

THE EFFECT OF STRUCTURAL INSTABILITY ON  
DYNAMIC RESPONSE OF SOIL-STRUCTURE SYSTEMS

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SUMMARY

The problem of soil-structure interaction including the effect of structural instability due to gravity load acting on the displaced configuration of the structure is studied. A direct formulation of the problem including structural embedment in an arbitrary seismic environment is presented. In order to have a clear understanding of the gravity load effect, a comprehensive parametric study is performed. Use is made of a stability parameter and a modified wave parameter in examining the basic characteristics of the soil-structure and gravity effects. The results are presented in terms of the normalized response spectra amplitudes, and the variations of peak responses and resonant frequencies are given for a wide range of parameters.

INTRODUCTION

The effect of soil-structure interaction on the seismic response of structures has been extensively investigated during the past decade. In defining the characteristic parameters which control soil-structure interaction, the effect of gravity load acting on the structure is usually neglected. This is due primarily to the opposing stiffness requirements for the structure in having simultaneously pronounced interaction effect and gravity effect. In fact, the soil-structure interaction is more important in tall but relatively rigid structures founded on soft soils, whereas the gravity effect becomes more pronounced in flexible structures. Conceptually, the gravity load effect may be regarded as an equivalent softening of the structure which could lead to a decreasing tendency in the soil-structure interaction. However, the overturning moment due to gravity load could cause a pseudo-softening of the soil, resulting in an interaction effect.

Very few studies have been made to examine the effect of gravity load on soil-structure interaction. While the importance of second-order analysis is generally emphasized in existing books on structural dynamics (1,2) and practical methods for analyzing fixed-base structures have been developed (3), only in recent years, results of studies of flexibly supported structures have been reported (4). In Ref. 4, the problem was formulated through an energy approach; however, the significance of gravity effect with respect to the main controlling parameters of soil-structure interaction was not thoroughly investigated.

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The objective of this paper is to present a direct formulation for the problem and to clarify the role of the gravity effect on soil-structure interaction through a parametric study.

#### FORMULATION OF THE PROBLEM

In general, the equations of motion of a soil-structure system including the second-order effect can be expressed as,

$$M\ddot{r} + C\dot{r} + (K-\Delta K)r = p \quad (1)$$

in which M, C and K represent the mass, viscous damping and the stiffness matrices of the system, respectively, and  $\Delta K$  refers to the geometric stiffness matrix of the structure representing the gravity effect. Referring to Fig. 1, partitioned forms of the relevant matrices can be written as,

$$K = \begin{bmatrix} K_{ss} & K_{sb} & 0 \\ K_{bs} & (K_{bb}^s + K_{bb}^g) & K_{bg} \\ 0 & K_{gb} & K_{gg} \end{bmatrix}, \quad \Delta K = \begin{bmatrix} \Delta K_{ss} & \Delta K_{sb} & 0 \\ \Delta K_{bs} & \Delta K_{bb}^s & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2)$$

in which symbol b refers to the degrees of freedom along the soil-structure interface (base), and symbols s and g indicate the degrees of freedom of the structure and the foundation soil, respectively, excluding those indicated by b. Although not given here, the partitioned forms of the mass and the damping matrices can be written in a similar manner. The definition of the effective force vector, p, in Eq. 1 depends essentially on the definition of the response vector, r. Based on the possible combinations of the total and relative response components throughout the system, different sets of the effective force vectors can be obtained for certain free-field conditions (5). In the following derivation, only one of those combinations for the response vector is considered as the basis of the formulation as follows;

$$r = \{r_s^t \ r_b^\Delta \ r_g^\Delta\}^T \quad (3)$$

where the motion of the structure is represented by its total response and the remaining response components are defined relative to the free-field motion; i.e.

$$r_b^\Delta = r_b^t - v_b \quad ; \quad r_g^\Delta = r_g^t - v_g \quad (4)$$

in which  $r_b^t$  and  $r_g^t$  represent the total response, and  $v_b$  and  $v_g$  represent the free-field response at the base of the structure and foundation soil, respectively. It is usually more convenient to transmit the free-field base motion,  $v_b$ , or total base motion,  $r_b^t$ , into the structure in pseudo-static manner so that the remaining dynamic part of the structural response can be defined relatively. The corresponding effective force vector components can be found in Ref. 5 and the details of pseudo-static transmission including the gravity effect are described in Ref. 6. In the following derivation, the base of the structure is assumed to be rigid and the structural response

is defined relative to the total base motion, i.e.,

$$\begin{aligned} r_b^\Delta &= T_{bo} r_o^\Delta ; & v_b &= T_{bo} v_o \\ r_s^t &= T_{so} (r_o^\Delta + v_o) + \delta_s \end{aligned} \quad (5)$$

in which  $r_o^\Delta$  and  $v_o$  represent the relative base motion and the free-field motion at the base in terms of the rigid body degrees of freedom of the base, and  $T_{bo}$  and  $T_{so}$  are the rigid body motion transformation matrices which relate these degrees of freedom to those of the base and the structure, respectively.  $\delta_s$  refers to the dynamic structural response relative to the total base motion.

The problem, in general, is geometrically nonlinear and the exact solution can be obtained only in the time domain. However, assuming the normal forces in members being conservative in space and also unchanged with respect to time, the problem can be linearized approximately. Under these assumptions, transformation of Eq. 1 (and making use of Eq. 2 through 5) into frequency domain followed by dynamic condensation of the soil degrees of freedom results in the following set of frequency dependent equations;

$$\begin{bmatrix} (\bar{K}_{ss} - \Delta K_{ss}) & (\bar{K}_{so} - \Delta K_{so}) \\ (\bar{K}_{os} - \Delta K_{os}) & (\bar{K}_{oo} - \Delta K_{oo}^s) \end{bmatrix} \begin{Bmatrix} \bar{\delta}_s \\ \bar{r}_o^\Delta \end{Bmatrix} = \begin{Bmatrix} \bar{p}_s \\ \bar{p}_o \end{Bmatrix} + \begin{Bmatrix} \Delta \bar{p}_s \\ \Delta \bar{p}_o \end{Bmatrix} \quad (6)$$

in which

$$\begin{aligned} \bar{K}_{ss} &= K_{ss} + i\omega C_{ss} - \omega^2 M_{ss} \\ \bar{K}_{so} &= -\omega^2 M_{ss} T_{so} \\ \bar{K}_{oo} &= S_{oo}^g - \omega^2 (T_{bo}^T M_{bb}^s T_{bo} + T_{so}^T M_{ss} T_{so}) \end{aligned} \quad (7)$$

$$\begin{aligned} \Delta K_{so} &= \Delta K_{sb} T_{bo} + \Delta K_{ss} T_{so} \\ \Delta K_{oo}^s &= T_{bo}^T \Delta K_{bb}^s T_{bo} + T_{bo}^T \Delta K_{bs} T_{so} + T_{so}^T \Delta K_{sb} T_{bo} + T_{so}^T \Delta K_{ss} T_{so} \end{aligned} \quad (8)$$

and

$$\begin{aligned} \bar{p}_s &= \omega^2 M_{ss} T_{so} \bar{v}_o \\ \bar{p}_o &= \omega^2 (T_{bo}^T M_{bb}^s T_{bo} + T_{so}^T M_{ss} T_{so}) \bar{v}_o \end{aligned} \quad (9)$$

$$\begin{aligned} \Delta \bar{p}_s &= \Delta K_{so} \bar{v}_o \\ \Delta \bar{p}_o &= \Delta K_{oo}^s \bar{v}_o \end{aligned} \quad (10)$$

In Eq. 7 to 10, the structural masses are assumed to be lumped and pseudo-static viscous damping forces are omitted as usually done in practice.  $\omega$  indicates the frequency of excitation and  $S_{00}^g$  represents the frequency dependent complex stiffness (impedance) matrix of the rigid base. In Eq. 9 and 10,  $\bar{v}$  denotes the vector of displacement (and/or rotation) amplitudes of input base motion in terms of rigid body degrees of freedom. The above equations are applicable to embedded and/or surface supported structures in any seismic environment provided that the proper impedance matrix and compatible input base motion are available. A careful inspection of Eq. 6 to 10 reveals that a coupling occurs between the structural and base degrees of freedom because of the gravity effect, in addition to the inertial coupling. Also, gravity effect produces an additional term in the effective force vector. However this term vanishes in surface supported structures subjected to vertically propagating seismic waves due to the lack of rotational component of the input motion.

#### PARAMETRIC STUDY

In order to understand the relative importance of the instability effects due to gravity load on the response of soil-structure systems, a parametric study is performed on a simple SDOF shear type structure (in its fixed-base condition) supported by an elastic half space at the surface (Fig. 2). The base of the structure is assumed to be a rigid circular plate. This model has been used in many of the investigations and has provided a simple but reasonable approximation for demonstrating the effects of soil-structure interaction. In this study, the input motion is specified at the surface as horizontal harmonic free-field motion with amplitude  $V_b$  and frequency  $\omega$ . Frequency dependent complex stiffness influence coefficients given in Ref. 7 are utilized to represent the dynamic soil stiffness. The details of the application of Eq. 6 to 10 to the model is given elsewhere (6). Note that the problem is treated as a linear problem based on the approximations explained above.

The nondimensional parameters defining the problem without considering gravity effects are well established (Ref. 8). With some notation differences these parameters are,

- 1) Relative stiffness parameter (wave parameter),  $\alpha = \omega_R h / V_s$ , where  $\omega_R$  is the natural frequency in fixed-base condition,  $V_s$  denotes the shear wave velocity of the supporting medium,
- 2) The aspect ratio,  $\beta = h/r$  (see Fig. 2),
- 3) The relative mass parameter;  $\eta = \rho \pi r^2 h / m$ , where  $\rho$  represents the mass density of the soil;
- 4) Relative frequency parameter,  $\omega_R / \omega$ ;
- 5) The viscous damping ratio of the structure;  $\xi$
- 6) The ratio  $m_b / m$  of the masses of the base and of the structure;
- 7) Poisson's ratio of the soil,  $\nu$ .

Introducing the gravity effect, a new parameter is defined in this study, the so-called stability parameter;

$$\theta = \frac{mg}{kh} \quad (11)$$

It defines the ratio of P-Δ moment to the elastic story moment in the fixed-base condition. It can also be defined as,

$$\theta = \omega_p / \omega_R \quad (12)$$

where  $\omega_p$  represents the natural frequency of a pendulum with the same height as the structure,

$$\omega_p = \sqrt{g/h} \quad (13)$$

in which  $g$  is the acceleration of gravity. Introducing the stability parameter, it was found that the wave parameter,  $\alpha$ , is no longer independent. Hence, a modified wave parameter is defined as,

$$\alpha_p = \omega_p h / V_s = \sqrt{gh} / V_s \quad (14)$$

The previously defined wave parameter,  $\alpha$ , can then be obtained as,

$$\alpha = \alpha_p / \sqrt{\theta} \quad (15)$$

This equation demonstrates clearly the contradictory character inherent in the problem. The soil-structure interaction is more pronounced as  $\alpha$  parameter increases. This requires high  $\alpha$  values and/or low  $\theta$  values which overshadow the gravity load effect. However, in relatively tall structures with high aspect ratios, the overturning moment due to gravity loads can increase the effective base shear even for relatively low  $\theta$  values. This fact is demonstrated in Figs. 3 to 5. Figure 3 represents the response spectra of the structure subjected to harmonic input base motion. The abscissa gives the relative frequency parameter,  $\omega_R / \omega$ , and ordinate represents the equivalent pseudo-acceleration of the structural mass,  $\omega^2 \delta_{eq}$ , normalized with respect to the acceleration amplitude of the input base motion,  $\omega^2 v_b$ , i.e.,

$$S_c = \frac{\delta_{eq}}{V_b} \left( \frac{\omega_R}{\omega} \right)^2 \quad (16)$$

This quantity also indicates the equivalent base shear,  $Q_{eq}$ , normalized with respect to the base shear of an infinitely rigid structure<sup>4</sup> with the same mass,  $m\omega^2 v_b$ , subjected to the same acceleration. The equivalent base shear is defined as the overturning moment divided by the height of the structure, i.e.

$$Q_{eq} = \frac{M}{h} = k \delta + \frac{mg}{h} (\phi h) \quad (17)$$

or

$$Q_{eq} = k \delta_{eq} \quad (18)$$

through which  $\delta_{eq}$  is defined as

$$\delta_{eq} = \delta + \theta(\phi h) \quad (19)$$

In Eq. 17 and 19,  $\delta$  represents the structural deformation amplitude based on second-order theory, and  $\phi h$  is the translation of the top mass due to rigid base rotation (Fig. 2). When gravity effect is not considered,  $\delta_{eq}$  in Eq. 19 reduces to the first term only, representing first-order deformation amplitude. In Fig. 3, the gravity effect on the response of soil-structure systems with relatively high aspect ratio ( $\beta = 10$ ) is shown. In the parametric study, the relative mass parameter  $\eta$  is taken constantly as 6, the Poisson's ratio of the soil is  $\nu = 0.45$  and the rotatory inertias as well as the mass of the base plate are neglected. The viscous damping ratio of structure is taken as  $\xi = 0.02$ . Three values of stability parameter,  $\theta$ , are considered. In each figure, response spectra is plotted for two values of the modified wave parameter,  $\alpha = 0.2$  and  $\alpha = 0.3$ , respectively. These values are for high-rise structures founded on relatively soft soils. The curves for  $\alpha = 0$  represents the fixed-base condition. In order to cover a wider range of parameters, the ratio of the peak spectral amplitude of the soil-structure system to that of fixed-base system ( $\alpha = 0$ ), i.e.,  $S/S_R$  is plotted against  $\alpha$  in Fig. 4. In each figure three values of aspect ratio are considered. Also plotted in Fig. 5 are the ratio of resonant frequencies of the corresponding systems. In all figures the solid lines represent the cases where the gravity effect is not considered and the dotted lines represent solutions including the gravity effect. Note that, in Figs. 4 and 5, for a specific  $\alpha$  value, the interaction effect is more pronounced as the stability parameter,  $\theta$ , decreases. Thus, figures with different  $\theta$  values correspond to different ranges of the  $\alpha$  parameter. An examination of the figures reveals that gravity loads tend to increase the peak responses and decrease the resonant frequencies. It is interesting to note that even for small or moderate  $\theta$  values ( $\theta = 0.05$  and  $\theta = 0.10$ ) a certain change in the peak response and in the resonant frequencies is observed. The significance of these changes, of course, depends on both  $\alpha$  and  $\beta$  parameters. Another significant effect of gravity loads is an apparent increase in the response spectra ordinates in the low frequency range (right-hand sides of Fig. 3). They become more significant as  $\theta$  value increases.

#### CONCLUSIONS

It is shown that in certain situations the effect of gravity load may influence significantly the response of soil-structure systems. In spite of the compensating character of soil-structure interaction and gravity effects, for high-rise buildings on relatively soft soils the gravity effect can increase the response noticeably. In addition to the detailed parametric study performed, the formulation of the problem presented includes the case of embedded structures in an arbitrary seismic environment. Further research is needed to investigate the problem in the time domain using real earthquake data recorded on different soils.

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#### FIGURES

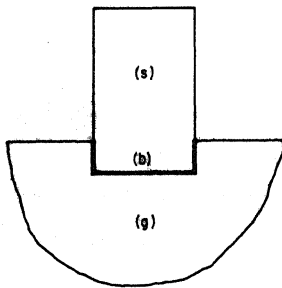


Fig. 1. Schematic representation of soil-structure system

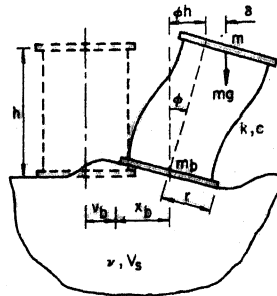


Fig. 2. System considered in parametric study

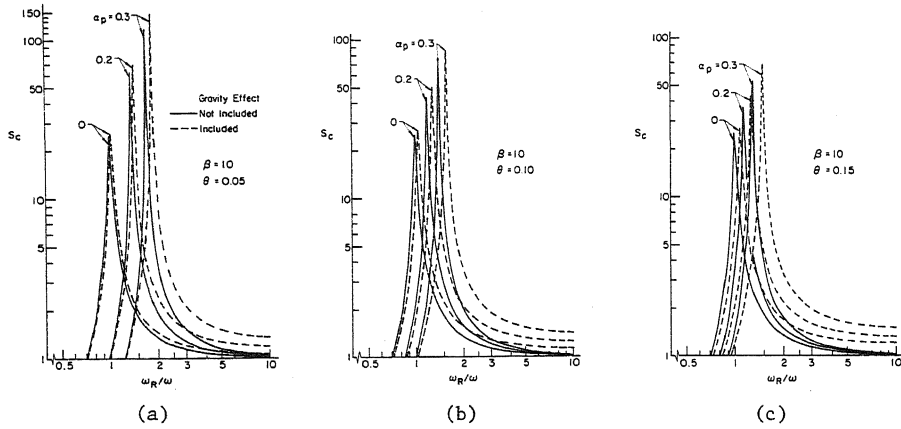


Fig. 3. Normalized response spectra with and without gravity effect

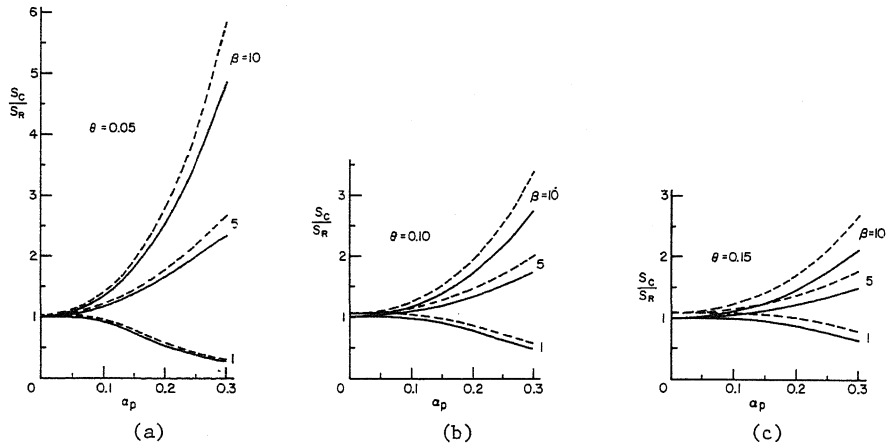


Fig. 4. Variation of normalized peak response versus modified wave parameter

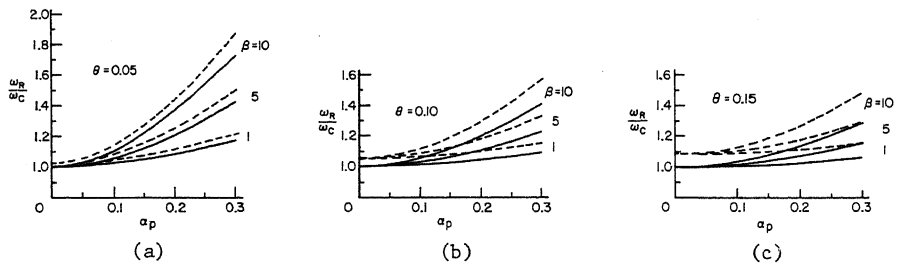


Fig. 5. Variation of resonant frequencies versus modified wave parameter