PROBABILISTIC ANALYSIS OF DAMAGE
FROM EARTHQUAKE GROUND MOTION

by

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SUMMARY

A methodology is developed to assess earthquake damage on buildings through the use of probabilistic models when characteristics of the building, seismicity and attenuation relationships are known. An example of its application is presented for high-rise buildings. As a result the logarithm model seems to give the best fit for high-rise buildings. Statistical tests reveal significant correlation between peak ground velocity and seismic damage for this type of structures. Primary contributions are believed to include (1) the use of instrumental strong ground motion data to develop empirical damage models, and (2) the use of both acceleration and velocity as damage estimator to account for the frequency dependence of low and high-rise buildings.

PROCEDURE

The overview of the proposed methodology consists of basically three steps:

(1) Development of a statistical damage model which computes, through a functional relationship, damage estimates using ground motion parameters (peak ground acceleration and velocity).

(2) Development of probability distribution functions for the ground motions parameters.

(3) Development of a derived probability distribution of damage through the integration of the two previous steps.

In the first step data from the 1971 San Fernando earthquake was used to perform the statistical analysis where the input data consisted of: (1) building vulnerability, expressed by the age, material of construction, and the number of stories, (2) building damage, quantified by the damage, and (3) the peak ground acceleration and velocity measured at the site (Ref. 1). In this study the application was restricted to that class of structures characterized by concrete construction built after 1947 (considered earthquake resistant) and having from 8 to 13 stories.

After several models were tested to correlate the data (Ref. 2) it was concluded that the best fit was achieved by a multiple linear regression in the log-log domain (base 10) as follows:

$$\log D = \alpha + \beta \log A + \gamma \log V + \varepsilon \qquad (1)$$

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D is the damage ratio; A and V are peak ground accelerations and velocity; α, β and γ are constants computed from the observed data; and ε represents the data scatter about the mean estimate, which is a function of the uncertainty associated with the damage evaluation. Table 1 shows the results of the regression analysis; the statistics were derived using a 99% confidence interval. By comparing the standard errors from the different models, the log-log model was found to give 42% better estimates than the linear or second-order models (Ref. 1). Statistical tests show that poor correlation is obtained using peak ground acceleration for the type of buildings used in the study, while a much better indication is given by the peak ground velocity. This can be visualized in Figure 1 where "Eq. 1" is plotted for different damage levels. A damage state is defined as the interval between two adjacent curves. Notice that for constant peak acceleration, the damage ratio is more susceptible to peak velocity than to peak acceleration.

The next step was to compute the probability distribution of peak acceleration and velocity in order to derive a joint distribution for the damage. This computation was performed using the approach followed by Vanmarke and Cornell (Ref. 3). The probability density function (PDF) for the damage is computed through a joint probability distribution assuming that the ground acceleration and velocity are independent variables.

$$f_D(d) = f_A(a)\ f_V(v) \tag{2}$$

This equation states that the damage PDF is equal to the product of the PDFs of the peak ground motion parameters. As a result we have that:

$$\text{Prob}\left[d_i \le D \le d_j\right] = \text{Prob}\left[a_i \le A \le a_j \text{ and } v_i \le V \le v_j\right] \tag{3}$$

developing the calculations we have:

$$\text{Prob}\left[d_i \le D \le d_j\right] = W \int_{a_i}^{a_j} \int_{v_k}^{v_j} (A)^{\eta}\ (V)^{\theta}\ dAdV \tag{4}$$

where

$$W = \left\{km_1\ e^{\beta m_0}\right\}^2\ E\left[R^{-\rho^a}\right] E\left[R^{-\rho^v}\right] \left(1/b_1^a\right)^{\beta/b_2^a} \left(1/b_1^v\right)^{-\beta/b_2^v} \left(\beta^2/b_2^a b_2^v\right)$$

$$\eta = -\beta/b_2^a - 1$$

$$\theta = -\beta/b_2^v - 1$$

with km being the normalization factor, β the slope of the Gutenberg-Richter equation, m_0 the mganitude below which the earthquake events were neglected, $E[R^{-\rho}]$ the expectation of a power of the distance to the earthquake epicenter, and $b_1 b_2 b_3$ being the constants of the attenuation relationships ("a" for acceleration, "v" for velocity). The value of d_i and d_j are given by "Eq. 1" by setting $d_i = g(a_i, v_i)$ and $d_j = g(a_j, v_j)$. "Eq.4" gives the probability of having a certain level of damage state for a given peak acceleration and velocity.

APPLICATION

The model was applied to the 1971 San Fernando earthquake and the results were displayed using the damage probability matrix form. To calculate the integral in "Eq. 4", the intervals of integration for peak acceleration and velocity were based upon results given by the logarithm model (Eq. 1) in the following manner: (1) ranges of integration for velocity were defined by the intersection of the damage level curves of Figure 1 with the V-axis, and (2) the upper and lower limit for acceleration (for a given range of velocity) were bounded by the damage level curves. The probability of having a certain damage state DSi, given a range of velocity Rj, is computed by integrating the acceleration between two consecutive damage curves delineating DSi. In order to evaluate $E[R^{-\rho}]$ the weighted average of the distances of the record stations from the epicenter was found equal to 40 kilometers. This coincides with downtown Los Angeles, where most of the buildings used in the study were located. The seismic source was modeled as a single point. The input data used in the example, together with the results of the application, are shown in matrix form in Table 2. It shows the joint probability of having damage state DSi and velocity range Rj for the site in consideration. The damage for the overall range of velocity used in the study is tabulated in a separate column. DSi appears to be the most likely damage state to occur, coinciding with the observed damage state of the site for which the application was performed. The matrix is site-dependent and can be computed for a particular location if the geometry and seismicity of the area are known.

CONCLUSIONS

The model is limited by the minimal amount of data available, the majority of which is just about the threshold of intensity-causing damage. The correlations of damage with acceleration and velocity should be considered up to the point where the repair cost of the building equals some fraction of its replacement cost. At that level of damage, owners are no longer willing to repair the structure, but would rather replace it. The contributions of the paper are believed to include (1) the use of instrumental strong motion data to develop empirical damage models, (2) the use of both acceleration and velocity in the regression analysis to account for the frequency dependence of buildings, and (3) the use of velocity ranges in the damage matrices as indicators of ground-shaking levels.

REFERENCES

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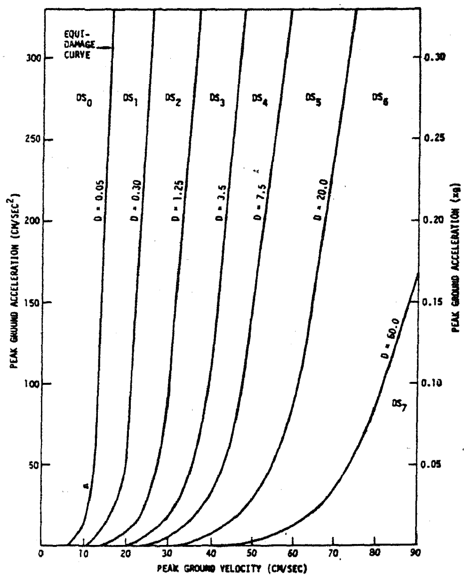

Figure 1. Correlation of Earthquake Damage with Peak Ground Acceleration and Velocity Using the Logarithmic Model

Table 1. Results of Multiple Regression Analysis for Logarithm Model

VARIABLE	MEAN	REGRESSION COEFFICIENT	ST. ERROR OF ESTIMATES	COMPUTED F-VALUE
LOG A	2.057	-0.603	1.000	0.767
LOG V	1.290	3.972	1.550	14.162
LOG D	-0.694			
INTERCEPT			-4.577	
ST ERROR OF ESTIMATE FOR $\bar{Y}$			0.050	

Table 2. Damage Probability Matrix Obtained from the Application of the Probabilistic Seismic Damage Model

INPUT DATA
$\beta = 2.3$
$m_0 = 4$
$m_1 = 8.5$
$b_1^a = 2000$
$b_2^a = 0.80$
$b_3^a = 2$
$b_1^v = 16$
$b_2^v = 1$
$b_3^v = 1.7$
R = 40km.

DAMAGE STATE \ VELOCITY RANGE	R_1 $6.70 \le V \le 10.52$	R_2 $10.52 < V \le 15.07$	R_3 $15.07 < V \le 19.53$	R_4 $19.53 < V \le 23.7$	R_5 $23.7 < V \le 30.3$	R_6 $30.3 < V \le 40.8$	R_7 $6.70 < V \le 40.8$
DS_0 $0 < d \le 0.05$	15.82	10^{-6}	2×10^{-12}	2×10^{16}	$\times10^{-19}$	2×10^{-23}	15.82
DS_1 $0.05 < d \le 0.3$	68.20	2.56	5×10^{-4}	4×10^{-10}	3×10^{-13}	4×10^{-17}	70.76
DS_2 $0.3 < d \le 1.25$		9.11	0.64	4×10^{-5}	3×10^{-8}	4×10^{-12}	9.75
DS_3 $1.25 < d \le 3.5$			1.60	0.16	2×10^{-4}	2×10^{-8}	1.76
DS_4 $3.5 < d \le 7.5$				0.5	0.08	9×10^{-6}	0.58
DS_5 $7.5 < d \le 20$					0.16	0303	0.19
DS_6 $20 < d \le 65$						1016	0.16
DS_7 $65 < d \le 100$							0.00

VELOCITY [cm/sec]