

Drift Components of Response

Once the solutions of the z_i -components are found the drift component of response η_i of each element may be evaluated likewise to SDOF structures [7]. The variance equation has the general form

$$\frac{d\sigma_{\eta_i}^2}{dt} = f(\sigma_{\eta_i}, \sigma_{z_i}, m_{\Delta_i}, \bar{c}_i, \bar{k}_i, \alpha_i) \quad (10)$$

and is solved numerically, which gives the transient and steady-state solution $\sigma_{\eta_i}(t)$. The mean yield increment m_{Δ_i} is obtained from averaging over all possible structural yielding configurations as

$$m_{\Delta_i} = \left(\sum_j m_{\Delta_{ij}} q_{ij} \right) / \sum_j q_{ij} \quad (11)$$

The function $f(\)$ may be found in [7]. The time step Δt applied in the solution of eq.(10) can be taken in the order of the nominal period of the structure. In case of vanishing upper yield slope ($\alpha_i=0$) eq.(10) can be solved analytically [7]. The frequency ω_{η_i} may be taken as a representative frequency

$$\omega_{\eta_i} = \frac{\sigma_{z_i}}{\sigma_{z_i} N_{c_i}} \exp(-x_i^*/6\sigma_{z_i}^2) \quad (12)$$

where N_{c_i} denotes the mean clump size of crossings of the yield displacement level x_i^* by the (effectively linear) z_i -component. Again, it is found in [7].

Hysteretic Energy Dissipation Rate

The hysteretic energy dissipation rate $\dot{E}_{h,i}$ is given by the number of yielding cycles per unit time and the mean yield increment as

$$\dot{E}_{h,i} = k_{i0} \omega_{\eta_i} m_{\Delta_i} x_i^* N_{c_i} / \pi \quad (13)$$

Inter-Storey Ductility Ratios

The inter-storey ductility ratio is defined as

$$\mu_i = \frac{\max_t (x_i(t) - x_{i-1}(t))}{x_i^*} \quad (14)$$

It may be computed from

$$\mu_{i,p} = \frac{\sigma_{x_i - x_{i-1}}}{x_i^*} r_{i,p} \quad (15)$$

The peak factors $r_{i,p}$ may be evaluated as for SDOF-systems [7]. The variance of the i -th inter-storey displacement is

$$\sigma_{x_i - x_{i-1}}^2(t) = \sigma_{\eta_i}^2(t) + \sigma_{z_i}^2 \quad (16)$$

It is assumed that the maximum response occurs at the end of the considered time interval.

Limitations of the Method

The method is an approximate procedure based on effective structural properties during stationary (elastic or yielding) response cycles. The method should therefore not be used for very short earthquake inputs with only one or two strong acceleration pulses, where even the z_i components reach stationary conditions. But the present approach can be used for non-stationary input motions if their intensity is slowly varying with time. Eq.(5) has then to be resolved after each time step in order to obtain the nonstationary z_i -components.

case 1 in Fig.3. The ductility ratios are then evaluated as for SDOF-structures [7] and mode superposition is applied. The following results are obtained:

$\langle \mu_1 \rangle$	SDOF-an. MDOF an.		$\langle \mu_2 \rangle$:	SDOF-an. MDOF an.	
$S_0 = 50x_1^*$	3.11	2.64		4.65	4.22
$S_0 = 150x_1^*$	8.69	3.27		14.40	8.01

Such a superposition method seems to be acceptable only for low values of input strength.

Optimization of Structural Parameters

The preceding and many other examples show that inelastic structural response is often concentrated in only one or a few members of the structure. However, for many structures it is sometimes desired to distribute the energy dissipation, about equally among all members. This is illustrated in Fig.4, where a two-storey frame is considered with fixed fundamental frequency $\omega_0 = 1.22$ cps and damping $\zeta_0 = 0.02$. The optimization criteria to be applied are either a) equal ductility ratios μ_i or b) equal energy dissipation rates \dot{E}_{p_i} in both storey. The ratio $\mathcal{K} = k_{10}/k_{20}$ is taken as design parameter. The optimum design value \mathcal{K}_{opt} is nearly identical for both criteria. \mathcal{K}_{opt} is about 1.55 for $S_0 = 50x_1^*$ and 1.8 for $S_0 = 150x_2^*$. It is also observed that even small deviations of \mathcal{K} around \mathcal{K}_{opt} may lead to rather large differences in the storey ductility ratios or energy dissipation rates. Depending on the input strength an optimal design value derived from a linear analysis may be far from an optimal design value therefore for a nonlinear structure. (In the present example \mathcal{K}_{opt} was 1.50 for a linear structure).

Also given in Fig.4 is a simulation result for the ductility ratio of a SDOF-system. As anticipated, the solution for $\langle \mu_1 \rangle$ is close to that value, if \mathcal{K} becomes very small.

CONCLUSIONS

A new method for the analysis of inelastic moment-resistant frames under random earthquake excitation has been presented. Bilinear hysteretic behavior has been assumed for the inelastic joints. After introducing effective stiffness and damping parameters a linearized system may be obtained, the response of which is superimposed to the drift response of each inelastic joint.

Based on the presented as well as other examples the following conclusions may be drawn:

1. The inelastic response tends to overproportionally concentrate in the weaker structural members. This implies that the mode shapes derived from the effective structural parameters may become very different from those of a linear structure.
2. The response is rather sensitive to small variations

