STUDIES OF THE EFFECT OF EARTHQUAKE AND STRUCTURAL UNCERTAINTIES ON OPTIMUM ASEISMIC DESIGN OF LONG SPAN SUSPENSION BRIDGES

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SUMMARY

Although practical aseismic design of structures is based on the best available knowledge, there still exist some uncertainties in both structural modeling and earthquake loading. In this paper, the author tried to evaluate the effect of uncertainties on the earthquake response of tower and pier systems of long span suspension bridges by dynamic analysis using linear statistical approximation. Optimum aseismic design with probabilistic constraints due to uncertainties of excitation using random vibration theory and dynamic reliability analysis is performed.

INTRODUCTION

Studies on aseismic design of long span suspension bridges are especially important in Japan because of Japan's well-known earthquake history. According to these studies[1)], aseismic design of the tower and pier system is very important, since this system and many earthquakes have similar frequencies. The aseismic design of these systems have a considerable safety factor since earthquakes and structures both involve large uncertainties. Thus, a more rational, safe and economical aseismic design is possible if we can clarify the effect of uncertainties of the earthquake response and use an optimization technique.

In the tower and pier system, the elastic modulus of foundation and the damping constant are, among many, the most important uncertainties. The effect of these two structural uncertainties on dynamic response has been calculated herein by random characteristic analysis and linear statistical approximation. The uncertainties of earthquake excitation are estimated by the power spectrum density obtained from a statistical analysis of an earthquake. As the earthquake is assumed to have zero mean and to be a stationary probabilistic process, variances of the displacement and velocity can be evaluated based on random vibration theory. Failure probability can be computed through dynamic reliability theory using displacement and velocity variances. Thus, it is possible to formulate optimization by a probabilistic approach using failure probability as the constraint.

EFFECT OF STRUCTURAL UNCERTAINTIES ON THE EARTHQUAKE RESPONSE

The solution of the eigenvalue problem in an N-degree-of-freedom system can be written as:

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$$\lambda_j M \phi_j = K \phi_j \qquad (j=1,\cdots,N) \qquad \cdots\cdots\cdots\cdots \qquad (1)$$

where M is the mass matrix, K is the stiffness matrix, λ_j is the j-th eigenvalue and ϕ_j is the j-th eigenvector. When M and K are considered to be functions of the random variables r, which are assumed to be stochastically independent of each other, λ_j and ϕ_j are functions of r, similarly. Now linear perturbations may be formed in order to expand these functions in Taylor's series about the mean value of $\bar{r}$, and truncating the series after the third term, the mean value of $\bar{\lambda}_j$ and $\bar{\phi}_j$ are finally given by[2)3)]:

$$\bar{\lambda}_j (r) = \lambda_j (\bar{r}) \qquad \cdots\cdots\cdots\cdots \qquad (2)$$

$$\bar{\phi}_j (r) = \phi_j (\bar{r}) \qquad \cdots\cdots\cdots\cdots \qquad (3)$$

The variances of λ_j and ϕ_j are then

$$\mathrm{Var}(\lambda_j) = \sum_l \left(\frac{\partial \lambda_j (\bar{r})}{\partial r_l} \right)^2 \mathrm{Var}(r_l) \qquad \cdots\cdots\cdots\cdots \qquad (4)$$

$$\mathrm{Var}(\phi_j) = \sum_l \left(\frac{\partial \phi_j (\bar{r})}{\partial r_l} \right)^2 \mathrm{Var}(r_l) \qquad \cdots\cdots\cdots\cdots \qquad (5)$$

The derivatives of the eigenvalues and eigenvectors have been obtained by Fox et al[4)]. The maximum response of the structure can be estimated, if the process is stationary random and a Gaussian distribution with zero mean value[3)].

$$|x|_{max} = \sqrt{2 \sigma_x^2 \ln \left(\frac{T}{\pi} \frac{\sigma_{\dot{x}}}{\sigma_x} \right)} \qquad \cdots\cdots\cdots\cdots \qquad (6)$$

where σ_x and $\sigma_{\dot{x}}$ represent the standard deviation of the response x, and its time derivative $\dot{x}$, and T is the duration time of the response. The mean values and variances of the maximum response $|x|_{max}$ can be obtained using eq. 6:

$$\overline{|x|_{max}(r)} = |x|_{max}(\bar{r}) \qquad \cdots\cdots\cdots\cdots \qquad (7)$$

$$\mathrm{Var}(|x|_{max}(r)) = \sum_l \left(\frac{\partial |x|_{max}(\bar{r})}{\partial r_l} \right)^2 \mathrm{Var}(r_l) \qquad \cdots\cdots\cdots\cdots \qquad (8)$$

The variances of x, $\dot{x}$ are obtained using white noise excitation.

$$\mathrm{Var}(x) = \sum_i \sum_k 4 \pi \phi_i \phi_k \Gamma_i \Gamma_k S_0 A_{ik} / B_{ik} \qquad \cdots\cdots\cdots\cdots \qquad (9)$$

$$\mathrm{Var}(\dot{x}) = \sum_i \sum_k 4 \pi \phi_i \phi_k \Gamma_i \Gamma_k S_0 C_{ik} / B_{ik} \qquad \cdots\cdots\cdots\cdots \qquad (10)$$

where

$$A_{ik} = \beta_i \omega_i + \beta_k \omega_k \quad \cdots\cdots\cdots\cdots \quad (11)$$

$$B_{ik} = (\omega_i^2 - \omega_k^2)^2 + 4 (\beta_i \omega_k + \beta_k \omega_i) \omega_i \omega_k A_{ik} \quad (12)$$

$$C_{ik} = \omega_i \omega_k A_{ik} \quad \cdots\cdots\cdots\cdots \quad (13)$$

where S_0 is the white noise level, Γ_i and Γ_k are i-th and k-th participation factors, ω_i and ω_k are i-th and k-th natural frequencies, β_i and β_k are i-th and k-th damping constant.

OPTIMUM ASEISMIC DESIGN WITH PROBABILISTIC CONSTRAINTS

Dynamic Response Analysis by Probabilistic Constraints

The equation of motion for a multi-degree-of-freedom system can be written as:

$$M \ddot{x} + C \dot{x} + K x = - M \ddot{z} \quad \cdots\cdots\cdots\cdots \quad (14)$$

where M, C and K are the mass, damping and stiffness matrix and x and $\ddot{z}$ are displacement and earthquake acceleration vectors. The earthquake excitation model is represented by a power spectrum density function. As the earthquake is assumed to have zero mean and to be a stationary probabilistic process, the response is assumed to be the same process when the time is long enough. The variances of the displacement and of the velocity can be evaluated based on random vibration theory:

$$\sigma_x^2 = \sum_i \phi_i^2 \Gamma_i^2 \int_{-\infty}^{\infty} | H_i (\omega) |^2 S_F (\omega) d\omega \quad \cdots \quad (15)$$

$$\sigma_{\dot{x}}^2 = \sum_i \phi_i^2 \Gamma_i^2 \int_{-\infty}^{\infty} \omega^2 | H_i (\omega) |^2 S_F (\omega) d\omega \quad (16)$$

where $H_i(\omega)$ is the i-th frequency response function and $S_F(\omega)$ is the power spectrum density function. Failure probability of the system can be computed through dynamic reliability theory using displacement and velocity variances:

$$P_f (\nu , -\nu) = 1 - \exp \left[- \frac{T}{\pi} \frac{\sigma_{\dot{x}}}{\sigma_x} \exp \left(- \frac{\nu^2}{2 \sigma_x^2} \right) \right] \quad (17)$$

where ν refers to the barrier value, $P_f(\nu,-\nu)$ refers to the failure probability and T is the duration time of the earthquake.

Optimization

Design Model To save calculation time and to improve the reliability of the solution, the moment of inertia of the tower and the longitudinal width of the pier are selected as design variables.

Objective Function The generalized cost, W, is selected to be the objective function:

$$W = W_T + \kappa W_P \quad \cdots\cdots\cdots\cdots \quad (18)$$

where W_T represents the weight of the tower and W_P, of the pier, and κ refers to the ratio of the unit cost of the pier to that of the tower.

Constraints The following constraints are considered:

(1) Failure probability of the pier top displacement does not exceed a given allowable failure probability.
(2) Failure probability of the tower shaft stress does not exceed a given allowable failure probability.
(3) The pier is safe against overturning.
(4) The tower shaft is safe against buckling.

Optimization Technique Objective function and constraints obtained in this way become nonlinear and non-differential, so SUMT by Powell's direct search method without differential is employed as the optimization technique.

NUMERICAL EXAMPLES

Results of the Effect of Structural Uncertainties

As a numerical example, the tower and pier system shown in Fig. 1 is considered. The structural uncertainties contained in the system have been investigated in Japan, but it is impossible to estimate their true randomness. The system has bulky dimensions and is constructed in water which increases the uncertainty of the system. From past investigations, the elastic modulus of foundation and the damping constant are the most important uncertainties (among several), and coefficients of variation of these uncertainties are 0.2 (lowest estimate). It is a well known fact that the system has the property of accession and separation of natural frequencies. In this model, the same property can be observed, as shown in Fig. 2. Foundations A to D are investigated in this study.

In order to illustrate the difference of linear statistical approximation and Monte Carlo simulation, natural frequencies were computed by both methods and the results are shown in Table 1. These computation were performed using, for the uncertainty of the system, the elastic modulus of foundation, which has 0.2 in coefficient of variation. In Table 1, $E[\omega]$, $\sigma[\omega]$, $COV[\omega]$ represent the mean, standard deviation and coefficient of variation of the natural frequencies, and the upper value are the results by linear statistical approximation and the lower ones are Monte Carlo simulation results. The values enclosed in heavy lines were the modes where the vibration of the pier was predominant. These modes are very important in order to know the dynamic response of the system. From Table 1, both results show a good agreement, especially for the enclosed in heavy line modes, while the variation of natural frequencies in such coupling modes are larger in proportion than the variation of other modes for all foundations. Thus, the variation of the elastic modulus of the foundation shows a large influence on the variation of natural frequencies which are predominant in the pier vibration.

The coefficient of variation of the maximum displacement and moment are shown in Tables 2 and 3. In making Table 2, we used white noise excitation and the same uncertainty as Table 1, and the following comments may be made. The soft foundation has more influence on the variation of the moment than the hard foundation. Foundation B has a larger influence on the variation of tower displacement and the other foundations have large influence on the pier top displacement. These variations are almost the same for the variation of inputs and are considered fairly large taking into account the filter effect of the structures. So, the probabilistic approach is needed not only for earthquake excitation but also for the foundation model. The results of computations using uncertainties of the damping constant are shown in Table 3. In making Table 3, all modes had 0.2 coefficient of variation of damping. This table shows that the influence of variation is about the same in foundations A to D, and these variation are about half of the input damping variation, which shows that the probabilistic approach is also needed for damping, because of the large damping variation. These tables show that the variation of dynamic response due to the change of elastic modulus of foundation is larger than for the damping constant if they have same variation.

The results of various earthquake excitations are shown in Figs. 3 and 4. These computations were performed using white noise, filtered white noise, El Centro 1940 NS, Taft 1952 N21E, and using the same uncertainties as Table 1. There is some difference in the dynamic response depending on the earthquake excitation used. This can be explained by resonance between the structure and the excitation. The first natural frequency of the system exists in the range lower than 5 rad/sec, as shown in Table 1. In this range, the spectrum of El Centro 1940 NS has a large peak, but the spectrum of Taft 1952 N21E has no such peak, and very small spectrum value. Other spectra are between these two spectra. The effect of uncertainties on the dynamic response are closely related to the predominant frequency range of the power spectrum density of the earthquake.

Results of Optimum Design

As an optimum design example, for the system shown in Fig. 1, the computation results are shown in Tables 4, 5 and 6. These computations were performed using the design model with three design variables, which were the moment of inertia of the upper and lower parts of the tower (I), and the longitudinal width of the pier (b_2). Other variables of the system were defined by approximation concepts[5)6)]. In making Tables 4, 5 and 6, the following data was used: expected maximum acceleration=180 gal, barrier of pier top displacement=0.10 m, barrier stress of steel=46000 ton/m^2, cost ratio=0.2, allowable failure probability=0.001 and Table 4, 5 and 6 used filtered white noise, El Centro 1940 NS and Taft 1952 N21E for the power spectrum density of earthquake excitation, respectively.

From these tables, the following conclusions may be made. When the elastic modulus of the foundation, Es, is small, the optimum design of the system is determined solely by the failure probability constraint of the pier top displacement, for all earthquake excitation models. When the value of Es is large, it tends to be determined by overturning of the pier and buckling of the tower, at places the failure probability of tower shaft stress, and the pier width tends to decrease. This shows that the pier width is closely related to Es. The generalized cost is greatly affected by the modulus of elasticity of the foundation. Thus, the investigation of the foundation is very important. When Es is large, the effect of earthquake response tends to decrease, and stiffness of the tower becomes uniform along the height of the tower. Such probabilistic results are almost the same as from the deterministic approach[5]. But, the probabilistic results obtained using various earthquake excitations have a fairly large difference, especially in the range of small Es. The reason can be explained by resonance between the system and the earthquake excitation. So, the predominant frequency range of the power spectrum density of the earthquake has a large influence on the optimization results.

CONCLUSION

In this paper, uncertainties in both structural modeling and earthquake excitation in aseismic design of tower and pier systems of long span suspension bridges were studied by using linear statistical approximation and optimization techniques. This study shows that the effect of structural uncertainties on the earthquake response are fairly large. So, the probabilistic approach is needed not only for earthquake excitation but also for structural uncertainties. The optimization results show that optimum aseismic design of the system is greatly affected by the modulus of elasticity of the foundation. Thus, the investigation of the foundation is very important. The predominant frequency range of the power spectrum density of the earthquake has a large influence both on the effect of uncertainties on the dynamic response and on the optimization results.

REFERENCES

1. Yamada, Y., Goto, Y. and Takemiya, H., "Studies on the Earthquake-Resistant Design of Suspension Bridge Tower and Pier System", Proc. of 5th WCEE, Vol. 1, pp. 1329-1338, 1974.
2. Hasselman, T.K. and Hart, G.C., "Modal Analysis of Random Structural Systems", ASCE, Vol. 98, No. EM3, pp. 561-579, June, 1972.
3. Yamazaki, Y., "Study on Earthquake Response of Structures by Considering Non-Deterministic Variables", Proc. of 4th Japan Earthquake Engineering Symposium, pp. 871-878, Nov., 1975.
4. Fox, R.L. and Kapoor, M.P., "Rates of Change of Eigenvalues and Eigenvectors", AIAA Journal, Vol. 6, No. 12, pp. 2426-2429, Dec., 1968.
5. Yamada, Y. and Furukawa, K., "Earthquake-Resistant Design of the Tower and Pier System of Suspension Bridges", The Preliminary Report of 10th Congress of IABSE, pp. 183-188, 1976.
6. Templeman, A.B., "Optimization Concepts and Techniques in Structural Design", Introductory Report of 10th Congress of IABSE, pp. 41-60, 1975.

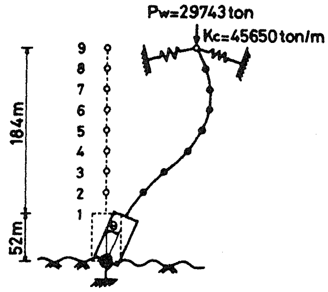

Fig. 1 Structural Model

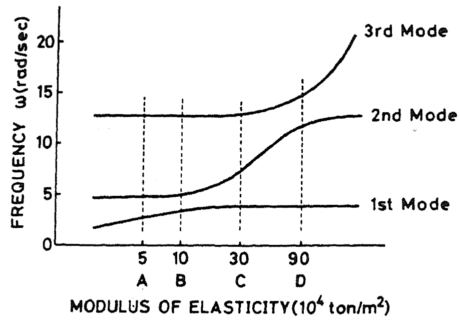

Fig. 2 Natural Frequencies

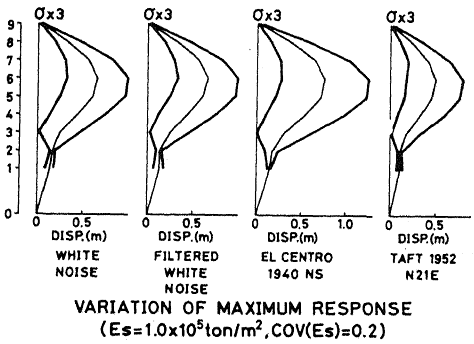

Fig. 3 Variation of Max. Response
[Foundation B]

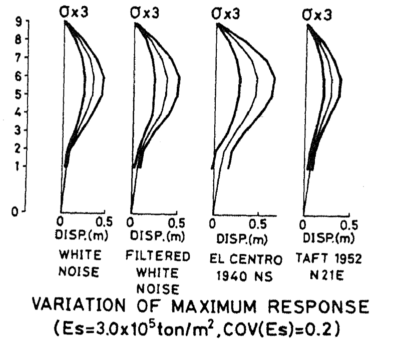

Fig. 4 Variation of Max. Response
[Foundation C]

Table 1 Variation of Natural Frequencies

	FOUNDATION A			FOUNDATION B		
	E[ω]	σ[ω]	COV[ω]	E[ω]	σ[ω]	COV[ω]
ω_1	2.727 2.713	0.2496 0.2551	0.9151×10^{-1} 0.9404×10^{-1}	3.490 3.449	0.1431 0.1739	0.4102×10^{-1} 0.5041×10^{-1}
ω_2	4.080 4.089	0.0477 0.0475	0.1168×10^{-1} 0.1162×10^{-1}	4.544 4.577	0.2746 0.2742	0.6042×10^{-1} 0.5900×10^{-1}
ω_3	12.64 12.64	0.0009 0.1349	0.6860×10^{-4} 0.1067×10^{-1}	12.65 12.65	0.0020 0.1263	0.1545×10^{-3} 0.9988×10^{-2}
	FOUNDATION C			FOUNDATION D		
	E[ω]	σ[ω]	COV[ω]	E[ω]	σ[ω]	COV[ω]
ω_1	3.771 3.768	0.0133 0.0157	0.3523×10^{-2} 0.4166×10^{-2}	3.807 3.807	0.0029 0.0031	0.7708×10^{-3} 0.8071×10^{-3}
ω_2	7.306 7.294	0.7024 0.7266	0.9614×10^{-1} 0.9962×10^{-1}	11.97 11.73	0.6872 0.7197	0.5741×10^{-1} 0.6136×10^{-1}
ω_3	12.67 12.67	0.0103 0.1181	0.8091×10^{-3} 0.9320×10^{-2}	13.29 13.50	0.6656 0.6936	0.5009×10^{-1} 0.5138×10^{-1}

Table 2 Coefficient of Variation of Maximum Response [COV(Es)=0.2]

FOUNDATION		A	B	C	D
DISPLACEMENT	POINT 6	0.081	0.115	0.105	0.051
	POINT 1	0.147	0.089	0.133	0.612
MOMENT	POINT 6	0.145	0.109	0.093	0.116
	POINT 1	0.191	0.089	0.080	0.024

Table 3 Coefficient of Variation of Maximum Response [COV(β)=0.2]

FOUNDATION		A	B	C	D
DISPLACEMENT	POINT 6	0.076	0.085	0.099	0.092
	POINT 1	0.091	0.083	0.097	0.063
MOMENT	POINT 6	0.078	0.088	0.090	0.076
	POINT 1	0.085	0.090	0.085	0.085

Table 4 Optimum Results Using Filtered White Noise

Es	I (m^4)		b_2(m)	W	Constraints					
					Pier		Tower			
($10^4 ton/m^2$)	Upper	Lower			(1)	(2)	Top	(3)	Base	(4)
10	27.06	40.18	45.20	60620	X					
30	7.26	11.66	23.07	31690	X					
50	4.75	9.45	16.65	23430	X					
70	4.75	5.53	14.33	20480		X			X	X
150	4.75	6.73	14.33	20510		X			X	X
300	4.75	4.75	14.33	20460		X				X

(1): Failure probability of the pier top displacement (2): Overturning
(3): Failure probability of the tower shaft stress (4): Buckling

Table 5 Optimum Results Using El Centro 1940 NS

Es	I (m^4)		b_2(m)	W	Constraints					
					Pier		Tower			
($10^4 ton/m^2$)	Upper	Lower			(1)	(2)	Top	(3)	Base	(4)
10	11.17	19.30	58.77	75860	X					
30	13.81	38.08	29.93	41090	X					
50	9.57	42.85	21.60	30570	X					
70	9.32	17.73	17.91	25730	X					
150	4.75	4.75	20.42	27890					X	X
300	4.75	4.75	14.33	20460		X				X

(1): Failure probability of the pier top displacement (2): Overturning
(2): Failure probability of the tower shaft stress (4): Buckling

Table 6 Optimum Results Using Taft 1952 N21E

Es	I (m^4)		b_2(m)	W	Constraints					
					Pier		Tower			
($10^4 ton/m^2$)	Upper	Lower			(1)	(2)	Top	(3)	Base	(4)
10	6.46	13.14	36.37	47860	X					
30	4.86	15.97	18.66	26040	X					
50	4.76	7.61	14.35	20570	X	X				X
70	4.75	4.75	14.33	20460		X				X
150	4.75	4.75	14.33	20460		X				X
300	4.75	4.75	14.33	20460		X				X

(1): Failure probability of the pier top displacement (2): Overturning
(3): Failure probability of the tower shaft stress (4): Buckling