STATISTICAL INTERPRETATION AND APPLICATION OF RESPONSE SPECTRA

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SUMMARY

This paper aims at extending the concept of response spectra and broadening their applications by investigating the relevant aspects of the maximum response based on the random vibration theory. The asymtotical distributions of both stationary and nonstationary maximum structural responses are established and compared with the results of computer simulations. Applications of the statistical distributions in the development of design response spectra of building-equipment systems and in the dynamic analysis of tall buildings are demonstrated.

INTRODUCTION

Building structures and equipments under earthquake excitations are commonly analyzed and designed according to the deterministic response spectra approach. While this approach is simple to follow, it has serious limitations in its ability to account for the uncertainties associated with various structural and earthquake parameters in the response prediction. To overcome this difficulty, both the ground motion and structural response have been modeled as stochastic processes, and treated as such [1,2].

This paper establishes the relationship between the conventional response spectra analysis approach and the random vibration analysis approach, thus permitting a statistical response prediction and rational interpretation. In the theory of nonstationary random vibration, the maximum structural response is a statistical variable, whose precise distribution function, unfortunately, has not been obtained to date. In this paper, first an asymptotic approximation is established for the distribution of the maximum nonstationary response using extreme value theories. This distribution is shown to degenerate into the familiar function commonly found in the literature when the excitation is modeled as a stationary process [2-5]. These asymptotic distributions are then compared with the average response spectra generated by computer simulations, and the applications of the statistical approach in the establishment of design spectra and dynamic analysis of buildings or building-equipment systems are also demonstrated.

DISTRIBUTION OF MAXIMUM RESPONSE

Let $y(t)$ be the response of a linear structure to earthquake ground

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acceleration $\ddot{x}_g(t)$, and $h(t)$ and $H(\omega)$ be the impulse response function and the frequency response function, respectively, of the structure. Note that $h(t)$ and $H(\omega)$ constitute a Fourier transform pair. We further assume that $\ddot{x}_g(t)=\psi(t)\ddot{x}(t)$ = a uniformly modulated nonstationary process [3-5] in which $\ddot{x}(t)$ is a stationary process with zero mean and a power spectral density $\Phi(\omega)$ and $\psi(t)$ is a deterministic modulating (or envelope) function.

Distribution of nonstationary maximum response

Let $\eta_1,\eta_2,\eta_3\ldots,\eta_n$ be n absolute values of local peaks and troughs of $y(t)$ in the time interval $[0,T]$. The sequence $\{\eta_j\}$ constitutes a discrete nonstationary point process referred to as the extreme point process. The distribution of these individual extreme values can be approximated by that of the envelope process of $y(t)$ as follows [6]

$$F_{\eta_j}(u) = P[\eta_j \le u] = 1 - \exp(-u^2/2\sigma_j^2) \tag{1}$$

in which $\sigma_j=\sigma_y(t_j)$ = the standard deviation of $y(t)$ at $t=t_j$, $\sigma_y^2(t)= \int_{-\infty}^{\infty}|M(t,\omega)|^2\Phi(\omega)d\omega$, and $M(t,\omega)=\int_0^t h(\tau)\psi(t-\tau)\exp(-i\omega\tau)d\tau$ where $|M(t,\omega)|^2\Phi(\omega)$ = the evolutionary power spectral density of $y(t)$. From Eq. 1 the mean value and the mean square value of η_j are: $E[\eta_j]=(\pi/2)^{1/2}\sigma_j$ and $E[\eta_j^2]=2\sigma_j^2$.

Now let $F_\eta(u;T)$ be the distribution of the extreme values (peaks and negative troughs) in $[0,T]$, then

$$F_\eta(u;T) = 1 - [M_u(T)/M(T)] \tag{2}$$

in which $M_u(T)$ = expected number of extremes (peaks and negative troughs) exceeding the level u in $[0,T]$ and $M(T)$ = expected number of all extremes in $[0,T]$. When $y(t)$ is stationary and ergodic, $F_\eta(u;T)$ defined in Eq. 2 degenerates into the distribution of peak (or trough) values given by Cramer and Leadbetter as $T\to\infty$ [7]. It is shown in Ref. 8 that $F_\eta(u;T)$ can be expressed by the Weibull distribution, e.g.,

$$F_\eta(u;T) = 1 - \exp\left[-\frac{1}{\alpha}\left(\frac{u}{\sigma}\right)^\alpha\right] \tag{3}$$

in which α and σ depend on T as well as the nonstationary characteristics of $y(t)$ as will be determined later. Note that when $y(t)$ is stationary, Eq. 3 reduces to the well-known Rayleigh's distribution with $\alpha=2.0$ and $\sigma=\sigma_y(t)$ = constant.

Let y_m be the absolute maximum of $y(t)$ in $[0,T]$. Assuming that (i) the extreme values $\eta_j(j=1,2,\ldots,n)$ are statistically independent and (ii) the total number n of the extreme values in [0,T] is large, then the distribution of y_m approaches asymptotically to [9]

$$F_{y_m}(u) = F_\eta^n(u;T) = \exp\left\{-\exp\left[-K^{\alpha-1}\left(\frac{u}{\sigma}-K\right)\right]\right\} \quad (4)$$

in which $K = (\alpha \ln n)^{1/\alpha} = (\alpha \ln \nu T)^{1/2}$, and ν is the expected zero crossing rate of y(t) from below as well as from above. From Eq. 4, the mean value and standard deviation of the absolute maximum y_m are, respectively,

$$\bar{y}_m = (K + 0.5772\, K^{1-\alpha})\sigma \quad (5)$$

$$\sigma_{y_m} = 1.28\sigma/K^{\alpha-1} \quad (6)$$

The Weibull parameters α and σ appeared above can be determined from the mean $\bar{\eta}$ and the coefficient of variation V_η of η using Eq. 3 as

$$V_\eta = \left[\Gamma\left(\frac{2}{\alpha}+1\right) - \Gamma^2\left(\frac{1}{\alpha}+1\right)\right]^{1/2} \Big/ \Gamma\left(\frac{1}{\alpha}+1\right)$$
$$\bar{\eta} = \sigma\alpha^{1/\alpha}\Gamma\left(\frac{1}{\alpha}+1\right) \quad (7)$$

in which $\Gamma(\cdot)$ is the gamma function, $V_\eta=\sigma_\eta/\bar{\eta}$, $\bar{\eta}=\frac{1}{n}\sum_{j=1}^{n} E[\eta_j]=\sqrt{\frac{\pi}{2}}\frac{1}{n}\sum_{j=1}^{n}\sigma_j$, and $\sigma_\eta^2=\frac{1}{n}\sum_{j=1}^{n}E[\eta_j^2]-\bar{\eta}^2=\frac{2}{n}\sum_{j=1}^{n}\sigma_j^2-\bar{\eta}^2$. Therefore, it follows from above expressions that α and σ can be determined once $\sigma_j=\sigma_y(t_j)$ is computed.

Distribution of stationary maximum response

It has often been assumed that strong shaking portion of typical earthquake accelerograms is stationary and so is the corresponding structural response. In this case $\psi(t)=1$ and the distribution of the absolute maximum reduces to the familiar expressions under stationary theory as follows:

$$F_{y_m}(u) = \exp\{-\exp[-K(\frac{u}{\sigma} - K)]\} \quad (8)$$

and the mean value and standard deviation are

$$\bar{y}_m = (K + 0.5772\, K^{-1})\sigma \quad (9)$$

$$\sigma_{y_m} = 1.28\sigma/K \quad (10)$$

in which $\sigma=\sigma_y(t)$ = constant.

Monte Carlo simulation

The validity of the asymptotic distributions in Eqs. 4 and 8 can be examined by comparison with the results of computer simulation. The power spectral density $\Phi(\omega)=S^2[1+4\lambda_g^2(\omega/\omega_g)^2]/\{[1-(\omega/\omega_g)^2]^2+4\lambda_g^2(\omega/\omega_g)^2\}$ with ω_g= 18.85 rad/sec., λ_g=0.65 and S^2=4.65x10^{-4} m^2/sec. has been used. The enveloped function used is $\psi(t)=(t/t_1)^2$ for $0\leq t\leq t_1$, $\psi(t)=1$ for $t_1\leq t\leq t_2$ and $\psi(t)=\exp[-C(t-t_2)]$ for $t>t_2$ where t_1=3 sec., t_2=13 sec., and C=0.26.

Figure 1 shows a comparison for the average response spectra, $\bar{y}_m$, among the simulation results (circles), the stationary theory (solid curve) of Eq. 9, and the nonstationary theory (dashed curve) of Eq. 5 for different damping values. A total of 24 sample functions have been simulated using the Fast Fourier transform technique (FFT) for each frequency and damping value.

From these results it can be noted that the theoretical approximation always predict higher average maximum response than the computer experiments; therefore Eqs. 4 and 8 give conservative results. This has been expected because of the assumption made in the theoretical approximations that the local extreme values (peaks and troughs) are statistically independent. As expected also, the nonstationary approximation (dashed curve) is more accurate than the stationary approximation (solid curve). However, when the frequency increases the discrepency between two approximations reduces and the accuracy of the theoretical approximations improves. This again is due to the reason that the number of half cycles $n=\nu T$ in [0,T] needs to be sufficiently large to develop the stationary response and to yield a justifiable asympototic distribution.

It is also observed that the accuracy of the theoretical approximations improve with the damping coefficient of the structure. This again is due to the assumption that the local extreme values are statistically independent. It is well-known that the correlation among local extreme values is stronger for smaller damping values [6]. Furthermore, the coefficients of variation of the response spectra using the theoretical approximations correlate very well with those of the simulation results. The accuracy is within 8%.

APPLICATIONS

Floor response spectra

The asymptotic distributions described above can be used to establish the floor response spectra required for the design of mechanical and electrical equipments or other systems housed in a building [1]. Using a two mass dynamic system representing the building-equipemnt assembly, the distribution of the floor response spectra can be obtained analytically using the stationary approximation as the input at the building base. To facilitate the comparison, the following values are used in the calculations:

dampings of building and oscillator = 5%, S^2=0.07 ft^2/sec^2 (which results in an average maximum ground acceleration of 0.28g). Furthermore, the natural frequency of the building is restricted to be greater than 0.5 hz. The average floor response spectra based on the stationary approximation is plotted in Fig. 2 as a dashed curve along with the floor response spectra suggested in [1] (solid curve). It is noticed that the trend of both spectra is similar but the theoretical approximation (dashed curve) is more conservative. This is due to the fact that the natural frequency of the building is tuned to be identical to the oscillator in the calculation in order to produce the maximum response. Such a conservatism is reasonable when the information of the building characteristic is not available or when the equipments have to be installed in a variety of different buildings in various cities.

Maximum response of building

The response of an 8-story building under earthquake ground excitations presented in [10] is considered. The mass center and the elastic center of the building are not coincident such that the building exhibits coupled lateral-torsional motions. The first three coupled natural frequencies are 0.866 hz (in x-direction), 1.038 hz (in y-direction) and 1.31 hz (torsion), respectively. The damping coefficient associated with the first mode is approximately 2%. The analysis is focused upon the excitation in the x-direction only.

Under stationary assumption, i.e., $\psi(t)$=1.0 for all t, the standard deviations of the base shear force (in x-direction) and the torsional moment are 1106 KN and 7516 KN·m, respectively. The nonstationary standard deviations are given in [10]. The distribution of the maximum response, normalized by the corresponding stationary standard deviation, based on the stationary approximation (curve 1), nonstationary approximation (curve 2), and Monte Carlo simulations (circles) are presented in Figs. 3(a) and 3(b). In nonstationary approximation the result using 25 sec. duration of the response process is plotted as Curve 3, while that based on 10 sec. duration is shown by Curve 2. Since the difference between Curves 2 and 3 is insignificant [Fig. 3(a)], Curve 3 is not shown in Figs. 3(b)-3(d). When the damping coefficient is increased to 6%, the corresponding distributions are plotted in Fig. 3(c) and 3(d) in which the stationary standard deviations of the based shear force and torsional moment are 702.8 KN and 4273.5 KN·m, respectively.

It is observed from Fig. 3 that the nonstationary approach generally provides more accurate results than the stationary approach and that the accuracy of the asymptotic approximations improves as the damping value increases. It is interesting to note that the agreement between the theoretical and simulation results is very good at the upper tail portion of the distribution (probability greater than 95%). This is evident because the events of exceeding a high level response tend to be statistically independent, being consistent with the assumption of the theoretical approximation.

CONCLUSION

The above analytical and computational analyses indicate that the asymptotic distributions in general provide good and slightly conservative estimates of the statistical response spectra, particularly when the damping or the frequency of the structure is high. The nonstationary approximation usually yields reasonable results, even when the damping or the frequency is low.

The procedure suggested can be a very useful and complementary tool to the conventional analysis method, and can provide a great deal of insight into the variability aspects of the earthquake response due to the inherent uncertainties in the problem. The investigation of tall building maximum response distribution and the determination of average design floor response spectra presented herein are merely two simple examples of the potential engineering applications of the statistical response approach.

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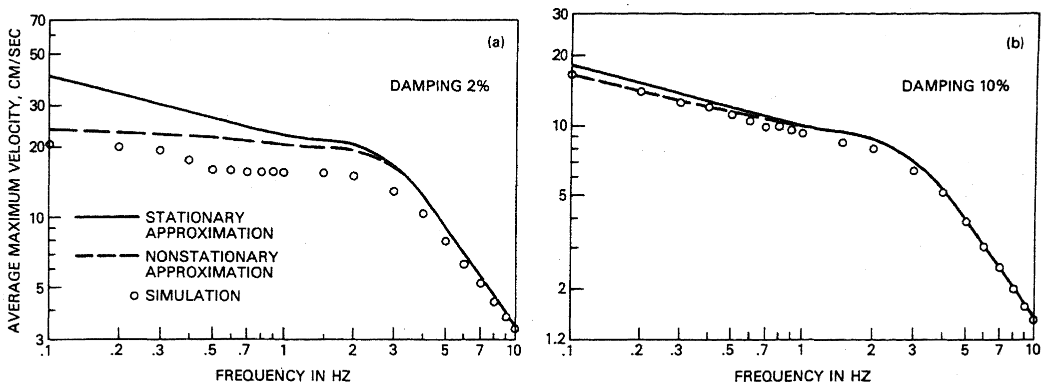

FIG 1: COMPARISON OF AVERAGE RESPONSE SPECTRA

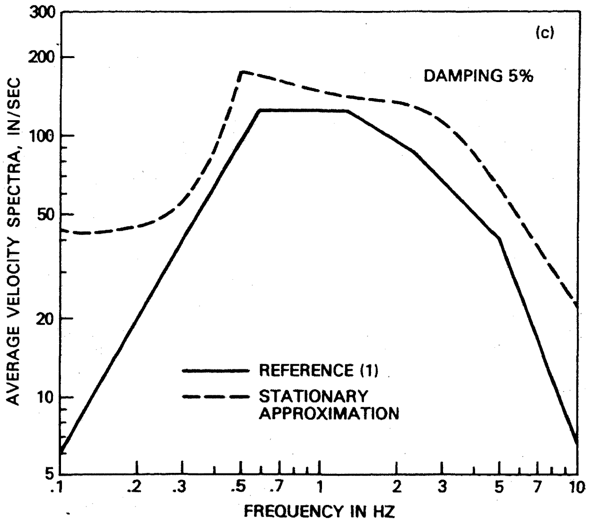

FIG 2: COMPARISON OF AVERAGE FLOOR RESPONSE SPECTRA

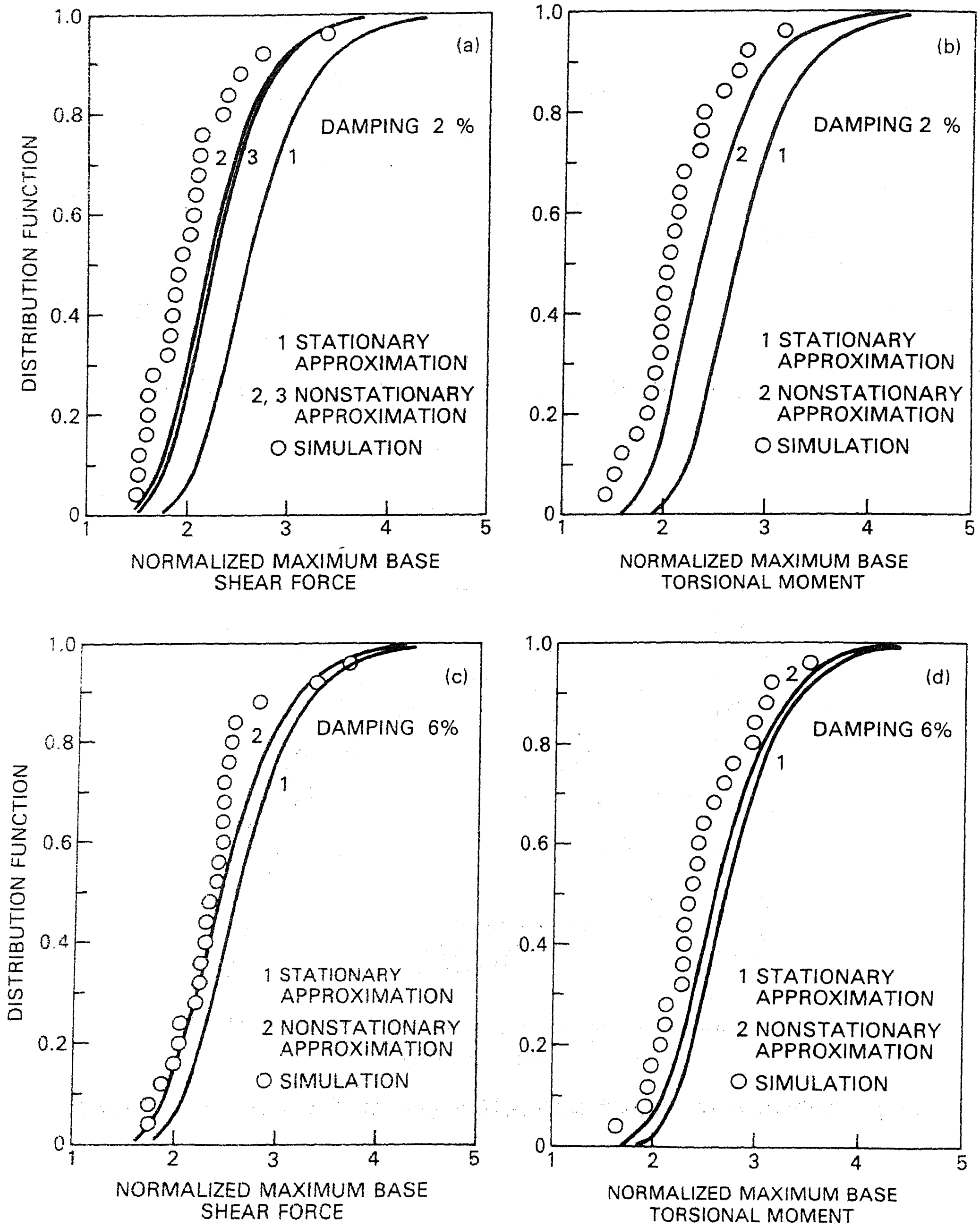

FIG 3: DISTRIBUTION OF MAXIMUM STRUCTURAL RESPONSE