ENERGY DISTRIBUTION CRITERIA FOR BRACED AND UNBRACED STRUCTURAL DESIGN SUBJECTED TO PARAMETRIC EARTHQUAKES

by
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I. SYNOPSIS

The optimality criteria method based on the energy distribution of the structural members is presented for five design conditions with multicomponent inputs of static loads and seismic excitations. Four 15-story frameworks of unbraced, single-, double-, and K-braced systems are designed using ODSEWS program for coupling ground motions of the El Centro, 1940 earthquake. The paper includes various design results among them the significant observation is that, for the same constraints, the K-braced system requires much less structural weight than other systems.

II. INTRODUCTION

In the past decade, a considerable amount of literature has been published in the area of optimum structural design. The increasing number of publications correspond closely to the rapid demand for economical and reliable structural design mainly in aircraft engineering.[4] The conventional design is based on the trial and error process and is recognized that this is an inadequate method, which yields solutions that cannot always satisfy safety and performance constraints and provide the lowest possible structural cost. The current computer applications in seismic structures is based on the conventional design and must have a preliminary assumption of member stiffnesses. If the initial stiffnesses are misjudged, repeated analysis, regardless of the program's sophistication, will not yield an improved design. The paper presents a powerful optimization technique of which the significant advantage is that the number of iterations required to converge on an optimum (or pseudo-optimum) design is largely independent of the number of variables in the problem. The structural formulation is based on the consistent mass method with the P-Δ effect[1] of the vertical gravity load and ground motions. Five versatile behavior constraints and various numerical examples are shown and discussed.

III. OPTIMALITY CRITERIA FOR MINIMUM WEIGHT DESIGN

The optimality criteria for five general design conditions are briefly presented. The objective is to minimize the weight of the structure with fixed configuration while satisfying the desired constraints under specific loading conditions. The total structure is discretized into m finite elements for which the total structural weight, W(x), may be expressed as

$$W(x) = \sum_{i=1}^{m} \rho_i \eta_i x_i \ell_i \qquad (1)$$

where ρ_i is the mass density, η_i is the ratio of the cross-sectional area, A_i, to the moment of inertia, x_i, and the product $\eta_i x_i \ell_i$ represents the volume of the element, i. For a structure subjected to both static loads and multicomponent ground motions, the equation of motion can be expressed as

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$$(\underline{M}_n+\underline{M}_s)\ddot{\vec{r}}(x,t)+\underline{C}\,\dot{\vec{r}}(x,t)+(\underline{K}_s-\underline{K}_g)(\vec{r}(x,t)+\vec{r}(x)) = -(\underline{M}_n+\underline{M}_s)\ddot{\vec{r}}_g(t)+\vec{R}, \quad (2)$$

in which M_n = nonstructural mass resulting from the superimposed weight, $\underline{M}_s$ = structural mass, $\underline{C}$ = viscous damping, $\underline{K}_s$ = structural stiffness, $\underline{K}_g$ = geometric stiffness, $\ddot{\vec{r}}(x,t)$, $\dot{\vec{r}}(x,t)$, $\vec{r}(x,t)$ = acceleration, velocity, and displacement vectors of system coordinates measured from the equilibrium position after deformation caused by static loads, $\vec{r}(x)$ = static displacement vector, $\ddot{\vec{r}}_g(t)$ = vector of ground accelerations, and $\vec{R}$ = vector of static loads.

If the behavior constraints are denoted by $y(x)$, then a new function can be written as

$$\psi(x,\lambda) = \sum_{i=1}^{m} \rho_i \eta_i x_i \ell_i \pm \sum_{j=1}^{n} \lambda_j y_j(x) \quad (3)$$

where λ_j is the undetermined Lagrangian multiplier for jth constraint. By using the necessary condition of Kuhn-Tucker to characterize any local minimum,

$$\frac{\partial\psi(x,\lambda)}{\partial x_i} = \rho_i \eta_i \ell_i \pm \sum_{j=1}^{n} \lambda_j \frac{\partial y_j(x)}{\partial x_i}, \; i = 1, 2, 3, \ldots m \quad (4)$$

After simplifying Eq. 4 the optimality criteria can be expressed as

$$\sum_{j=1}^{n} \lambda_j e_{ji} = 1, \qquad i = 1, 3, 2, \ldots m \quad (5)$$

where e_{ji} can be interpreted as an energy density function in the following form

$$e_{ji} = \frac{\partial y_j(x)}{\partial x_i} \Big/ \frac{\partial W(x)}{\partial x_i} \quad (6)$$

The energy density function is derived on the basis of the following individual design conditions (behavior constraints). In the design, the constraints can be considered separately as well as in combination.

1. Flexibility Constraints for Static Loads. The flexibility constraints of a system result from the displacement limitations of either certain nodes or all the nodes. The displacement constraint function may be expressed by using the following virtual work at any nodal point:

$$y_j(x) = \vec{Q}_j^T \vec{r}(x), \quad (7)$$

in which $\vec{Q}_j$ = load vector with unit value at the jth direction and zero values for others, and $\vec{r}(x)$ = vector of generalized displacements attributable to the static load, $\vec{R}$. Substituting Eqs. 1 and 7 into 5 and 6 and then simply considering a single displacement constrain in the jth direction,

$$e_{ji} = \frac{\vec{q}_j^T(x)\, \underline{a}_i^T\, \underline{K}_{si}\, \underline{a}_i\, \vec{r}(x) - P_i'\, \vec{q}_j^T(x)\, \underline{a}_i^T\, \underline{K}_{gi}\, \underline{a}_i\, \vec{r}(x)}{\rho_i \eta_i x_i \ell_i} = \frac{1}{\lambda_j} \quad (8)$$

in which P_i' consists of half the structural mass of member i ($\eta_i \rho_i \ell_i/2$) that must have the influence on the geometric stiffness matrix established on the basis of column members. $\underline{a}_i$ is the compatibility matrix connecting the generalized coordinates of a structure and those of the constituent members.

2. Stiffness Constraints for Static Loads. The stiffness constraints are used to measure the limitations of the allowable shear stress and the allowable combined stress of axial and bending. The stiffness of the structure can be described by the work caused by the static load, $\vec{R}$, multiplied by the generalized displacement, $\vec{r}(x)$, in the form of

$$y(x) = \frac{1}{2} \vec{R}^T \vec{r}(x), \tag{9}$$

because the product, $\vec{R}^T \vec{r}(x)$, is an inverse measure of the stiffness. Thus, y(x) may be called a measuring function of static stiffness. The optimality criteria of Eqs. 5 and 6 for a single loading condition, j, can be expressed as

$$e_{ji} = \frac{1}{2} \frac{\vec{r}(x)\ \underline{a}_i^T\ \underline{K}_{si}\ \underline{a}_i\ \vec{r}(x) - P_i' \vec{r}(x)\ \underline{a}_i^T\ \underline{K}_{gi}\ \underline{a}_i\ \vec{r}(x)}{\rho_i \eta_i x_i \ell_i} = \frac{1}{\lambda_j} \tag{10}$$

3. Flexibility Constraints for Dynamic Loads. The dynamic displacement constraint function can be expressed in a form similar to Eq. 7 in terms of virtual work as follows:

$$y_j(x,t) = \vec{Q}_j^T \vec{r}(x,t), \tag{11}$$

where $\vec{Q}_j$ = load vector with unit force and unit time function at the jth direction only, and $\vec{r}(x,t)$ = vector of generalized displacements attributable to the dynamic load, $\vec{R}(t)$. For a single displacement constraint at jth direction, Eqs. 5 and 6 yield

$$e_{ji} = -\frac{1}{x_i} [\vec{q}_j^T(x,t) \underline{a}_i^T\ \underline{K}_{si}\ \underline{a}_i\ \vec{r}(x,t) - P_i'\ \vec{q}_j^T(x,t) \underline{a}_i^T\ \underline{K}_{gi}\ \underline{a}_i \cdot \vec{r}(x,t)$$

$$-p^2\ \vec{q}_j^T(x,t) \underline{a}_i^T\ \underline{M}_{si}\ \underline{a}_i\ \vec{r}(x,t)] / \rho_i \eta_i x_i \ell_i \tag{12}$$

in which p is the vibrating frequency approximately obtained by using the nodal displacements, $\vec{r}(x,t)$, in the Rayleigh quotient (see Eq. 15), and $\vec{q}_j(x,t)$ is the displacement vector resulting from the application of $\vec{Q}_j$.

4. Stiffness Constraints for Dynamic Loads. The measuring function of dynamic stiffness may be established in a manner similar to that of Eq. 9 as follows:

$$y(x,t) = \frac{1}{2} \vec{R}_0^T\ \vec{r}(x,t), \tag{13}$$

in which $\vec{R}_0$ represents the magnitude of the dynamic load vector. For a single loading condition, j,

$$e_{ji} = -\frac{1}{2x_i} [\vec{r}^T(x,t) \underline{a}_i^T\ \underline{K}_{si}\ \underline{a}_i\ \vec{r}(x,t) + \vec{r}^T(x,t) \underline{a}_i^T\ \underline{K}_{gi}\ \underline{a}_i\ \vec{r}(x,t)$$

$$-p^2\ \vec{r}^T(x,t) \underline{a}_i^T\ \underline{M}_{si}\ \underline{a}_i\ \vec{r}(x,t)] / \rho_i x_i \eta_i \ell_i \tag{14}$$

5. Natural Frequency Constraints. The natural frequency of any mode, ω_j, of a structure can be obtained by using the Rayleigh quotient expressed in modal displacements

$$\omega_j^2 = \frac{\vec{\phi}_j^T\ \underline{K}\ \vec{\phi}_j}{\vec{\phi}_j^T\ \underline{M}\ \vec{\phi}_j}, \tag{15}$$

in which ω_j^2 represents the constraint function $y(x,t)$, $\underline{K} = K_s - \underline{K}_g$, and $\underline{M} = M_s + M_n$. Substituting Eqs. 1 and 15 into Eq. 6, then the optimality criteria for jth mode became

$$e_{ji} = \frac{\vec{\phi}_j^T \underline{a_i^T} \underline{K_{si}} \underline{a_i} \vec{\phi}_j - P_i' \vec{\phi}_j^T \underline{a_i^T} \underline{K_{gi}} \underline{a_i} \vec{\phi}_j - \omega_j^2 \vec{\phi}_j^T \underline{a_i^T} \underline{M_{si}} \underline{a_i} \vec{\phi}_j}{\rho_i n_i x_i \ell_i} = \frac{1}{\lambda_j} \quad (16)$$

The energy density functions, e_{ji}, derived above reveal that the optimum design is obtained when the ratio of the strain energy (static cases) or the strain energy combined with kinetic energy (dynamic cases) to the mass density is the same for all members.[2,3] If there is more than one active constraint in any of the five cases, then Eq. 5 must be satisfied.

IV. RECURSION RELATIONSHIP AND NUMERICAL PROCEDURES

The recursion relationship provides a means of numerical procedures to resize structural members on the basis of the optimality criteria. Let $\tau_i' = W(x)/(\Lambda x_i)$, $x_i = \Lambda\alpha_i$, and $e_{ji}' = \Lambda e_{ji}$, then Eq. 5 can be arranged as

$$\Lambda\alpha_i = \alpha_i \left[\frac{\sum_{j=1}^{n} \lambda_j e_{ji}'}{\tau_i'}\right]^{1/2}, \quad i = 1, 2, \ldots, m \quad (17)$$

in which α_i = relative design variables corresponding to x_i and Λ = scaling factor. Equation 17 suggests the following recursion relation for determining the design variable in each cycle:

$$(\Lambda\alpha_i)_{v+1} = (\alpha_i)_v \left[\frac{\sum_{j=1}^{n} \lambda_j \mu_{ji}'}{\tau_i'}\right]_v^{1/2}, \quad i = 1, 2, \ldots, m \quad (18)$$

where the subscripts v and v+1 denote the cycles of iteration.

The numerical procedures for the recursions may be briefly explained as follows: 1) the structure is analyzed with initial relative design vector consisting of equal sizes for all members, 2) the constraint surface is located by scaling the design to satisfy the specified frequencies, stresses, and displacements resulting from static loads, dynamic forces, or combined loadings, 3) the weight of the feasible design is determined, 4) the active constraints are identified and then the structure is resized according to the iterative algorithm shown in Eq. 18, 5) steps 1 through 4 are repeated (except the areas used in step 1 are based on the results from the previous cycle) as long as the design improves.

The bracings are bar elements and beams and beam-columns are built-up sections as shown in Fig. 1 for which the moment of inertia, I_0, is considered as a primary design variable, and the depth, d, flange thickness, t_f, and web thickness, t_w, are the secondary design variables. The cross-sectional area, A_0, and the shear flow, V_0, are expressed in terms of the secondary design variables. The upper and lower bounds of the cross-section can be specified and the ratios of the minimum moment of inertia to the maximum moment of inertia of both girders and columns can be imposed.

V. NUMERICAL RESULTS AND CONCLUSIONS

1. Numerical Results. The 15-story buildings shown in Figs. 2-5 are designed for Models I and II of the accompanying figures. The span length is 21 ft (6.40 m), the floor height, 12 ft (3.66 m), the dead load on each floor as nonstructural mass, w, 180 lbs/in. (178.74 N/m), the modulus of elasticity, E, 29,000 ksi (200.1 GN/m^2), and the mass density of the construction material ρ = 0.283 lbs/in^3 (783.34 kg/m^3). The dynamic excitation is due to the horizontal and vertical earthquake accelograms of El Centro, 1940 for which the acceleration spectra with 5% damping are given in Fig. 6. The allowable stress for bending combined with the axial force is assumed to be $\sigma \leq 29$ ksi (200.1 MN/m^2) and the allowable shear stress, σ_V, should be less than or equal to 0.65 σ. Although different allowable deflections may be imposed at any particular nodes, the allowable deflection considered herein is based on the general code provision, i.e., the relative displacement between floors is limited to 0.005 times the story height. Other constraints are b = 12 in. (30.48 cm), d_{max} = 75 in. (190.50 cm), d_{min} = 8 in. (20.32 cm), $(t_f/d)_{max}$ = 0.045, $(t_f/d)_{min}$ = 0.023, and t_w/d = 0.02.

Four cases of the final design results for Fig. 2 are shown in Figs. 7 and 8 in which Case (a), H, signifies the design resulting from the horizontal ground motion only, Case (b), H+P-Δ(DL), is due to the horizontal ground motion plus the P-Δ effect of static load of nonstructural mas acting on girders, Case (c) of H+V indicates the horizontal and vertical earthquake components, and Case (d) represented by H+V+P-Δ(DL+V) corresponds to the design obtained by considering horizontal and vertical earthquake components as well as the P-Δ effect of the vertical inertia forces associated structural and nonstructural masses. The final design results of the moments of inertia and the cross-sectional areas for Models I and II of Fig. 3 are shown in Figs. 9, 10, and 11 for all the four leading cases. The design-result comparisons corresponding to Case (d) of Model II for the braced frames are shown in Figs. 12 through 17. The final results of the structural weight, natural periods and the displacements at top floor are given in Table 1. The observations of the results are included in the conclusions.

2. Conclusions. (1) The optimality criteria method is presented for five versatile design conditions of braced and unbraced frames with multicomponent inputs of static loads and seismic excitations. (2) The inclusion of the vertical seismic component and the P-Δ effect in Model II yields the heaviest design among all the four cases. (3) the moments of inertia of columns of all the structures are the largest at the base and then become gradually smaller from the bottom to the top floor. However, the moment of inertia of the first floor girder of the unbraced frame is smaller than those of the next three upper floors and those of the sixth through eleventh floor are almost the same. The moments of inertia of girders of double-and K-braced systems are governed by the lower bound constraints. (4) The K-system demands the lightest structural design and seems to be the most favorable system among these four. (5) The designs are controlled by the combined stress of axial and bending of the columns at the support. All other members are not fully stressed. The displacements of the K- and double-braced systems are similar but larger than those of the single-braced frame.

VI. ACKNOWLEDGMENTS

The author gratefully acknowledges the financial support of a National Science Foundation Grant: PFR78-5694. Appreciation is also extended to Mr. L.H. Shen for his assistance of running the numerical examples based on the ODSEWS program developed at UMR.

VII. REFERENCES

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TABLE 1. FINAL WEIGHTS, NATURAL PERIODS, AND DISPLACEMENTS AT TOP FLOOR (A=Unbraced, B=Single-Braced, C=Double-Braced, D=K-Braced) (1 kip=453 kg, 1 in.=2.54 cm)

Group	Case	Final Weight (kips)	Natural Period (sec.)					Disp. at Top Floor (in.)
			1	2	3	4	5	
A	a	60.44	2.152	0.694	0.404	0.282	0.212	10.80
	b	62.21	2.179	0.671	0.386	0.268	0.202	10.80
	c	60.77	2.158	0.692	0.402	0.281	0.217	10.80
	d	62.21	2.152	0.677	0.395	0.276	0.212	10.80
B	a	48.32	1.879	0.490	0.256	0.247	0.187	9.00
	b	47.69	1.876	0.479	0.274	0.252	0.209	9.05
	c	48.76	1.874	0.468	0.291	0.255	0.224	9.02
	d	50.01	1.874	0.485	0.288	0.253	0.221	9.02
C	d	31.55	2.134	0.509	0.398	0.361	0.335	10.80
D	b	28.61	2.111	0.519	0.274	0.269	0.178	10.55

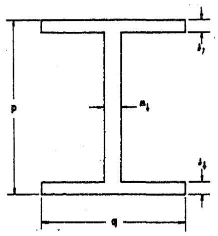

Fig. 1.Built-up Section

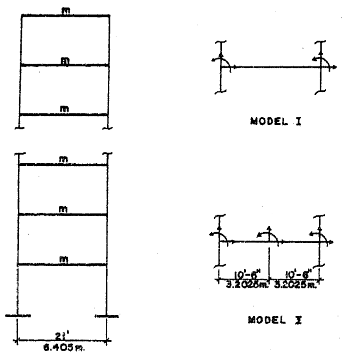

Fig. 2.Unbraced Frame

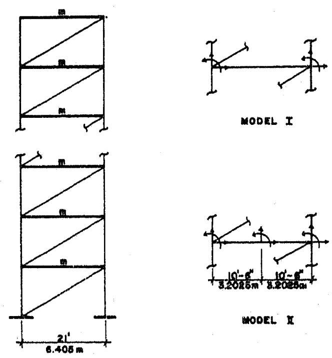

Fig. 3.Single-Braced Frame

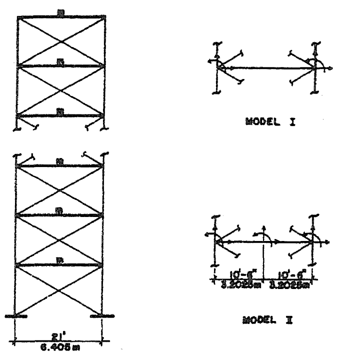

Fig. 4.Double-Braced Frame

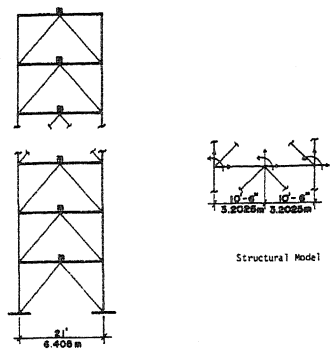

Fig. 5.K-Braced Frame

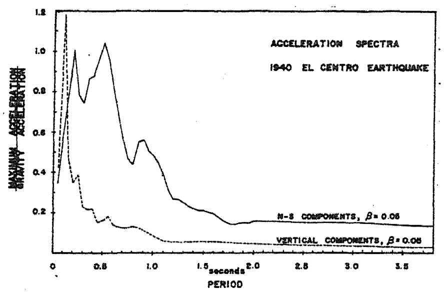

Fig. 6.Acceleration Spectrum

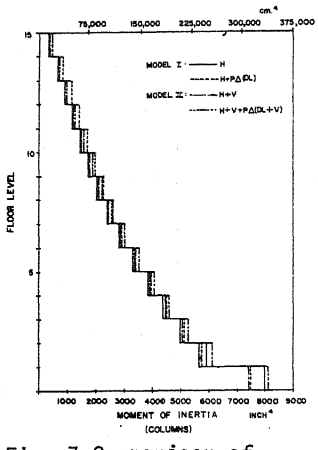

Fig. 7.Comparison of
Fig. 2

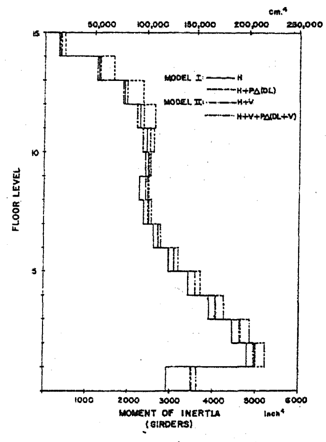

Fig. 8.Comparison of
Fig. 2

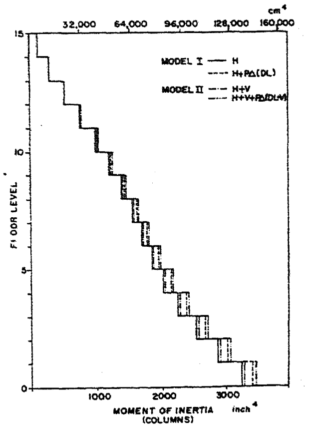

Fig.9.Comparisons of Fig. 3

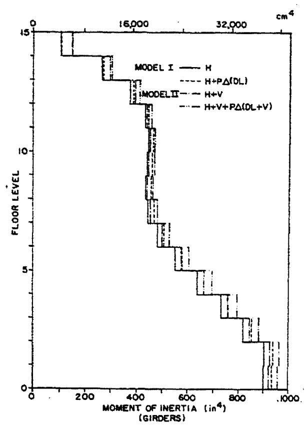

Fig. 10.Comparison of Fig. 3

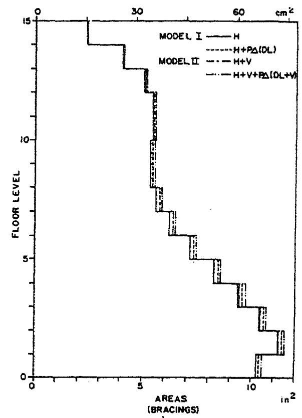

Fig. 11.Comparisons of Fig. 3