CROSS-INTERACTION BETWEEN TWO EMBEDDED STRUCTURES IN EARTHQUAKES

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SUMMARY

The purpose of this paper is to evaluate both the vibrational characteristics and the earthquake responses of a cross-interaction system between two building structures embedded in a visco-elastic soil stratum. The available information concerning cross-interaction between two embedded structures is summarized: for two identical structures, the maximum coupled response of the cross-interaction system subjected to earthquake is generally smaller than that of uncoupling system, but for two different structures, the maximum coupled response becomes mostly greater than that of single structure.

INTRODUCTION

A considerable amount of work has been done in recent years to obtain responses of a cross-interaction system between two structures resting on a layered soil stratum or a visco-elastic half-space for three dimensional problem. This paper presents both the dynamic characteristics and the earthquake responses of a cross-interaction system between two building structures embedded in a visco-elastic soil stratum. It is assumed that each structure consists of the lumped mass and cylindrical embedded foundation, and that a three-dimensional space model of the soil ground is subdivided by several horizontal planes, shown in Figure 1. First, the dynamic characteristics of two embedded cylindrical building structures are presented by making use of two approaches; *i.e.* one, in the horizontal direction, is based on the wave propagation theory in two local cylindrical coordinates, and the other, in the vertical direction, is the Finite Element Method applied on the several thin layered elements. Secondly, the transient coupled responses of two building structures are calculated by using the Fast Fourier Transform techniques. The earthquake response calculations are done using two components of the 1952 Taft earthquake ground motion and two components of the 1940 El Centro earthquake ground motion as the bed rock excitation.

EQUATIONS OF MOTION AND ITS SOLUTION

Before developing the theoretical approach of this cross-interaction system, the assumptions should be described as follows:
(1) The cross-section of each cylindrical foundation and its subsoil is not deformed.
(2) The soil stratum consists of several thin layered elements and is isotropic, homogeneous and visco-elastic.
(3) The displacement in the each thin layered element change linearly along vertical axis.
(4) Each foundation is welded to its surrounding ground.
(5) The vertical displacement of foundation is ignored, but its rotation about a horizontal axis is considered.

A soil-structure interaction model is shown in Fig. 1. For each thin

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layered element, the equations of motion in cylindrical coordinate are expressed as

$$\{(\lambda+2\mu)+(\lambda'+2\mu')\frac{\partial}{\partial t}\}\frac{\partial\Theta}{\partial r}-\frac{2}{r}(\mu+\mu'\frac{\partial}{\partial t})\frac{\partial\omega_z}{\partial\theta}+2(\mu+\mu'\frac{\partial}{\partial t})\frac{\partial\omega_\theta}{\partial z}=\rho\frac{\partial^2 u_r}{\partial t^2}$$

$$\{(\lambda+2\mu)+(\lambda'+2\mu')\frac{\partial}{\partial t}\}\frac{1}{r}\frac{\partial\Theta}{\partial\theta}-2(\mu+\mu'\frac{\partial}{\partial t})\frac{\partial\omega_r}{\partial z}+2(\mu+\mu'\frac{\partial}{\partial t})\frac{\partial\omega_z}{\partial r}=\rho\frac{\partial^2 u_\theta}{\partial t^2} \quad \cdots(1)$$

$$\{(\lambda+2\mu)+(\lambda'+2\mu')\frac{\partial}{\partial t}\}\frac{\partial\Theta}{\partial z}-\frac{2}{r}(\mu+\mu'\frac{\partial}{\partial t})\frac{\partial}{\partial r}(r\omega_\theta)+\frac{2}{r}(\mu+\mu'\frac{\partial}{\partial t})\frac{\partial\omega_r}{\partial\theta}=\rho\frac{\partial^2 u_z}{\partial t^2}$$

where λ, μ : Lamé's constants, λ', μ': viscous constants
$\{u_r, u_\theta, u_z\}$: displacement vector

$$\Theta=\frac{1}{r}\frac{\partial}{\partial r}(ru_r)+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{\partial u_z}{\partial z}, \qquad 2\omega_r=\frac{1}{r}\frac{\partial u_z}{\partial\theta}-\frac{\partial u_\theta}{\partial z}$$

$$2\omega_\theta=\frac{\partial u_r}{\partial z}-\frac{\partial u_z}{\partial r}, \qquad 2\omega_z=\frac{1}{r}\frac{\partial}{\partial r}(ru_\theta)-\frac{1}{r}\frac{\partial u_r}{\partial\theta} \quad \cdots\cdots(2)$$

Two local cylindrical coordinates are used as shown in Fig. 1, *i.e.* (r_I, θ_I, z) and (r_{II}, θ_{II}, z) coordinates. Provided that (u_{rI}, $u_{\theta I}$, u_{zI}) and (u_{rII}, $u_{\theta II}$, u_{zII}) are associated with the waves scattered by the first and second foundation, respectively, and the cross-interaction model is excited in the horizontal direction parallel to line through the two fundations, then the each displacements of the ground for a symmetric situation may be expressed by

$$u_{rI}(r_I,\ \theta_I,\ z)=\sum_{n=0}^{\infty}\{\phi_I^n H_{n+1}^{(2)}(\kappa r_I)+\psi_I^n H_{n-1}^{(2)}(\kappa r_I)\}\cos n\theta_I$$

$$u_{\theta I}(r_I,\ \theta_I,\ z)=\sum_{n=0}^{\infty}\{\phi_I^n H_{n+1}^{(2)}(\kappa r_I)-\psi_I^n H_{n-1}^{(2)}(\kappa r_I)\}\sin n\theta_I \quad \cdots(3)$$

$$u_{zI}(r_I,\ \theta_I,\ z)=\sum_{n=0}^{\infty}\chi_I^n H_n^{(2)}(\kappa r_I)\cos n\theta_I$$

and

$$u_{rII}(r_{II},\ \theta_{II},\ z)=\sum_{n=0}^{\infty}\{\phi_{II}^n H_{n+1}^{(2)}(\kappa r_{II})+\psi_{II}^n H_{n-1}^{(2)}(\kappa r_{II})\}\cos n\theta_{II}$$

$$u_{\theta II}(r_{II},\ \theta_{II},\ z)=\sum_{n=0}^{\infty}\{\phi_{II}^n H_{n+1}^{(2)}(\kappa r_{II})-\psi_{II}^n H_{n-1}^{(2)}(\kappa r_{II})\}\sin n\theta_{II} \quad \cdots(4)$$

$$u_{zII}(r_{II},\ \theta_{II},\ z)=\sum_{n=0}^{\infty}\chi_{II}^n H_n^{(2)}(\kappa r_{II})\cos n\theta_{II}$$

in which $H_m^{(2)}(\kappa r)$ is second Hankel function of order m, and ϕ_K^n, ψ_K^n and χ_K^n (K = I, II) are eigenvectors which obtain from the eigenvalue equation for the thin layered elements. In Eqs. 3 and 4, the time facter $\exp(i\omega t)$ is omitted. In order to impose boundary condition at the cross-section of each cylindrical foundation, it is necessary to express (u_{rII}, $u_{\theta II}$, u_{zII}) in the coordinate system (r_I, θ_I, z); similarly, it is necessary to express (u_{rI}, $u_{\theta I}$, u_{zI}) in the coordinate system (r_{II}, θ_{II}, z). These motions (u_{rI}, $u_{\theta I}$, u_{zI}) may be represented by a series of circular cylindrical foundations in the (r_{II}, θ_{II}, z) coordinates.

$$u_{rI}(r_{II},\ \theta_{II},\ z)=-\sum_{m=0}^{\infty}\frac{\varepsilon_m}{4}\Big[\sum_{n=0}^{\infty}(\phi_I^n-\psi_I^n)\{H_{n+m}^{(2)}(\kappa D_x)+(-1)^m H_{n-m}^{(2)}(\kappa D_x)\}$$
$$\cdot\{J_{m-1}(\kappa r_{II})-J_{m+1}(\kappa r_{II})\}+\sum_{n=0}^{\infty}(\phi_I^n+\psi_I^n)\{H_{n+m}^{(2)}(\kappa D_x)$$

$$-(-1)^m H_{n-m}^{(2)}(\kappa D_x)\}\{J_{m-1}(\kappa r_{II})+J_{m+1}(\kappa r_{II})\}]\cdot\cos m\theta_{II}$$

$$u_{\theta I}(r_{II}, \theta_{II}, z) = \sum_{m=0}^{\infty}\frac{\varepsilon_m}{4}\Big[\sum_{n=0}^{\infty}(\phi_I^n-\psi_I^n)\{H_{n+m}^{(2)}(\kappa D_x)+(-1)^m H_{n-m}^{(2)}(\kappa D_x)\} \quad \cdots(5)$$

$$\cdot\{J_{m-1}(\kappa r_{II})+J_{m+1}(\kappa r_{II})\}+\sum_{n=0}^{\infty}(\phi_I^n+\psi_I^n)\{H_{n+m}^{(2)}(\kappa D_x)$$

$$-(-1)^m H_{n-m}^{(2)}(\kappa D_x)\}\{J_{m-1}(\kappa r_{II})-J_{m+1}(\kappa r_{II})\}\Big]\cdot\sin m\theta_{II}$$

$$u_{zI}(r_{II}, \theta_{II}, z) = \sum_{m=0}^{\infty}\frac{\varepsilon_m}{2}\sum_{n=0}^{\infty}\chi_I^n\{H_{n+m}^{(2)}(\kappa D_x)+(-1)^m H_{n-m}^{(2)}(\kappa D_x)\}J_m(\kappa r_{II})\cdot\cos m\theta_{II}$$

where $\varepsilon_0 = 1$ and $\varepsilon_m \neq 2$ for $m = 0$, D_x is the separation distance between two foundations, and $J_m(\kappa r)$ is Bessel function of order m. The total displacement (u_r, u_θ, u_z) in the coordinate system (r_{II}, θ_{II}, z) can be expressed by

$$u_r(r_{II}, \theta_{II}, z) = u_{rI}(r_{II}, \theta_{II}, z) + u_{rII}(r_{II}, \theta_{II}, z) \equiv \sum_{n=0}^{\infty} v_{rII}^n \cos n\theta_{II}$$

$$u_\theta(r_{II}, \theta_{II}, z) = u_{\theta I}(r_{II}, \theta_{II}, z) + u_{\theta II}(r_{II}, \theta_{II}, z) \equiv \sum_{n=0}^{\infty} v_{\theta II}^n \sin n\theta_{II} \quad \cdots(6)$$

$$u_z(r_{II}, \theta_{II}, z) = u_{zI}(r_{II}, \theta_{II}, z) + u_{zII}(r_{II}, \theta_{II}, z) \equiv \sum_{n=0}^{\infty} v_{zII}^n \cos n\theta_{II}$$

Similarly, the total displacement (u_r, u_θ, u_z) in the coordinate system (r_I, θ_I, z) can be expressed by

$$u_r(r_I, \theta_I, z) = u_{rI}(r_I, \theta_I, z) + u_{rII}(r_I, \theta_I, z) \equiv \sum_{n=0}^{\infty} v_{rI}^n \cos n\theta_I$$

$$u_\theta(r_I, \theta_I, z) = u_{\theta I}(r_I, \theta_I, z) + u_{\theta II}(r_I, \theta_I, z) \equiv \sum_{n=0}^{\infty} v_{\theta I}^n \sin n\theta_I \quad \cdots(7)$$

$$u_z(r_I, \theta_I, z) = u_{zI}(r_I, \theta_I, z) + u_{zII}(r_I, \theta_I, z) \equiv \sum_{n=0}^{\infty} v_{zI}^n \cos n\theta_I$$

The modal displacement v^n must satisfy the following boundary conditions,

$$\left.\begin{array}{lll} v_{rI}^n|_{r_I=R_I}=0 & v_{\theta I}^n|_{r_I=R_I}=0 & v_{zI}^n|_{r_I=R_I}=0 \\ v_{rII}^n|_{r_{II}=R_{II}}=0 & v_{\theta II}^n|_{r_{II}=R_{II}}=0 & v_{zII}^n|_{r_{II}=R_{II}}=0 \end{array}\right\} \quad n \geq 2$$

$$v_{rI}^0|_{r_I=R_I}=0 \qquad v_{rII}^0|_{r_{II}=R_{II}}=0 \qquad \cdots(8)$$

$$v_{rI}^1|_{r_I=R_I}=-v_{\theta I}^1|_{r_I=R_I} \qquad v_{rII}^1|_{r_{II}=R_{II}}=-v_{\theta II}^1|_{r_{II}=R_{II}}$$

and from the assumption (5), we have

$$v_{zI}^0|_{r_I=R_I}=0 \qquad v_{zII}^0|_{r_{II}=R_{II}}=0 \qquad \cdots\cdots(9)$$

It is necessary for the flexible vibration to evaluate the angle of cross section because of subdivision of the cylindrical foundation. Therefore, the equations of motion for the each element of its foundation are given by

$$\frac{\partial}{\partial z}[\kappa_t(G+G'\frac{\partial}{\partial t})\, A\,(\frac{\partial v}{\partial z}+\frac{w}{R})] = \rho A\,\frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial}{\partial z}[\,(E+E'\frac{\partial}{\partial t})\frac{I}{R}\frac{\partial w}{\partial z}]-\kappa_t(G+G'\frac{\partial}{\partial t})\, A\,(\frac{\partial v}{\partial z}+\frac{w}{R}) = \frac{\rho I}{R}\,\frac{\partial^2 w}{\partial t^2} \quad \cdots\cdots(10)$$

in which, A and I are the cross-sectional area and moment of inertia for the cylindrical foundation, respectively, R is the radius, ρ is the density, E and G are the Young modulus and shear modulus, respectively, E' and G' are the viscous constants corresponded to E and G , respectively, and κ_t is the coefficient of cross-section, $\kappa_t = 0.85$ for circular section. Provided that the super-structure is represented by N_K-lumped mass system, its moment m_K and its shear force S_K acting on the K-th foundation may be written by

$$m_K = -\omega^2 \sum_{j=1}^{N_K} (H_K^j - H_K^{j-1}) \sum_{n=j}^{N_K} M_K^n ({}_K G_{Bu}^n\, \tilde{u}_K^B + {}_K G_{B\phi}^n\, \tilde{\phi}_K) - \omega^2 \sum_{j=1}^{N_K} I_K^j\, \tilde{\phi}_K$$

$$S_K = -\omega^2 \sum_{j=1}^{N_K} M_K^j ({}_K G_{Bu}^j\, \tilde{u}_K^B + {}_K G_{B\phi}^j\, \tilde{\phi}_K) \qquad K = \mathrm{I}, \mathrm{II} \quad \cdots\cdots(11)$$

in which ${}_K G_{Bu}^j$ and ${}_K G_{B\phi}^j$ are the displacement transfer functions of the j-th mass to horizontal translation $\tilde{u}_K^B$ and rotation $\tilde{\phi}_K$ at the upper surface of the K-th foundation, respectively, H_K^j , M_K^j and I_K^j are the height, mass and mass moment of inertia at the j-th mass of the K-th foundation, respectively, and ω is the angular frequency.

Provided that the bed rock of the stratum is excited by horizontal harmonic motion $u_g e^{i\omega t}$, the stress wave propagates in the cylindrical foundation and in its surrounding soil ground. Therefore, the equation of motion for the foundation is expressed by

$$([\,F\,] + [\,P\,] - \omega^2 [\,M\,])\{\,V\,\} = u_g \omega^2 [\,M_H^*\,]\{\,E\,\} + [\,P\,]\{\,V_G\,\} \quad \cdots\cdots(12)$$

where, $[\,F\,]$ and $[\,M\,]$ are the stiffness matrix and mass matrix for two thin sliced foundations, respectively, $[\,P\,]$ is the reaction matrix acted on the side of foundations, $[\,M_H^*\,]$ is the mass matrix for two sliced foundations of which rotational components are ignored, $\{\,V\,\}$ is the displacement vector of two thin sliced foundations, $\{V_G\}$ is the displacement vector of the thin layered soil ground excited by the bed rock motion $u_g e^{i\omega t}$, and $\{\,E\,\}$ is the vector which consists of elements equal to unity for horizontal translation and elements equal to zero for rotation.

NUMERICAL RESULTS AND DISCUSSIONS.

The cross-interaction model considered is shown in Fig. 2. Each structure model is represented by a single-mass-foundation embedded in a visco-elastic stratum. The following dimensionless parameters are introduced:

$a_0 = \omega R_0 / V_S$	: frequency	$h = H_{\mathrm{I}}/R_0 = H_{\mathrm{II}}/R_0$	: height of upper mass
$d = D/R_0$	: layer depth	$e = D_{e\mathrm{I}}/R_0 = D_{e\mathrm{II}}/R_0$	: depth of basement
$m_K = M_K/\rho R_0^3$	: mass	$x = D_x/R_0$	: separation distance
ρ	: density	$\eta_P = \frac{V_S}{R_0}\frac{\lambda' + 2\mu'}{\lambda + 2\mu}$, $\eta_S = \frac{V_S}{R_0}\frac{\mu'}{\mu}$, $V_S = \sqrt{\frac{\mu}{\rho}}$	
λ, μ	: Lamé's constants		
λ', μ'	: viscous constants	ν	: Poisson's ratio
G	: shear modulus	R_0	: reference value of length
E	: Young modulus	L	: numbers of subdivisions

And common values of these parameters for numerical example are expressed in Table 1. The mass moment of inertia of the upper mass about its center of gravity is ignored.

Table 1 Values of parameters for cross-interaction system.

Soil Media	Upper Mass	Basement	Under Ground
$\nu = 0.25$ $\eta_p = \eta_s = 0.1$ $d = 2.0$ $L = 8$	$m_K = 1.0$ $\zeta_K = 0.05$ $h_K = 1.0$	$\rho_{bK}/\rho = 0.4$ $G_{bK}/\mu = 6.0$ $E_{bK}/\mu = 14.0$ $\eta_{bK} = 0.1$	$\rho_{gK}/\rho = 1.0$ $G_{gK}/\mu = 1.0$ $E_{gK}/\mu = 2.5$ $\eta_{gK} = 0.1$

(K = I, II)

Figs. 3(a) and 3(b) show the magnification factor of the upper mass M_I of structure I in the cross-interaction system subjected to horizontal harmonic excitation at the bed rock, versus the dimensionless frequency a_0. Two different structures with natural frequency $\pi/4$ and $\pi/2$, respectively, are located at distances $x = 3$, $x = 4$, $x = 8$ and $x = \infty$ from each other. Fig. 3 (a) corresponds to the magnification for $\omega_I = \pi/4$ and Fig. 3(b) for $\omega_I = \pi/2$. In both Figures, the solid line corresponds to the magnification when only one structure is present, or, equivalently, when the separation between the structures tends to infinite. It may be observed that the additional interaction effects due to the presence of a second structure are more important in the vicinity of its resonant frequency, and that the larger the separation between two structures, the smaller the additional interaction effects.

The acceleration time history of the earthquake response of the cross-interaction system due to the typical earthquake ground motion is evaluated using the Fast Fourier Transform technique. The acceleration responses at the upper mass of two structures with natural frequency $\omega_I = \pi/4$ and $\omega_I = \pi/2$, respectively, due to Taft N21E, 1952 earthquake ground motion are shown in Fig. 4. The time history at distance $x = 8$ is similar to that of single structure, but that at distance $x = 3$ is different from that of single one.

Figs. 5(a) and 5(b) show the maximum earthquake response due to El Centro NS, 1940 and Taft N21E, 1952, respectively, versus the separation distance, for the natural frequency of second structure $\omega_{II} = \pi/4$, $\omega_{II} = \pi/2$ and $\omega_{II} = \pi$. In these Figures, the dotted line corresponds to the maximum response of single structure. It may be observed that the additional interaction effects become more important when the response of the second structure is greater than that of structure considered.

To illustrate the effect of a second structure on the maximum earthquake response, the ratios of the maximum response of coupled structure to that of uncoupled one due to the four typical earthquake ground motions are evaluated; *i.e.* El Centro NS 1940, El Centro EW 1940, Taft N21E 1952 and Taft S69E 1952. The largest value of the four ratios is shown in Fig. 6, against separation distance $x = 3$, $x = 4$, $x = 6$ and $x = 8$. In these Figures, two circles connected with the solid line corresponds to the two structures considered, and if a pair of circles belongs to one of the domains associated with the natural frequency of super-structure $\omega = \pi/4$, $\omega = \pi/2$ or $\omega = \pi$, two structures are identical each other, on the other hand, if a pair of circles crosses over the two domains, these are different from each other. It is pointed out that for two identical structures, the maximum coupled response of the cross-interaction system subjected to earthquakes is apt to be smaller than that of uncoupling system, but that for two different structures, the maximum coupled response becomes mostly greater than that of single structure.

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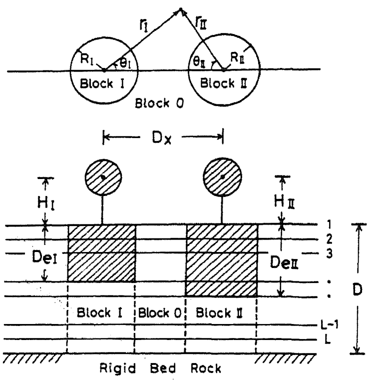


Fig. 1 Description of the model.

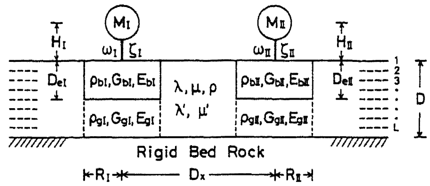
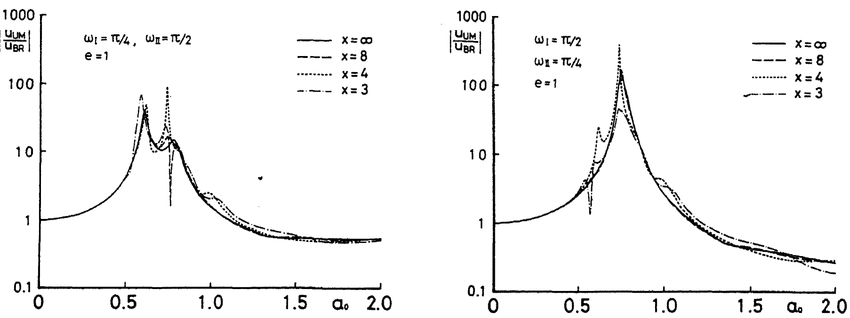
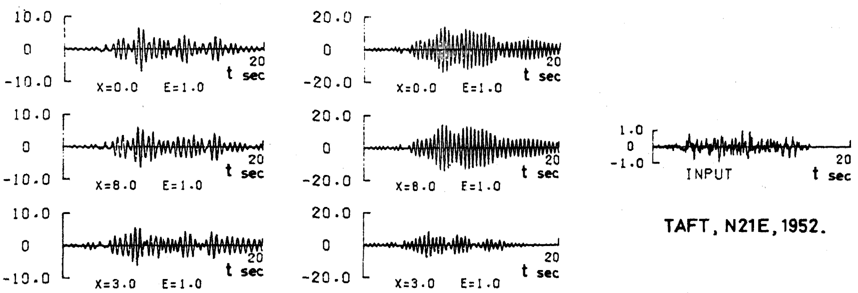


Fig. 2 Two-structures-soil system.



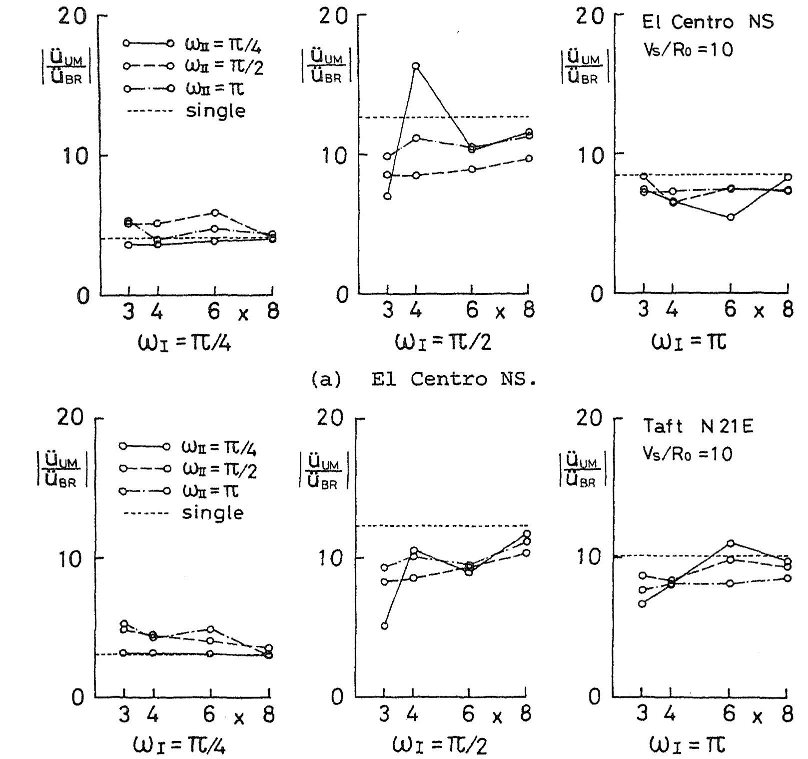
(a) $\omega_I = \pi/4$, $\omega_{II} = \pi/2$. (b) $\omega_I = \pi/2$, $\omega_{II} = \pi/4$.

Fig. 3 Magnification factor of cross-interaction system, $d=2$, $e=1$.



(a) $\omega_I = \pi/4$. (b) $\omega_I = \pi/2$.

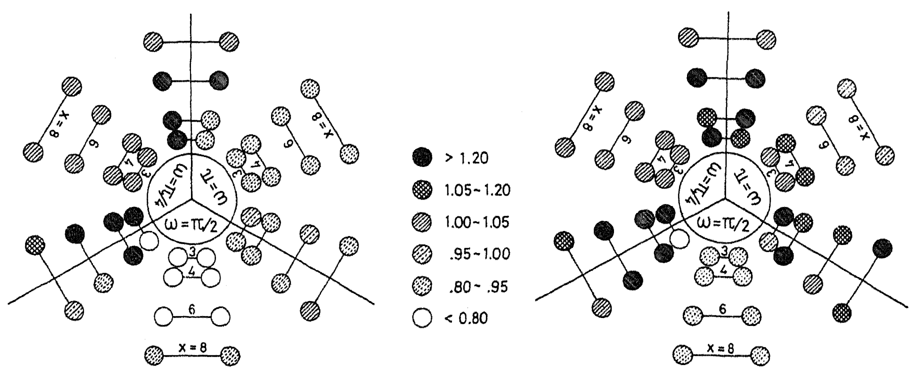
Fig. 4 Time histories of two defferent structures ($\pi/4$, $\pi/2$) due to Taft N21E earthquake ground motion.



(a) El Centro NS.

(b) Taft N21E.

Fig. 5 Maximum responses of upper mass against separation distance, $d=2$, $e=1$, $V_s/R_0=10$ (1/sec).



(a) $V_s/R_0=10$ (1/sec). (b) $V_s/R_0=20$ (1/sec).

Fig. 6 Ratios of maximum response for two coupled structures to that for single structure, $d=2$, $e=1$.