RE-EVALUATING THE EFFECT OF SEISMIC GROUND MOTION CORRELATION ON STRUCTURAL RESPONSE

by

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SUMMARY

The seismic analysis of critical structures requires the consideration of the simultaneous action of the radial, tangential, and vertical components of earthquake induced ground motions. Both response spectra and time-history techniques often use the root-sum-square (RSS) method to approximate total peak structural response. This paper examines the implications of using the RSS approach to estimate structural response, and its inherent assumption that the ground motion components are uncorrelated.

Results of this study suggest a simple method for determining whether the effect of ground motion correlation on a particular structure will have a significant effect on its response.

BACKGROUND

Chu (Ref. 1) showed that the RSS method for combining peak structural responses could be used if the ground motion components could be shown to be statistically independent stationary random processes with zero means. Actual strong ground motion accelerograms are, however, non-stationary and always have some degree of statistical dependence and correlation between their three orthogonal components. The problem of determining non-stationary peak response can be overcome by assuming the strong motion segments of the ground motion to be stationary and applying a peaking factor to the RMS (root-mean-square) stationary response levels (Ref. 2).

Chen (Ref. 3) suggested that the ground motion components could be considered statistically independent if their cross-correlation coefficients were less than 0.16. Penzien (Ref. 4) and Hadjian (Ref. 5) demonstrated that a simple axis rotation of the ground motion could be found which would insure that these coefficients would be zero. Finding a rotation that makes the ground motion components statistically independent, however, simply forces the integral over frequency of the cross-spectral density or the integral over time of the cross-correlation function to zero. Making this integral zero does not eliminate the effect that ground motion correlation has on structural response.

STRUCTURAL RESPONSE AND GROUND MOTION CORRELATION

For a stationary random process, the power (i.e., auto) and cross-spectral density of linear structural response due to multiple inputs (e.g., x, y, and z ground motion) are given by (Ref. 5):

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$$[\Phi(\hat{i}f)] = [H^*(\hat{i}f)][S(\hat{i}f)][H(\hat{i}f)]^T \quad (1)$$

The Φ-matrix contains the power and cross-spectral densities of response while the ij-th element of the transfer function matrix (H) relates structural response at coordinate i to ground motion input of frequency, f, in direction j. The diagonals of the S-matrix contain information on power versus frequency within each ground motion component while the off-diagonals contain phase (correlation) information between the components. Fig. 1 shows the amplitude of the power and cross-spectral density functions for the Kern County earthquake of 1952.

Considering only the horizontal (x,y) components of ground motion and structural response at a single coordinate, the integral over frequency of Eq. 1 yields the mean-square response:

$$\sigma_i^2 = \int |H_{ix}(\hat{i}f)|^2 |S_{xx}(\hat{i}f)| df + \int |H_{iy}(\hat{i}f)|^2 |S_{yy}(\hat{i}f)| df \quad (2)$$

$$+ \int \left[f_i\left(H_{ix}(\hat{i}f),\ H_{iy}(\hat{i}f),\ \theta_{xy},\ |S_{xy}(\hat{i}f)|\right)\right] df$$

where: $S_{xy}(\hat{i}f) = |S_{xy}(\hat{i}f)|(\cos\theta_{xy} + \hat{i}\sin\theta_{xy})$

Simplifying notation and solving for the root-mean-square (RMS) response yields:

$$\sigma_i = \left\{ \left[\sigma_{ix}^2 + \sigma_{iy}^2\right] + \int \left[f_1(H_{ix}, H_{iy}) \sin\theta_{xy} + f_2(H_{ix}, H_{iy}) \cos\theta_{xy} \right] |S_{xy}| df \right\}^{1/2} \quad (3)$$

The remaining integral gives the contribution that correlation between the x and y ground motion components ($|S_{xy}|$, θ_{xy}) has on structural response. Eq. 3 reduces to the traditional RSS approach only if this "correlation integral" vanishes. Making the cross-correlation coefficient of the x and y ground motion zero ($\int S_{xy} df = 0$) does NOT necessarily make this integral zero; the interaction of the ground motion with the structure must also be considered.

THE CORRELATION INTEGRAL

The correlation integral contains four factors which determine whether ground motion correlation will affect a particular structure:

1) The degree to which the x and y motion of the structure are coupled;

2) The dynamic characteristics of the structure (modal frequencies, damping, and participation factors);

3) The amplitude ($|S_{xy}|$) and phase (θ_{xy}) of the ground motion cross-spectral density in the range of the structural frequencies; and,

4) The phasing between the x and y ground motion and structural motion in its sensitive frequency ranges.

The effect of the correlation integral increases as the x-y coupling (twist) of the structure increases. For highly irregular structures, the ground motion and structural characteristics as well as their interaction should be considered before neglecting the effect of correlation and adopting the RSS approach.

NUMERICAL EXAMPLE

Figure 2 shows a simple building-type structure with a rigid floor diaphragm supported by four columns of equal stiffness. The center of mass has been offset from the center of rigidity to provide coupling between x and y motion. The structure parameters were selected to place its natural frequencies in the 0.25 to 0.45 hertz range, where the amplitude of the cross-spectral density function (Fig. 1) is comparable to those of the power spectral densities. X and y responses were computed at the corner of the structure for a range of modal damping using both the traditional RSS approach and the correlation integral method. Fig. 3 shows that ground motion correlation may produce smaller or greater structural responses. The magnitude of this effect, for this simple example, ranges from +15% for a very lightly damped structure to -12% for 10% structural damping.

CONCLUSIONS

The RSS approach for combining peak structural response assumes that correlation between the three components of ground motion can be neglected. A simple axis rotation which forces the cross-correlation coefficients between ground motion components to be zero does not guarantee that structural response will not be affected; characteristics of the structure as well as those of the ground motion should be considered before the effect of ground motion correlation is ignored.

The results of this study suggest a simple method for determining whether ground motion correlation will affect the response of a particular structure. The phasing between the ground motion and structural motion is critical when evaluating the contribution of the correlation integral. The importance of the correlation integral can be determined by simply comparing the phase angle of the ground motion (θ_{xy}) with the phase angle of structural response (given by its transfer function) in the frequency bands around its resonances.

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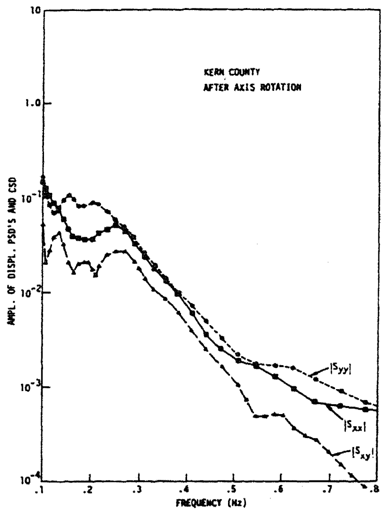

Figure 1. Comparison of the Amplitude of the Displacement Power and Cross Spectral Density Functions for Kern County.

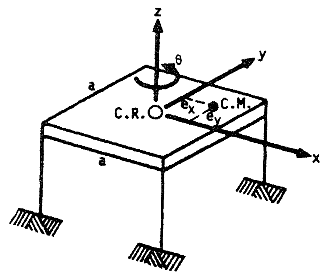

Figure 2. Simple Building Structure with Coupling of x and y Motion.

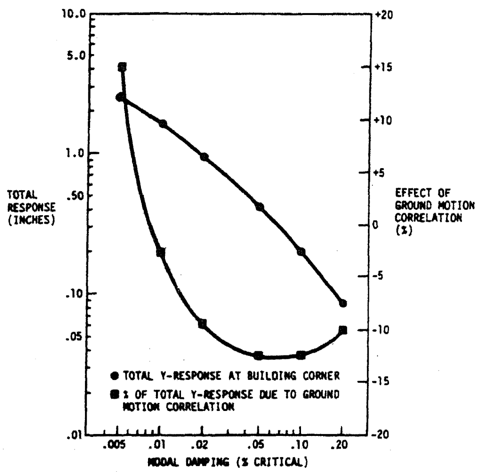

Figure 3. Effect of Ground Motion Correlation on Structural Response.