A STATISTICAL TECHNIQUE FOR RELATING EARTHQUAKE TIME HISTORIES AND RESPONSE SPECTRA

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SUMMARY

A new statistical analysis technique is presented for estimating the peak response of a linear single-degree-of-freedom system subjected to a transient random excitation. The technique may be used in a forward sense to predict the peak response of a simple structural system subjected to an earthquake-like transient random process. It may also be used in a backward sense to define an ensemble of earthquake-like transient random processes which correspond to given peak response data. The technique thereby provides a consistent and statistically sound framework for generating "artificial" records from given design response spectra.

INTRODUCTION

One of the most important and widely discussed problems in earthquake resistant design is the determination of the maximum response of a structure subjected to an earthquake excitation. If the structure is linear and the earthquake time history is known, the problem is straightforward and can be handled by a variety of analytical and numerical techniques. For very simple systems, the desired result is often graphed in the form of a response spectrum.

The analysis problem is greatly simplified when the earthquake is taken to be a known deterministic function of time. However, it has become more and more common to consider earthquakes to be nondeterministic stochastic functions of time. The problem of determining the peak response of randomly excited linear structures is much more formidable. Although considerable progress has been made in this area, there is still a need for accurate and simple solution techniques which relate the statistics of the peak response of the system to a probabilistic description of the excitation and vice versa.

Consider the simple structural model represented by the single-degree-of-freedom equation of motion

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = \theta(t)n(t) \tag{1}$$

where ζ is the fraction of critical damping of the structure or mode, ω_0 is the undamped natural frequency, $n(t)$ is a random noise process with spectral density S_0 and $\theta(t)$ is a modulating function of time. First, consider the case where $n(t)$ is a white noise and $\theta(t)$ is equal to unity for all time.

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The probability W(T) that the magnitude of the response x remains less than some value b throughout a time interval [0,T] will obviously be dependent upon the initial conditions imposed on Eq. 1. However, it has been observed that the effects of the initial conditions tend to die out as T becomes large compared to the natural period of the oscillator [1]. Specifically, it has been observed that W(T) approaches a decaying exponential of the form

$$W(T) \sim e^{-\alpha T} \tag{2}$$

regardless of the initial conditions [2]. The parameter α is referred to as the limiting decay rate of the first crossing density.

Numerous approximate solutions have been proposed for α and thereby W(T). Of these, the most accurate schemes generally involve the generation of an approximate solution for the conditional transition probability density governing first passage. Such approaches are, however, quite involved numerically and therefore rather costly.

For sufficiently small damping, the sample functions of x(t) have an approximately sinuosidal appearance. Hence, the magnitudes of the peaks of these quasi-harmonic sample functions may be treated as a one-dimensional continuous-state, discrete time Markov process. Mark [3] has suggested the approximation that the peaks are separated by intervals of exactly one half the structural natural period. A conditional transition probability density for the magnitude of a peak given the value of the preceding peak is then derived. This leads to an integral equation which must be solved numerically. This approach requires substantial numerical computation but is somewhat more efficient than Monte Carlo simulation or diffusion of probability methods.

A number of simpler approximate solutions for the limiting decay rate also exists. These require much less effort to implement but are generally less accurate. Of these, the simplest involves the assumption that the level crossings are statistically independent events. This will be approximately true when the value of b is large compared with the root mean square value of the response σ so that the average interval between successive up-crossings of b becomes very long. Making this assumption, the times at which up-crossings occur constitutes a Poisson process, and the intervals between up-crossings will be exponentially distributed. The rate of down-crossings will be the same as the rate of up-crossings. Thus, the resulting approximation for α will be a function of the structural natural frequency, the root mean square value of the response and the level b.

For barrier levels of practical interest, the Poisson approximation is usually overly conservative. For very high barrier levels, however, the approximation becomes quite good and is in fact asymptotic to the limiting decay rate as $b \to \infty$ [4].

Corotis, Vanmarcke and Cornell [5] have proposed a scheme to obtain improved approximations to α from the average frequency of up-crossings of the level b. The stationary envelope response process a(t) is treated as a two-state, continuous time, Markov process in which the state 0 corresponds to $a < b$, and state 1 corresponds to $a > b$. The intervals of time spent in states 0 and 1, respectively, are assumed to be independent random variables with exponential distributions. Employing a physical argument to estimate the fraction of envelope crossings that are immediately followed by a level crossing leads to an approximation for α which is a function not only of the natural frequency of the structure but also its damping. This approximation is more accurate than the Poisson process approximation and predicts the correct qualitative behavior of α with variations in crossing level b. However, it is possible to obtain a further improvement in accuracy with little additional computational effort as indicated below.

A NEW APPROACH TO THE FIRST PASSAGE PROBLEM

It is widely agreed that the basic reason the Poisson process approximation for α breaks down for low barrier levels is that the crossings of the barrier level are, in effect, not statistically independent events. Since the envelope varies slowly, when a peak occurs above the threshold level the probability is higher than usual that the next peak will also be above the threshold. Thus, level crossings tend to occur together in clumps. If allowance were made for the clumping tendency, one would expect a more accurate estimate for α to result.

Let the clump duration T_1 be defined as the number of successive peaks that occur outside the level b with no intervening peak inside this level divided by twice the natural frequency of the structure in Hz. Then, assuming that the response process is ergodic, that the time between clumps has an exponential distribution, and that the clump size and the time between the beginning of successive clumps are independent, it may be shown that the limiting decay rate α is inversely proportional to the expected value of the clump duration $E[T_1]$. Hence, the problem of determining the limiting decay rate is reduced to the determination of $E[T_1]$.

Let $P(n+1|n)$ be the conditional probability that a clump which already contains n level crossings will continue for at least one more crossing. Then, $P(n+1|n)$ will increase rapidly with n for n small. However, as n becomes large, $P(n+1|n)$ will approach a limiting value P^*. It is assumed that the probability that T_1 is greater than some value s is of the form of a negative exponential in s multiplied by s. Then, it may be shown that [6]

$$\alpha = \nu_b \left[1 - \exp\left(- \frac{b^2}{2\sigma^2} \right) \right]^{-1} \ln (1/P^*) \qquad (3)$$

where ν_b is the well-known Poisson up-crossing rate of the level b. P^* is estimated by considering the response of the system for one-half cycle of oscillation after a peak greater than b has occurred.

Suppose the oscillator is at a peak during a clump in which k level crossings have already occurred and let $p_k(r)$ be the conditional probability density of such peaks, given that $r \geq b$. Due to the narrow-bandedness of the response, it will be assumed that any two successive peaks will be separated by an interval of π/ω_d, and that x changes sign during this interval. With this assumption, $p_{k+1}(r)$ may be expressed as a convolution of $p_k(r)$ and the conditional probability $q(x_1|x_2{:}\pi/\omega_d)dx_2$ that a response trajectory which starts at x_1 reaches the differential element of a measure dx_2 centered at x_2 at a time π/ω_d later. As $k \to \infty$, $p_k(r)$ approaches a stationary density which is assumed to be of the form of a clipped Gaussian distribution. This yields

$$P^* = \int_b^\infty \frac{\exp\left(-\frac{r^2}{2\sigma^2}\right)}{\sigma\sqrt{2\pi}\ \mathrm{erfc}\left(\frac{b}{\sqrt{2}\ \sigma}\right)}\ \mathrm{erfc}\left\{\frac{b - r\exp\left(-\frac{\omega_0}{\omega_d}\right)}{\sqrt{2}\ \sigma\left[1 - \exp\left(-\frac{\omega_0}{\omega_d}\right)\right]^{\frac{1}{2}}}\right\} dr \qquad (4)$$

The integral of Eq. 4 is approximated analytically by assuming a tri-linear representation of the complementary error function which matches the function exactly at its two asymptotes and at the point where the argument is equal to 0. The results of the present approach are less conservative than either the Poisson approximation or Corotis, Vanmarcke and Cornell's estimate for α and correspond well with the simulation results.

ESTIMATION OF RESPONSE SPECTRA

Using the results for stationary response, the nonstationary response problem may be treated approximately by applying the approach proposed by Corotis, et al. [5]. The Fourier transform of the autocorrelation of the excitation will be $\theta^2(t)S_0$ which may be though of as a time dependent intensity or "spectral density." It is assumed that the probability density of the response is a slowly varying function of t so that it may be approximated by a stationary density over one period of the system. Then it is assumed that the first level crossing during a period of the system will occur with approximately the same frequency as if the response were truly stationary. Making these assumptions, an expression for the instantaneous first crossing rate $\alpha(t)$ may be derived in a fashion entirely analogous to the derivation of the limiting decay rate given by Eq. 3. The form will be identical except for the functional dependence of each term on t. $\nu_b(t)$ may be evaluated in a straightforward manner using the quasi-stationary assumption. The procedure to obtain $P^*(t)$ is similar to that followed to determine the corresponding quantity P^* for the stationary case except that the excitation intensity is now $\theta^2(t)S_0$ which changes the form of Eq. 4 somewhat. With $\nu_b(t)$ and $P^*(t)$ specified, the first passage probability is given by

$$W(t) = \exp\left[-\int_0^t \alpha(s)ds\right] \qquad (5)$$

Let $|x_m(\omega_0,\zeta)|$ represent the maximum displacement of the structure due to the earthquake-like excitation $\theta(t)n(t)$. Then, $|x_m(\omega_0,\zeta)|$ must occur during a time interval τ equal to the duration of the excitation plus one half period of the structure. For a given sample function of the excitation, $|x_m(\omega_0,\zeta)|$ would be a deterministic function. However, since the excitation is assumed to be a stochastic process, $|x_m(\omega_0,\zeta)|$ will be a random variable. The response spectrum therefore takes on a probabilistic form. Let the confidence limit P_s be defined as the probability that $|x_m(\omega_0,\zeta)|$ does not exceed some value $SD(\omega_0,\zeta,P_s)$. Then, the response spectrum displacement $SD(\omega_0,\zeta,P_s)$ may be defined probabilistically by the relationships

$$SD(\omega_0,\zeta,P_s) = b \tag{6}$$

and

$$W(\tau) = P_s \tag{7}$$

APPLICATION TO THE EARTHQUAKE RESPONSE PROBLEM

The previous analysis may be employed in a number of different ways. If the excitation parameters $\theta(t)$ and $n(t)$, the structural parameters ω_0 and ζ, and the confidence limit P_s are given, Eqs. 3-5 may be used to determine the probabilistic spectral displacement SD. If the structural parameters are varied, a result analogous to the usual earthquake response spectrum graph may be obtained. This is the forward problem.

The problem may also be cast in a backward sense. That is, given the structural parameters, the confidence limit and the spectral displacement, find a class of stochastic excitations which will generate this peak response. This form of the problem can be useful in generating ensembles of time histories which correspond to a given probabilistically defined response spectrum.

In order to define the stochastic excitation process, it will herein be assumed that $\theta(t)$ is a known smooth function of t. In particular, it is assumed that [7]

$$\begin{aligned} \theta(t) &= \theta_0 \quad &;\quad t \le t_1 \\ \theta(t) &= \theta_0 \exp\left[-\theta_1(t-t_1)\right] \quad &;\quad t \ge t_1 \end{aligned} \tag{8}$$

where θ_0, θ_1, and t_1 are parameters to be defined later. It is further assumed that $n(t)$ is a gray noise with smoothly varying spectral density $S(\omega)$.

If the structural system is lightly damped, the covariance matrix of the response may be approximated as [8]

$$Q(t) \approx S(\omega_0) \int_{-\infty}^{\infty} \underset{\sim}{F}(\omega,t)\, \underset{\sim}{F}^{*T}(\omega,t)\,d\omega \tag{9}$$

where F is the deterministic 2n-space solution of the response of the structure to a modulated harmonic excitation. The components of the covariance matrix Q may then be used to solve for ν_b and subsequently $W(t)$. The backward problem may be solved by finding the value of $S(\omega_0)$ which gives $W(t) = P_s$ according to Eq. 6. This may be accomplished numerically with no difficulty. If the procedure is repeated for different values of ω_0, the spectral density of the process $n(t)$ is generated.

EXAMPLE OF APPLICATION

As an example of the application of the present approach, consider the problem of determining a class of nonstationary random processes which correspond to the response spectra specified by the United States Nuclear Regulatory Commission Guide 1.60 [9]. The 2% and 5% damped spectra for a nominal 1g peak acceleration will be modeled. It will be assumed that the confidence limit for these spectra is $P_s = 84.2\%$. The envelope $\theta(t)$ will be specified by the parameters $\theta_0 = 1$, $\theta_1 = 0.0992$/sec and $t_1 = 15$ sec. Then, except for the absence of an initial buildup and final decay region, these parameters correspond to the Caltech B-type earthquake envelope [7].

The target response spectra are shown in Fig. 1. Application of the technique herein described to the two spectra yields random processes with the power spectral densities also shown in Fig. 1. It is seen that the different response spectra correspond to different power spectra. Hence, within the context of the present analytical framework, the different response spectra of the Regulatory Guide can not be associated with a single probabilistically defined excitation process. However, the differences between the two power spectra for 2% and 5% damping is less than 20% for the frequency range of 0.2-10.0 Hz.

Sample functions were generated for the processes indicated by a superposition of harmonic functions with prescribed amplitudes and randomly distributed phases. Two representative sample functions are shown in Fig. 2. These time histories were generated by a superposition of 99 harmonic components. It is seen that the sample function time histories exhibit many of the features of real earthquakes.

An ensemble of 200 sample functions of the type shown in Fig. 2 was generated for each spectrum and used as the input for a simulation study of the peak response of a damped single-degree-of-freedom system. The results for the 84.2% level of exceedance are shown as data points in Fig. 1. It is observed that the simulation results lie very close to the target spectra over the range of 0.3 to 12 Hz. The agreement appears to be as good as that attainable by any other technique [10].

Based on the results of this numerical example, it is concluded that the present analytical model is capable of accurately predicting the peak response of single-degree-of-freedom systems subjected to nonstationary, non-white random excitation. This provides a relatively simple basis for relating earthquake time histories and response spectra in either a forward or backward sense. It is hoped that this new analytic approach will prove useful in the study of the response of structures to earthquake and other types of transient random excitation.

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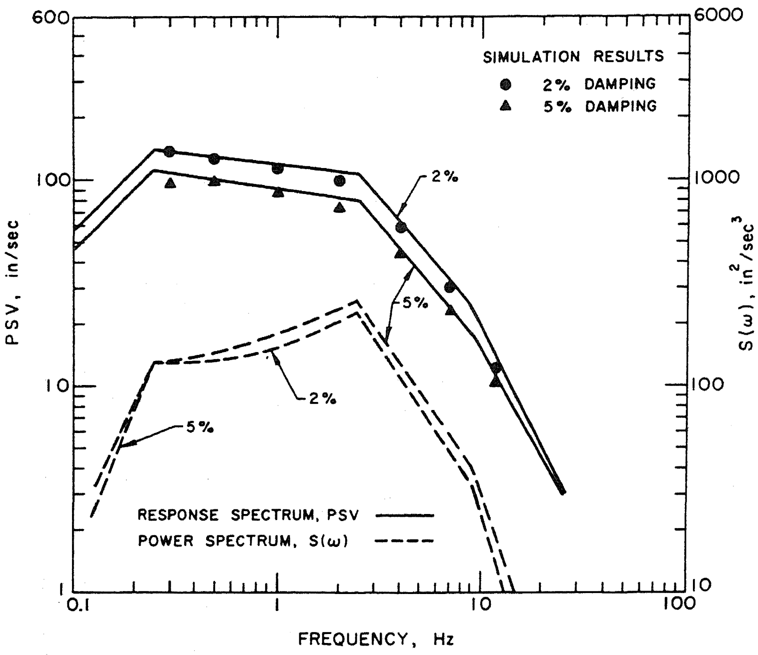

Figure 1

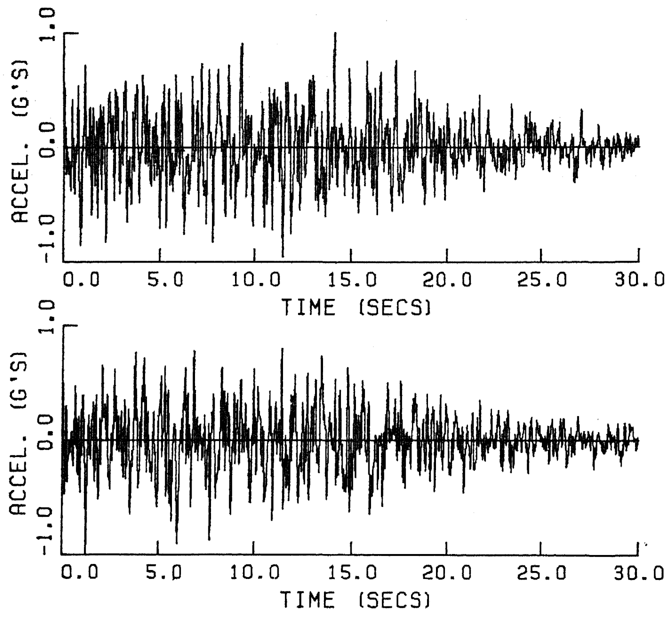

Figure 2