A SIMPLIFIED RANDOM VIBRATION ANALYSIS OF EARTHQUAKE EXCITED INELASTIC MOMENT-RESISTANT FRAMES

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SUMMARY

Inelastic plane frames are modelled as a collection of lumped masses, which are connected by elastic and inelastic, hysteretic spring elements. A random vibration procedure is outlined for such systems, which a) replaces each inelastic spring by an effectively linear one and adds effective viscous damping due to hysteresis, b) takes care of the drift being developped in each inelastic element and c) takes care of any possible combination of elastic or yielding states of inelastic spring elements.

The procedure is applied to a two-storey frame with various system parameters. In an example the frame is optimized in order to obtain equal ductility ratios or equal energy dissipation rates in each storey.

INTRODUCTION

During severe earthquakes many structures are loaded beyond their elastic strength capacity. The additional strength capacity due to inelastic structural behavior might be considered as additional safety against ultimate failure but economic considerations require it be taken into account in design.

A variety of investigations about inelastic structural seismic response is known for different types of structural models e.g.,[1-3]. The method of analysis adopted in such investigations is time integration method. It may be used for nearly any type of inelastic behavior. The main disadvantiages of this method are found in its costs, which become particularly high if several input motions are considered.

Random vibration techniques were found to be useful with linear structures [4,5]. The problem of inelastic structural random response has been mainly considered with single-degree-of-freedom (SDOF) structures [4,6,7] and rarely with multi-degree-of-freedom (MDOF) systems[8,9] . The present paper outlines a procedure for MDOF moment resistant frames with inelastic joints. The method is much less time-consuming than time integration and is therefore well suited for parameter studies. The method seems appropriate for even strongly yielding systems.

EQUATIONS OF MOTION

Fig.1 shows a model of a multi-storey plane frame. During strong motion earthquakes plastic hinges may develop

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near the joints of columns and girders. Due to axial load effects plastic hinges in the columns may ultimately lead to total collapse of the structure, if not designed properly. Some design procedures therefore require that yielding should primarily take place in the girders. In other cases the girders are very heavy and rigid and the structural behavior is similar to that of a shear-beam. Plastic hinges develop only in the columns of such structures.

For the dynamic analysis of a frame like in Fig.1 it is assumed that a) the mass is lumped at the joints of column and girders and b) the moment-curvature relationship of joints is given as an inelastic, bilinear hysteretic relationship. The frame may then be considered as a collection of lumped masses m_i being connected by inelastic joints and elastic spring elements. The latter ones model the elastic stiffness of the column and girder beam elements, respectively. For symmetric frames there is one rotational (φ_i) and one translational (x_i) degree of freedom for each storey. Neglecting inertia of rotation the following equations of motion are obtained

$$
\begin{aligned}
m_i\ddot{x}_i &= k_{c,i}(x_{i-1} - x_i) + \widehat{k}_{c,i}(\varphi_{i-1} - \varphi_i) \\
&\quad - k_{c,i+1}(x_{i+1} - x_i) - \widehat{k}_{c,i+1}(\varphi_{i+1} - \varphi_i) - m_i\ddot{u}
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
0 &= M_{c,i}+M_{c,i+1}+M_{g,i}+\widehat{k}_{c,i}(x_{i-1}-x_i)-\widehat{k}_{c,i+1}(x_{i+1}-x_i) \\
M_{c,i} &= \widehat{\widehat{k}}_{c,i}(\varphi_i-\varphi_{i-1}) \\
M_{g,i} &= \widehat{\widehat{k}}_{g,i}\,\varphi_i
\end{aligned}
\tag{2}
$$

Herein $M_{c,i}$ and $M_{g,i}$ denote the moments in the joints of the i-th column and girder, respectively. The coefficients of the stiffness matrix are hysteretic function of their corresponding inter-storey displacements or rotations. The elastic stiffness coefficients may be found in Ref.1. The rotational degrees of freedom can only be eliminated in Eq.1 if the (elastic or yielding) state of each inelastic joint is known. For the sake of simplicity only shear-beam type frames are considered in the following. Eq.1 then simplifies to

$$
\begin{aligned}
m_i\ddot{x}_i &= k_i(x_{i-1}-x_i)+c_i(\dot{x}_{i-1}-\dot{x}_i) - \qquad\qquad i=1,..n-1 \\
&\quad + k_{i+1}(x_{i+1}-x_i)+c_{i+1}(\dot{x}_{i+1}-\dot{x}_i) - m_i\ddot{u} \\
m_n\ddot{x}_n &= k_n(x_{n-1}-x_n)+c_n(\dot{x}_{n-1}-\dot{x}_n)-m_n\ddot{u}
\end{aligned}
\tag{3}
$$

where damping forces c_i have been included. The index c has been omitted. Bilinear hysteretic behavior (Fig.2) will be assumed.

METHOD OF SOLUTION

Basic Equations

The present method is an extension of a linearization scheme for SDOF systems [7] to MDOF systems. For MDOF-structures interaction between different inelastic or elastic spring elements has to be considered as a new feature in the analysis. Each inter-storey displacement is written as

$$x_i - x_{i-1} = z_i + \eta_i \tag{4}$$

where η_i denotes the low-frequent drift component being developped due to yielding of the inelastic spring element i. The restoring force of this element may considered to be effectively proportional to the z_i component. The solution for the z_i components is therefore obtained from a system of linearized equations

$$
\begin{aligned}
&m_i\ddot{z}_i+\bar{c}_i\dot{z}_i+\bar{k}_i z_i=+\bar{k}_{i+1}z_{i+1}+\bar{c}_{i+1}\dot{z}_{i+1}-m_i\ddot{u}-m_i z_{i+1} \qquad (i=1,..n-1) \\
&m_n\ddot{z}_n+\bar{c}_n\dot{z}_n+\bar{k}_n z_n=-m_n\ddot{u}
\end{aligned}
\tag{5}
$$

where $\bar{c}_i$ and $\bar{k}_i$ denote the effective damping force and effective stiffness, respectively, of inelastic spring element i.

For stationary random inputs the covariance matrix with elements $< z_i \dot{z}_j >$ may be obtained from a system of n linear algebraic equations using the state vector approach [10]. The earthquake excitation has to be modelled as filtered white noise for that purpose (e.g., with a Kanai-Tajimi power spectral density function $S_{\ddot{u}}(\omega)$).

The effective parameters are obtained from temporal averages as

$$\begin{aligned} \overline{k}_i &= k_{io}(1 - \sum_j q_{ij}) + \sum_j k_{ij,y} q_{ij} \\ \overline{c}_i &= c_{io}(1 - \sum_j q_{ij}) + \sum_j c_{ij,y} q_{ij} \end{aligned} \tag{6}$$

where q_{ij} denotes the mean fraction of time during which element i is in a yieldings cycle, if a structural yielding configuration j is assumed. A structural yielding configuration is specified by assuming each inelastic element to be in an either elastic or yielding response cycle. k_{io} and c_{io} denote the nominal stiffness and damping force of element i. The weighs q_{ij} are evaluated from

$$q_{ij} = \exp\left(-\frac{1}{2} \sum_k \sum_l u_{kl} x_k^* x_l^*\right) \tag{7}$$

where x_i^* are the yield displacements and u_{kl} the elements of the inverse matrix of the covariance matrix $<\zeta_k \zeta_l>$. Only those inelastic elements have to be considered in eq.(7), which are assumed to be during a yielding cycle in structural yielding configuration j. The solutions for $<\zeta_k \zeta_l>_j$ are obtained from Eq.5 if c_i and k_i are replaced by c_{ij}^* and k_{ij}^* as follows

$$(k_{ij}^*, c_{ij}^*) = \begin{cases} (k_{io}, c_{io}) & \text{if element i is not in a yielding cycle} \\ (k_{ij,y}, c_{ij,y}) & \text{if element i is in a yielding cycle} \end{cases}$$

The yielding stiffness ($k_{ij,y}$) and damping force ($c_{ij,y}$) which are effective during a yielding cycle, depend upon the absolute yield increment $m_{\Delta_{ij}}$ [7] during structural yielding configuration j.

$$\begin{aligned} k_{ij,y} &= k_i\left[(\theta_{ij} - \sin\theta_{ij} \cos\theta_{ij})(1-\alpha_i)/\pi + \alpha\right] \\ \theta_{ij} &= \cos^{-1}(m_{\Delta_{ij}} - 2x_i^*)/(m_{\Delta_{ij}} + 2x_i^*) \\ c_{ij,y} &= \frac{8k_i \, m_{\Delta_{ij}} \, x_i^*(1-\alpha_i^*)}{\pi\sqrt{\overline{k}_i}\,(m_{\Delta_{ij}} + 2x_i^*)^2} \end{aligned} \tag{8}$$

Likewise to SDOF system [7] an approximation for the average yield increment is found from an energy balance as

$$\begin{aligned} m_{\Delta_{ij}} = & \Big[(S_{\ddot{u}}(\omega_o)\pi + \frac{c_{i+i,j}^*}{m_i} < \dot{\zeta}_{i+1} \dot{\zeta}_i >_j + \frac{k_{i+i,j}^*}{m_i} < \zeta_{i+1} \dot{\zeta}_i > \\ & - m_{\Delta_{i-1,j}} k_{i-1,j} / (k_{io} - k_{ij,y}) \cdot \sigma_{z_i} / \sqrt{k_{ij,y}} \; \gamma_1 \left[1 - c_{io}/(2\overline{c}_i)\right]\Big] \end{aligned} \tag{9}$$

A coefficient γ_1 has been introduced in order to account for filtered white noise input spectra $S_{\ddot{u}}(\omega)$ as in [11]. ω_o denotes the first natural frequency of the associated linear structure. It is observed from eq.9 that the amount of yielding does not only depend upon the input strength but is also affected by interaction with neighbouring elements. Smaller yield increments are expected, if two neighbouring elements simultaneously yield than if only one is in the yielding regime.

The eqs.(6) to (9) have to be solved iteratively. Only a few iteration steps are usually required.

Drift Components of Response

Once the solutions of the z_i-components are found the drift component of response η_i of each element may be evaluated likewise to SDOF structures [7]. The variance equation has the general form

$$\frac{d\sigma^2_{\eta_i}}{dt} = f(\sigma_{\eta_i}, \sigma_{z_i}, m_{\Delta_i}, \bar{c}_i, \bar{k}_i, \alpha_i) \tag{10}$$

and is solved numerically, which gives the transient and steady-state solution $\sigma_{\eta_i}(t)$. The mean yield increment m_{Δ_i} is obtained from averaging over all possible structural yielding configurations as

$$m_{\Delta_i} = (\sum_j m_{\Delta_{ij}} \; q_{ij})/\sum_j q_{ij} \tag{11}$$

The function f( ) may be found in [7]. The time step Δt applied in the solution of eq.(10) can be taken in the order of the nominal period of the structure. In case of vanishing upper yield slope ($\alpha_i=0$) eq.(10) can be solved analytically [7]. The frequency ω_{η_i} may be taken as a representative frequency of the drift component η_i

$$\omega_{\eta_i} = \frac{\sigma_{\dot{z}_i}}{\sigma_{z_i} \; N_{c_i}} \exp(-x_i^{*}/\sigma^2_{z_i} 2) \tag{12}$$

where N_{c_i} denotes the mean clump size of crossings of the yield displacement level x_i^* by the (effectively linear) z_i-component. Again, it is found in [7].

Hysteretic Energy Dissipation Rate

The hysteretic energy dissipation rate $\dot{E}_{h,i}$ is given by the number of yielding cycles per unit time and the mean yield increment as

$$\dot{E}_{h,i} = k_{io}\omega_{\eta_i} m_{\Delta_i} \; x_i^* \; N_{c_i}/\pi \tag{13}$$

Inter-Storey Ductility Ratios

The inter-storey ductility ratio is defined as

$$\mu_i = \frac{\max_t (x_i(t)-x_{i-1}(t))}{x_i^*} \tag{14}$$

It may be computed from

$$\mu_{i,p} = \sigma_{x_i - x_{i-1}} \; r_{i,p} \tag{15}$$

The peak factors $r_{i,p}$ may be evaluated as for SDOF-systems [7]. The variance of the inter-storey displacement is

$$\sigma^2_{x_i - x_{i-1}}(t) = \sigma^2_{\eta_i}(t) + \sigma^2_{z_i} \tag{16}$$

It is assumed that the maximum response occurs at the end of the considered time interval.

Limitations of the Method

The method is an approximate procedure based on effective structural properties during stationary (elastic or yielding) response cycles. The method should therefore not be used for very short earthquake inputs with only one or two strong acceleration pulses, where even the z_i components reach stationary conditions. But the present approach can be used for nonstationary input motions if their intensity is slowly varying with time. Eq.(5) has then to be resolved after each time step in order to obtain the nonstationary z_i-components.

The procedure takes care of all possible yielding configuration. For structures with many degrees of freedom the total number of structural yielding configurations becomes very high and the iteration procedure might then become rather lengthy. A practical methodology for restricting the analysis to the most important structural yielding configurations has to be adopted for such cases.

No simulation results are available at this point in order to determine the accuracy of the method. However, because the procedure is an extension of a linearization scheme recently proposed for SDOF structures [7], the accuracy might be expected to be similar to there. The results for SDOF systems show good agreement to simulation estimates.

APPLICATIONS

The method is illustrated with a two storey frame. The ground motion is assumed to have a power spectral density function of the form

$$S_{\ddot{u}}(\omega) = S_o \frac{1 + 4\zeta_g^2(\omega/\omega_g)^2}{(\omega^2-\omega_g^2)^2+4\zeta_g^2(\omega/\omega_g)^2} \tag{17}$$

The parameters are selected as $\zeta_g = 0.6$ and $\omega_g = 18$. The duration of motion is 20 sec.

There are three different structural yielding configuration that might occur (1.first storey yielding and second storey elastic; 2. both storeys yielding; 3.first storey elastic and second storey yielding). The ratio of nominal stiffnesses k_{1o}/k_{2o} determines which one is dominating. The calculations for systems with stiffness ratios k_{1o}/k_{2o} about one and equal yield displacements $x_1^*=x_2^*$ show that yielding configuration 2 becomes more and more dominating with increasing excitation strength. For $k_{2o} \gg k_{1o}$ the structure behaves like a SDOF system and good agreement was found to results known for such systems. In this case the energy is mainly absorbed in the first storey, which leads to small amplitudes of response in the upper storey (like in a base isolation device).

Fig.3 gives results for the r.m.s. inter-storey displacements $\sigma_{x_i - x_{i-1}}$ for various combinations of structural parameters. The value S_o may be related to the mean maximum ground acceleration A, which, for the given parameters, yields

$$A = 34.3 \; S_o^{1/2} \tag{18}$$

It is observed from Fig.3 that the normalized response of the first floor for $\zeta_o \neq 0$ approaches a minimum with increasing input strength, whereas no such minimum is found for the upper floor. The weaker storey seems to absorb more and more energy as the excitation strength increases. This has been also observed in [9]. The results for constant input spectra ($S_{\ddot{u}}=S_o$=const) are similar in shape to those for Kanai-Tajimi input spectra. This does generally not hold for structures with nominal fundamental frequencies ω_o above ω_g. The effect of nominal viscous damping ($c_{io}=2\zeta_o m_i k_{io}^{1/2}$) is particularly severe for low input strength.

Comparison to SDOF systems

The SRSS superposition rule in connection with inelastic response spectra is sometimes applied to even nonlinear structures. For the purpose of comparison the elastic mode shapes are therefore computed for the structure with parameters of case 1 in Fig.3. The ductility ratios are then evaluated as for SDOF-structures [7] and mode superposition is applied. The following results are obtained:

$<\mu_1>$	SDOF-an.	MDOF an.	$<\mu_2>$:	SDOF-an.	MDOF an.
$S_o = 50x_1^*$	3.11	2.64		4.65	4.22
$S_o = 150x^*$	8.69	3.27		14.40	8.01

Such a superposition method seems to be acceptable only for low values of input strength.

Optimization of Structural Parameters

The preceeding and many other examples show that inelastic structural response is often concentrated in only one or a few members of the structure. However, for many structures it is sometimes desired to distribute the energy dissipation, about equally among all members. This is illustrated in Fig.4, where a two-storey frame is considered with fixed fundamental frequency ω_o=1.22 cps and damping ζ_o=0.02. The optimization criteria to be applied are either a) equal ductility ratios μ_i or b) equal energy dissipation rates $\dot{E}_{h,i}$ in both storey. The ratio $\varkappa=k_{1o}/k_{2o}$ is taken as design parameter. The optimum design value $\varkappa_{opt}$ is nearly identical for both criteria. $\varkappa_{opt}$ is about 1.55 for $S_o=50x_1^*$ and 1.8 for $S_o=150x_2^*$. It is also observed that even small deviations of $\varkappa$ around $\varkappa_{opt}$ may lead to rather large differences in the storey ductility ratios or energy dissipation rates. Depending on the input strength an optimal design value derived from a linear analysis may be far from as optimal design value therefore for a nonlinear structure. (In the present example $\varkappa_{opt}$ was 1.50 for a linear structure).

Also given in Fig.4 is a simulation result for the ductility ratio of a SDOF-system. As anticipated, the solution for $<\mu_1>$ is close to that value, if $\varkappa$ becomes very small.

CONCLUSIONS

A new method for the analysis of inelastic moment-resistant frames under random earthquake excitation has been presented. Bilinear hysteretic behavior has been assumed for the inelastic joints. After introducing effective stiffness and damping parameters a linearized system may be obtained, the response of which is superimposed to the drift response of each inelastic joint.

Based on the presented as well as other examples the following conclusions may be drawn:

1. The inelastic response tends to overproportionally concentrate in the weaker structural members. This implies that the mode shapes derived from the effective structural parameters may become very different from those of a linear structure.

2. The response is rather sensitive to small variations

in α_i, if α is small.

3. The method represents an efficient tool in order to design optimal structures with respect to given design criteria.

4. Even for nonstationary structural response, the numerical efforts required in the present analysis are always substantially smaller than those for a time integration analysis, where much smaller integration time steps have to be applied.

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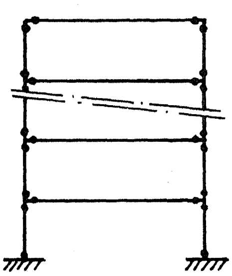


FIG. 1

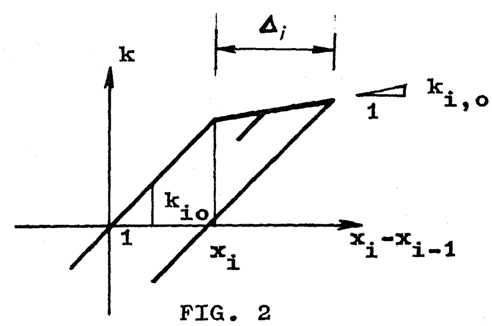
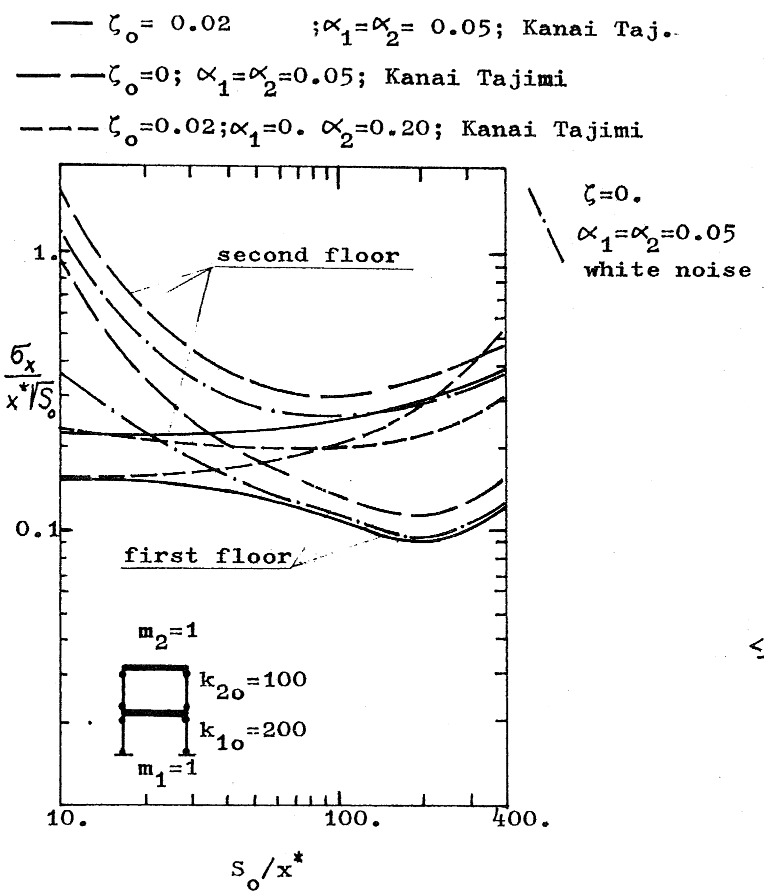
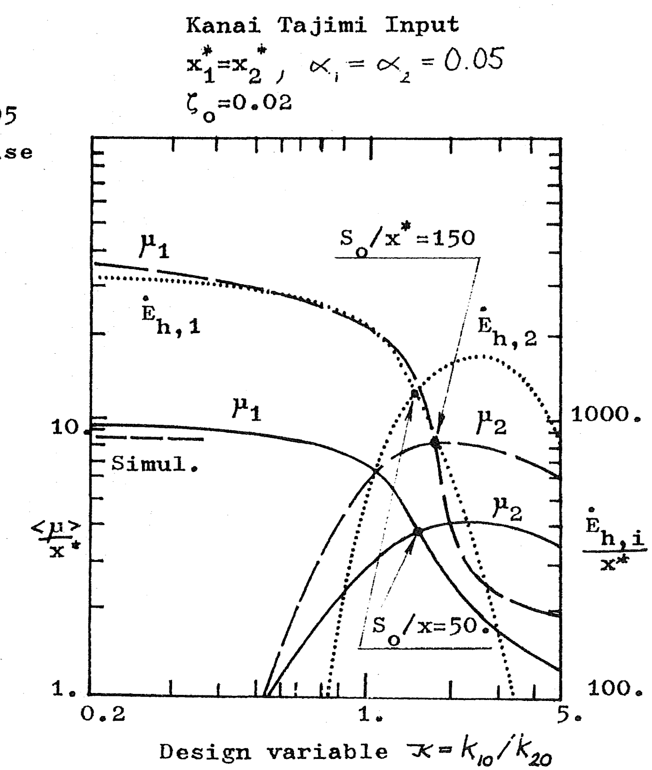


FIG. 2



FLOOR RESPONSE
FIG.3



OPTIMIZATION OF FRAME
FIG.4