad (13)$$

$$\{q\} = \{q_f\} + \{q_b\} = \frac{\{Q_f\}}{B/L} + \cos\theta \cdot \frac{\bar{L}}{H} \cdot \frac{\{N_b\}}{B/H} \quad (14)$$

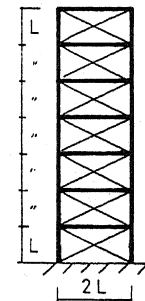


Fig.3 Model of Braced Frame

in which,  $\{\gamma_b\}$  and  $\{\gamma_j\}$  are the ductility ratios of brace and member joint

$\{q_f\}$  and  $\{q_b\}$  are the nondimensional shear forces acting on the frame and brace,  $\{n\}_r$  and  $\{n\}_b$  are relative horizontal displacement and relative displacement of brace. In the case of  $\xi=0$ , the model designates a pure frame structure and in the case of  $\xi=0.1$ , the elastic limit shear force ratio of the frame to the brace at the base story is 1:1.49 when the strength ratio  $\xi$  is 0.6 and 1:1.19 when  $\xi$  is larger than 1, where  $\mu=0.25$ ,  $\bar{L}/\bar{H}=20/3$  and  $\cos \theta=2/\sqrt{5}$ .

"Fig.4" shows the distribution of root mean square responses due to random excitation for the different values of stiffness distribution parameters  $\lambda_c$  and  $\lambda_b$ , which are selected as the values making the local responses comparatively uniform. In this figure, mark  $\text{---}$  shows relative displacement,  $\circ\text{---}\circ$ ,  $\Delta\text{---}\Delta$  and  $x\text{---}x$  represent the maximum ductility ratio responses of column, girder and brace, respectively, and solid line represents the shear force normalized by the shear force of the first story, the left part of which is the shear force acting on the frame and the right part of which is the shear force acting on the brace. The ductility ratio responses of the braces are larger than those of columns and girders for each story in the case of  $\beta=0.6$ ,  $\xi=0.02$ ,  $\nu_c=\nu_b=1.5$ . Namely, the aseismic elements effectively control the responses of the members and columns behave elastically till the braces fall into the plastic stage. In the case of the structure with stiff girders ( $\beta=5$ ), ductility ratio responses uniformly distribute when  $\lambda_c=\lambda_b=0.7$ . The responses of lower columns, however, have a tendency to fall into plastic range earlier than the braces because of the interaction effect of axial force caused by the axial force of braces.

In the elasto-plastic response analysis, the following parameters are introduced to represent the dynamic characteristics of the structures and the excitations. The strength parameter  $\alpha$  is defined as the ratio of the maximum acceleration  $A$  of the excitation to the elastic limit shear strength per unit mass of the base story and is chosen to have the elastic responses similar to those of pure frames due to five kinds of filtered artificial earthquakes<sup>5)</sup>, corresponding to the frequency parameter  $\psi$ , which is defined by the ratio of the fundamental period  $T_1$  of the structure multiplied by 100 to the duration time  $T_d$  of the earthquake.

"Fig.5" shows the time history of nondimensional displacements of 1st, 4th and 7th story in the case of  $\psi=60, 120$  and  $\xi=0, 0.05, \bar{\beta}=0.6$ . The frame structure without braces, after yielding, behaves with similar frequency as the initial state owing to the feature of the perfectly elasto-plastic hysteretic characteristics of the joints, while the frequency of the braced frame, after some plastic behavior, decreases due to post-buckling behavior of the braces. The difference of the maximum displacements between the structure with and without braces is rather small in the lower stories but becomes larger in the higher stories because of the axial deformation of the column members. The displacement responses of the pure frame are made one-sided by the plastic deformation, while the responses of the braced frame seems to behave around the initial position.

As mentioned already in the previous paper<sup>5)</sup>, it is very important for the aseismic safety of the whole structure to keep the safety of column members under gravity force and to make clear the capacity of plastic deformation of the columns. The hysteretic characteristics of the moment-rotation relation and the axial force-elongation relation of the elasto-plastic joint at the base column are compared in "Fig.6". Ductility ratio response of the

base column joint decreases considerably by introducing the adequate braces into the frame structure.

In the following three figures, the distributions of the relative displacement and the maximum ductility ratios of column and girder joints of the structure are presented in order to make clear the effects of area ratio  $\xi$ , frequency parameter  $\psi$ , slenderness ratio of the brace  $\lambda$  and the excitation. The result is similar to that of the elastic response in that the greater the cross-sectional area of the brace, the larger the response of the relative displacement of the higher part of the structure becomes as shown in "Fig.7", in which solid line represents the average distribution of No.6-10 earthquakes. Responses of the braced frame are not so affected by the frequency characteristics of the excitations as those of the structure without brace because of the variation of the fundamental frequency produced by the post-buckling behavior of the braces. The ductility ratio response of the base column decreases corresponding to the increase of the value of  $\xi$  as shown in "Fig.8". Ductility ratio responses of the girder joints in the lower part of the structure with soft girders are rather affected by the area ratio  $\xi$  as shown in "Fig.9". In the case of the fundamental frequency close to the predominant frequency of the excitation ( $\psi=30, 60$ ), braces are very effective for the aseismic safety of the girder joints. While, the responses in the higher part are controlled by the brace when the fundamental frequency becomes smaller than the predominant frequency of the excitation.

#### CONCLUDING REMARKS

From the results of the elastic and elasto-plastic response analyses of the frame structures with and without braces, the following concluding remarks were obtained :

1. From the results of the root mean square responses due to random excitation, it becomes clear that the ductility ratio responses of the column joints and the braces of the structure with stiff girders, are the same order and uniformly distributed when the stiffness distribution parameters,  $\lambda_c, \lambda_b$ , are about 0.7 and that the responses of the relative displacement of the higher part of the structure become larger than those of the lower part. This means that the relative displacement is only a restricted measure of the aseismic safety while the local ductility ratio may be more important and direct measure of structures.
2. The braces are expected to behave in plastic range prior to frame members except for the base column when the structure has soft girders. This fact explains that the braces are very effective for the aseismic safety of a whole structure. However, it should be mentioned that engineers must pay attention on the safety of the base column.
3. From the nonlinear response analyses considering the post-buckling behavior of braces, it becomes clear that the accumulation of the plastic deformation in the base column is considerably restrained. This fact is partially interpreted by the elongation of the fundamental period due to plastic behavior of the braces.
4. Ductility ratio response at the base column of a braced frame structure decreases as cross-sectional area of the brace increases. This tendency is remarkable when the fundamental frequency of the structure is close to the predominant frequency of excitation.

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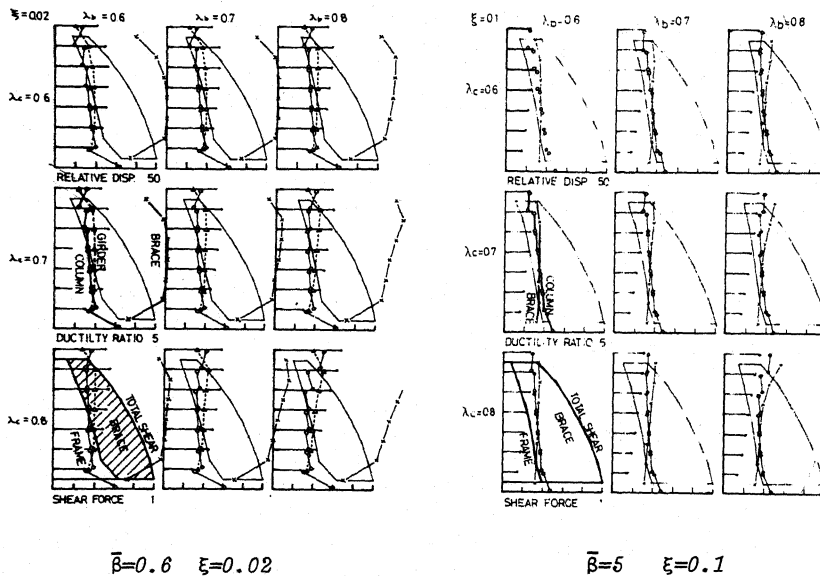


Fig. 4 Elastic Responses of Relative Displacement, Ductility Ratio and Shear Force :  $\mu=0.25$ ,  $v_c=v_b=1.5$

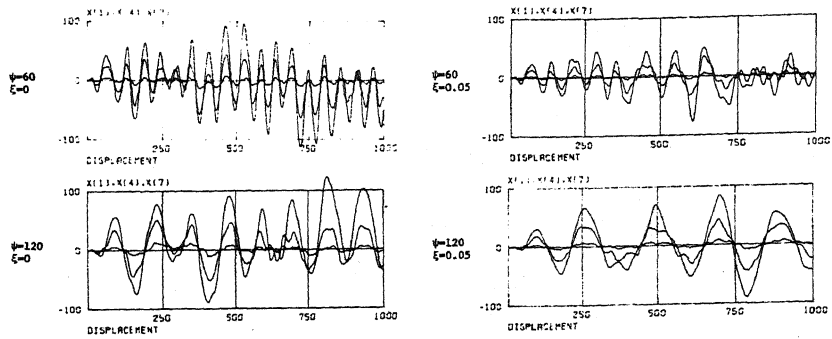


Fig.5 Nondimensional Displacement of 1, 4 and 7th Story

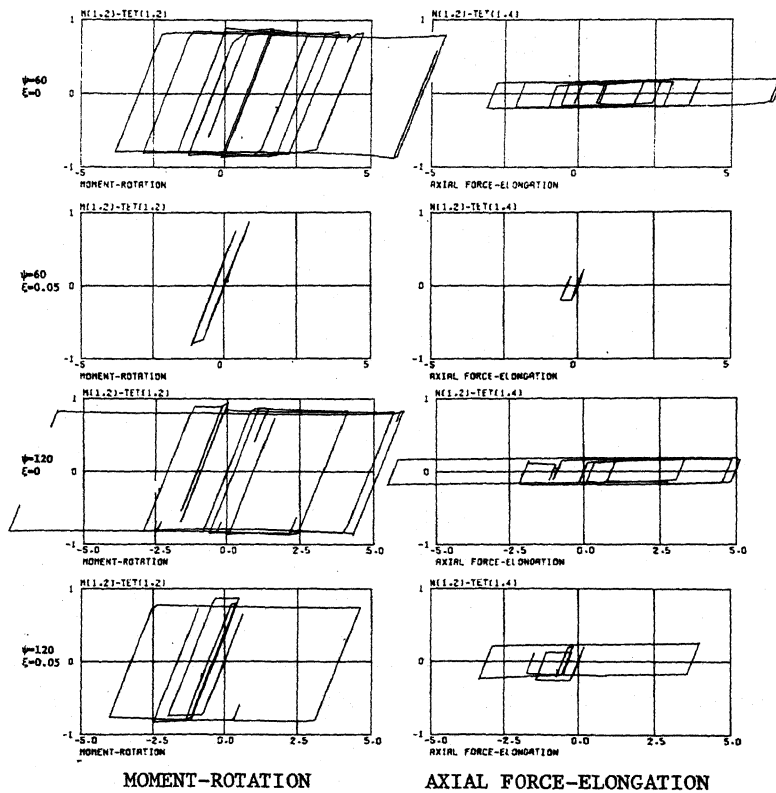
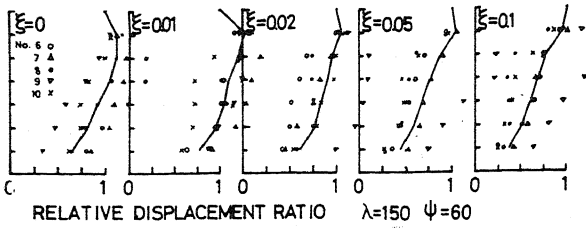


Fig.6 Local Hysteresis for Base Column Joint  
 $\bar{\beta}=0.6$ ,  $\lambda_c = \lambda_b = 0.7$ ,  $\nu_c = \nu_b = 1.5$ ,  $\mu = 0.25$ , Quake No.8



$$\bar{\beta}=0.6, \mu=0.25, \lambda_c=\lambda_b=0.7, \nu_c=\nu_b=1.5$$

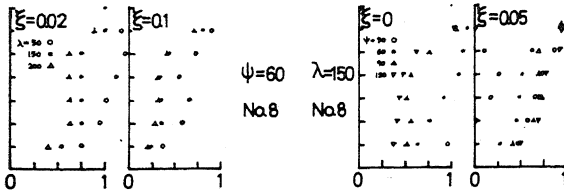


Fig.7  
Distribution of  
Relative Displacement

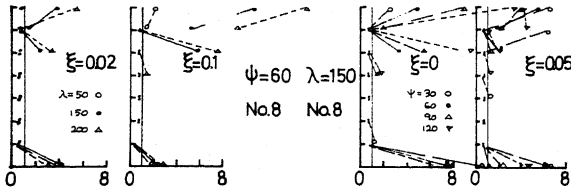
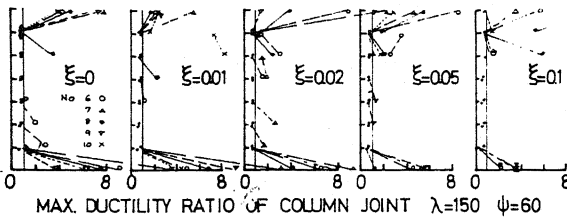


Fig.8  
Distribution of  
Ductility Ratio  
of Column Joints

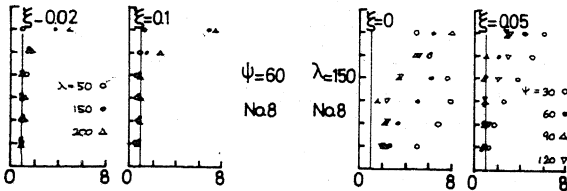
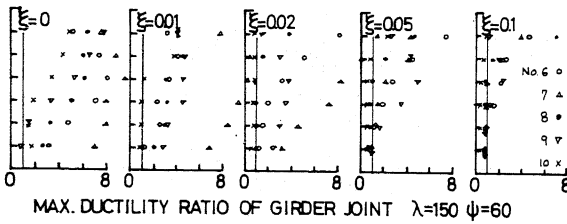


Fig.9  
Ductility Ratio  
of Girder Joints