STOCHASTIC RESPONSE ANALYSIS OF NONLINEAR STRUCTURES SUBJECTED TO STRONG MOTION EARTHQUAKES

by

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SUMMARY

The purpose of this paper is to present an analytical method on random responses of nonlinear systems. The response process is described by Marcov chain and the response problem is replaced by a threshold problem. The nonlinear response is described by discrete states at which the response is of linear behavior. And the response moves from one state to other states with the transition probability that the response exceeds the threshold value. The probabilities of displacement and maximum displacement of a nonlinear system are obtained. The numerical results are compared with empirical and analytical results, and the comparisons are generally satisfactory.

INTRODUCTION

In evaluation of structural safety during earthquakes, the stochastic responses of a structure in the nonlinear range are required because the earthquakes are random phenomena and the structure generally exhibits nonlinear behavior. But analytical studies of random vibrations of nonlinear systems are generally difficult, and empirical methods are useful in obtaining the solutions of such problems.

The purpose of this paper is to present a theory of nonlinear random vibration. In this theory the response process is considered as Marcov chain and the problem is replaced by a threshold problem. The probability densities of displacement and maximum displacement of the nonlinear system are obtained, and the results are compared with empirical results.

METHOD OF ANALYSIS

It is assumed that the nonlinear response process is a discrete type of Marcov chain. In this case the probability distribution of the response is dependent on the transition probability and the initial distribution. The probability that the response process of a single-degree-of-freedom system is in the j-th state at time n is

$$P(y_n=j) = \sum_i P(y_n=j\,|\,y_{n-1}=i)P(y_{n-1}=i) = \sum_i q_{ij}^{n-1}\, P(y_{n-1}=i) \qquad (1)$$

where $P(y_n=j\,|\,y_{n-1}=i)$ is the conditional probability of $y_n=j$ assuming $y_{n-1}=i$, and q_{ij}^{n-1} is the transition probability of i to j. And the initial distribution $P(y_0=j)$ is the one before the force excites and is described by

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$$P(y_0=j) = \begin{cases} P(y_0=0) = 1 \\ P(y_0=i) = 0 , \ i \neq 0 \end{cases} \tag{2}$$

Then the probability of response at time n is described by

$$\begin{aligned} \{P(y_n=j)\}^T &= \{P(y_0=j)\}^T \prod_{m=0}^{n-1} [\ q_{ij}^m\ ] \\ &= \{P(y_{n-1}=j)\}^T [\ q_{ij}^{n-1}\ ] \end{aligned} \tag{3}$$

where $[\ q_{ij}^m\ ]$ is the transition probability matrix and T denotes the transposition. And the probability of response during time n is given by

$$\{P(y_m=j\ ;\ 0 \leq m \leq n)\} = \frac{1}{n} \left(\sum_{m=1}^{n} \{P(y_m=j)\} \right) \tag{4}$$

The probability of maximum response is as follows. The partial matrixes are denoted by

$$\begin{aligned} [\ q_0^m\ ] &= [\ q_{00}^m\ ] \\ [\ q_1^m\ ] &= \begin{bmatrix} q_{00}^m & q_{01}^m \\ q_{10}^m & q_{11}^m \end{bmatrix} \\ &\text{-----------------} \\ [\ q_k^m\ ] &= \begin{bmatrix} q_{00}^m & \cdots & q_{0k}^m \\ \cdots & \cdots & \cdots \\ q_{k0}^m & \cdots & q_{kk}^m \end{bmatrix} \end{aligned} \tag{5}$$

The probability that the response experienced at 0, 1,... k-th states during time n is denoted by $P^n(k)$, then $P^n(k)$ is the probability distribution function of maximum response and is described by

$$\begin{aligned} P^n(0) &= P(y_{m,max} \leq 0\ ;\ 0 \leq m \leq n) = P(y_0=0) \prod_{m=0}^{n-1} [q_0^m] \\ P^n(1) &= P(y_{m,max} \leq 1\ ;\ 0 \leq m \leq n) = \begin{Bmatrix} P(y_0=0) \\ P(y_0=1) \end{Bmatrix}^T \prod_{m=0}^{n-1} [q_1^m]\{I\} \\ &\text{--} \\ P^n(k) &= P(y_{m,nax} \leq k\ ;\ 0 \leq m \leq n) = \begin{Bmatrix} P(y_0=0) \\ \vdots \\ P(y_0=k) \end{Bmatrix}^T \prod_{m=0}^{n-1} [q_k^m]\{I\} \end{aligned} \tag{6}$$

Then the probability density function is given by

$$\begin{aligned} p^n(j) &= P^n(j) - P^n(j-1) \\ p^n(0) &= P^n(0) \end{aligned} \tag{7}$$

The displacement response of a hysteretic nonlinear system is denoted by the states at which the response shows linear behaviors. The response behaves linearly at each state and it moves to other states with the probability that it crosses the threshold value. Then the transition probability is estimated by the threshold probability of response. In this paper the threshold probability is based on the probability of the linear response process because the probability density of elastic-nonlinear response is an approximately linear one that is bent at its yielding point. Fig. 1 shows the concept of the stochastic model of the bilinear response.

The threshold value is estimated based on energy with the elastic energy of a linear system equal to the elastoplastic energy of a nonlinear system. Fig. 2 shows the restoring force-displacement relation in which displacement states are denoted by ⓪, ①, ②, ..., and the interval between displacement states by Δy. The points A and B are determined so that the elastic energies $\triangle OA'A$ and $\triangle OB'B$ are equal to elastoplastic energies $\square OO'a'a$ and $\square OO'b'b$ as indicated below.

$$\left.\begin{aligned} \square OO'a'a &= \triangle OA'A \\ \square OO'b'b &= \triangle OB'B \end{aligned}\right\} \qquad (8)$$

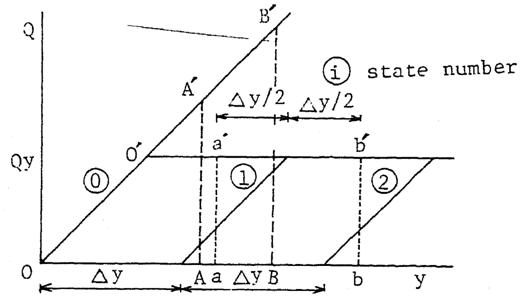

Fig. 2 Calculation of the Threshold Value

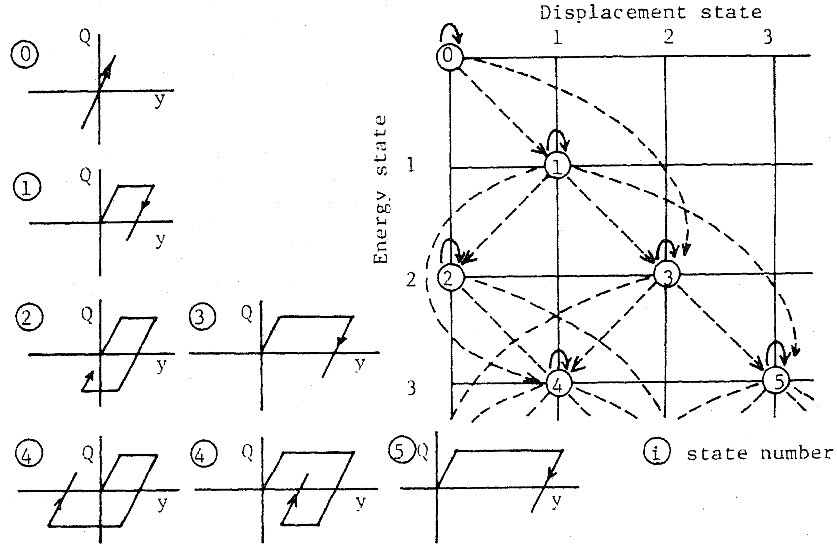

Fig.1 Stochastic Model of Bilinear Response

And the points A and B are threshold values that the response moves from the 0-th state to the 1st state and from the 0-th state to the 2nd state. Then the transition probability that the response moves from the 0-th state to the 1st state is given by

$$q_{01} = P(A \leq y < B) \tag{9}$$

If it is assumed that the response is a stationary Gaussian narrow-band process at each state, the threshold probability that the response crosses the line y = A in unit time is given by the next equation from Rice's theorem:

$$P(y \geq A) = \frac{\nu_A^+}{2f_e} = \frac{\nu_A^+}{2\nu_0^+} = \frac{1}{2} \exp\left(-\frac{A^2}{2\sigma_y^2}\right) \tag{10}$$

where f_e is the expected frequency, ν_A^+ is the expected crossing number of threshold value A in unit time and σ_y is the variance of displacement. The variance of stationary response is given by

$$\sigma_y^2 = \int_{-\infty}^{\infty} |H(\omega)|^2 \, S(\omega) d\omega \tag{11}$$

where $S(\omega)$ is power spectral density of input waves and $H(\omega)$ is harmonic response function. Therefore, the transition probability q_{01} is described by

$$q_{01} = \frac{1}{2} \left[\exp\left(-\frac{A^2}{2\sigma_y^2}\right) - \exp\left(-\frac{B^2}{2\sigma_y^2}\right) \right] \tag{12}$$

The assumption that the response is a stationary Gaussian process gives a good approximation when the yielding point is high and the crossing number of threshold value is a low one. But it gives an overestimation of the transition probability when the yielding point is low. Therefore, the number of states that the response can jump to in unit time is controlled by the total energy and the maximum displacement in each state, and hereinafter, this will be called jumping number. The jumping number is decided so that the response dissipates the average energy in unit time because the total energy transmitted to the system is almost constant and is dependent on the mass and natural period of the system. However, this gives a large displacement when the yielding force is low; the jumping number is limited by the maximum displacement so that the response cannot move.

The responses of a multi-degree-of-freedom system are determined by the probability distribution of story displacements. They behave linealy at their states and move other states with the probabilities that they cross thier threshold values. The transition probabilities are estimated by the threshold probabilities of linear systems and the interactions of stories are estimated from thier jumping numbers that are determined from thier yielding force distributions.

NUMERICAL EXAMPLES

As a check of the theory, comparisons were made with empirical and analytical results as shown in the figures. The seismic excitations are the simulated waves of two types which are band limited white noise and stationary waves. The power spectral density of a stationary wave is given by

$$S_x(\omega) = \frac{1 + 4h_g^2 \dfrac{\omega^2}{\omega_g^2}}{\left(1 - \dfrac{\omega^2}{\omega_g^2}\right)^2 + 4h_g^2 \dfrac{\omega^2}{\omega_g^2}} \qquad (13)$$

and the values of ω_g and h_g are 5π and 0.6. The root mean square values of the two types of waves are the same and the duration times are also the same at 15 sec. The structures are single-degree-of-freedom and three-degree-of-freedom systems of shear frame type with bilinear hysteretic force displacement relations. Thier natural periods both are 0.5 sec and damping ratios are 0.05. In the figures the abscissae show displacement and the coordinates probability density (p). The analytical results are shown by ●---● (j=1), ○--○ (j=2), and ■--■ (j=3) according to jumping number (j). The empirical results are shown by solid lines and the mark ▲.

Fig. 3 and 4 show the comparisons for various yielding force (Qy/W) systems of which the ratio of plastic to elastic stiffness (K_2/K_1) is zero. Fig. 5 shows the comparisons for various ratios of plastic to elastic stiffnesses. Fig. 6 shows the comparisons for various duration times. The empirical results are for every 3 sec of responses. Fig. 7 shows the comparisons for the three-degree-of-freedom system. This system has a yielding force distribution such that all stories yield simultaneously. These comparisons of empirical and analytical results show generally good agreement. Fig. 8 shows the comparisons between the response for a real earthquake record and the analytical results which are expected when an earthquake of the same statistical properties excites the system. The record is the ElCentro 1940 NS component (Imperial Vallry Earthquake). The earthquake wave has a nonstationary property, but in the analysis the transition probability is estimated under a condition that the expected value of linear response at each state is constant. The linear response by the step-by-step method was used for this expected value. That is, it is possible to evaluate the expected value of response when the system is excited by a wave having the same properties.

CONCLUSION

The random response process of a nonlinear system is described by Marcov chain and the response problem is replaced by a threshold value problem. The nonlinear response is described by discrete displacement states at which the response behaves linearly. And the transition probability is estimated by the probability that the response exceeds the threshold value. In this way the probabilities of displacement and maximum displacement are obtained. The numerical results are compared with empirical and analytical results, and the comparisons are generally satisfactory.

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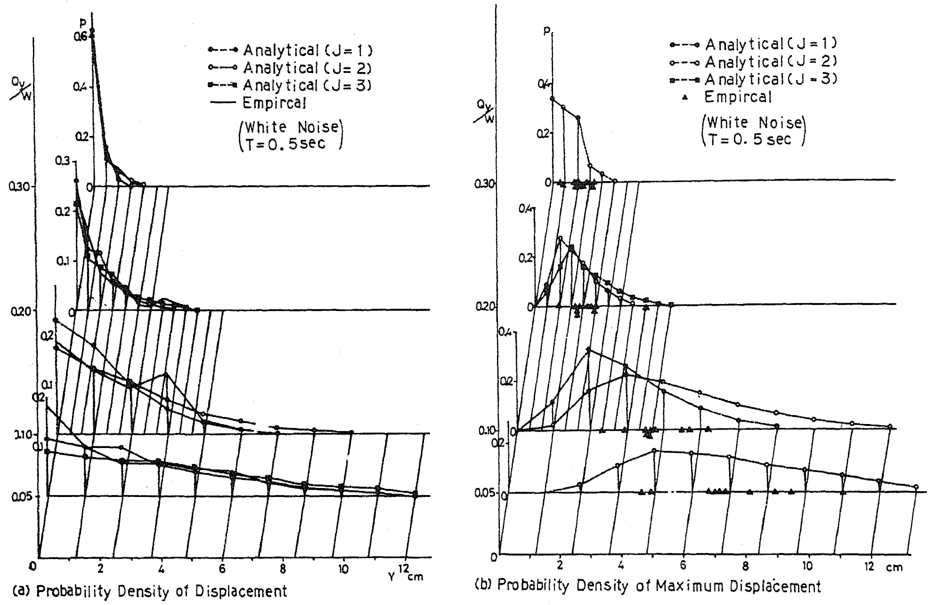


Fig. 3 Comparisons of Displacements and Maximum Displacements for Various Yielding Force Systems (White Nise, T=0.5 sec)

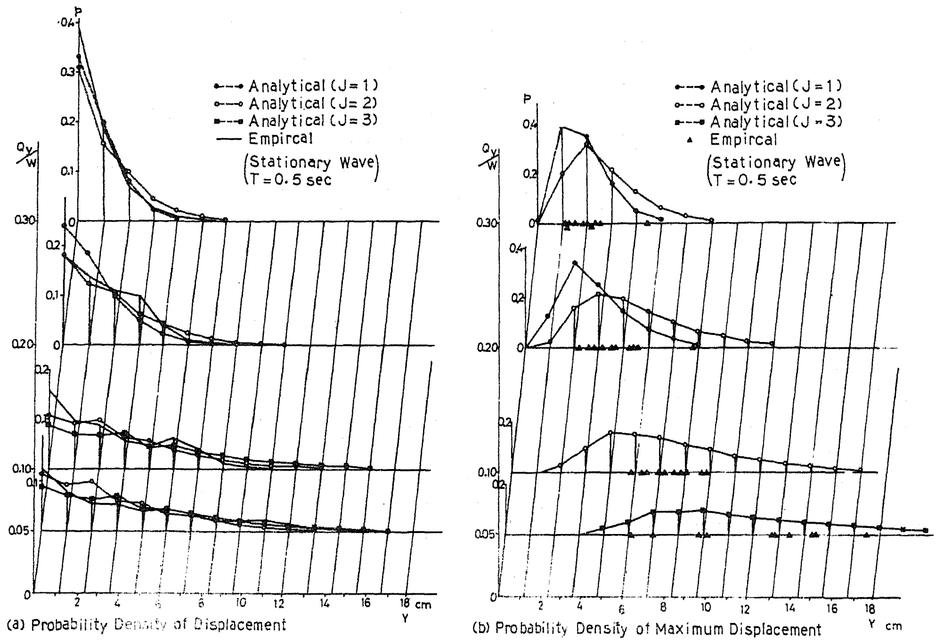
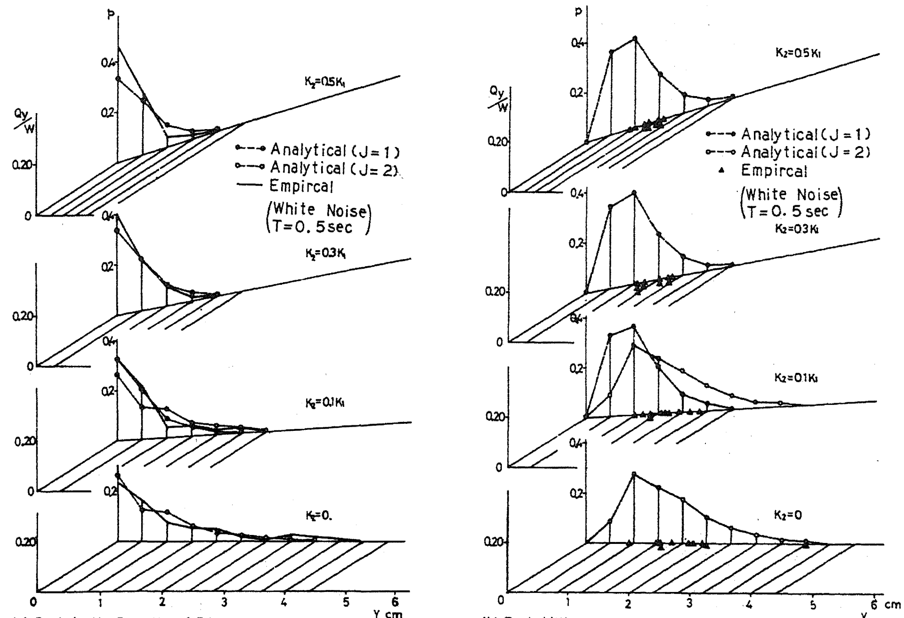
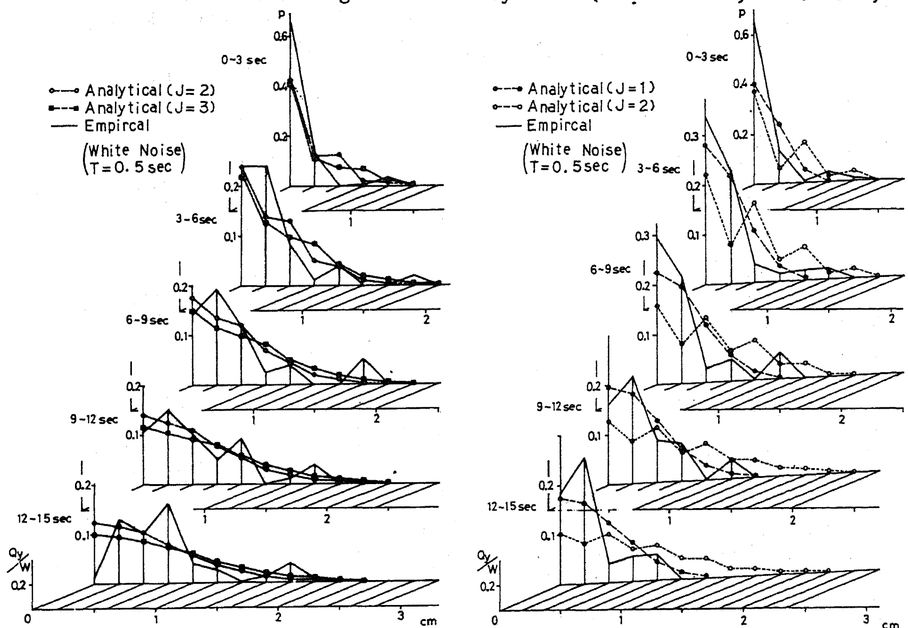


Fig. 4. Comparisons of Displacements and Maximum Displacements for Various Yielding Force Systems (Stationary Wave, T=0.5 sec)



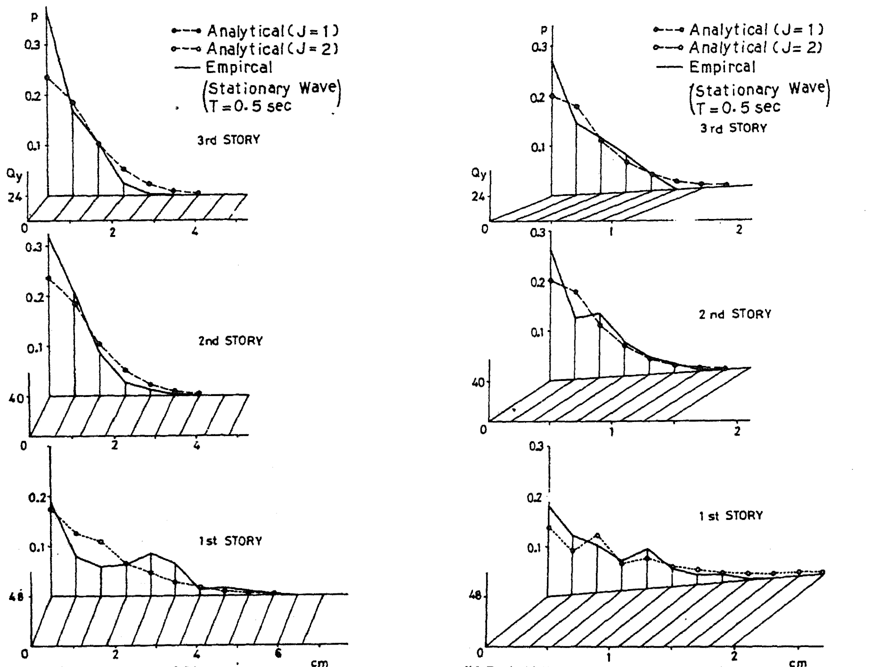
(a) Probability Density of Displacement (b) Probability Density of Maximum Displacement

Fig. 5 Comparisons of Displacements and Maximum Displacements for Various Yielding Stiffness Systems (White Noise, T=0.5 sec)



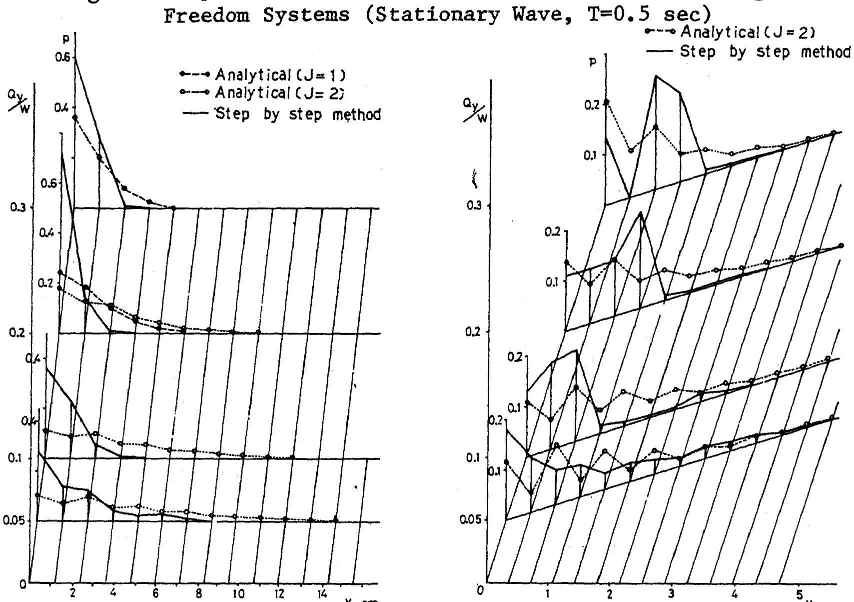
(a) Probability Density of Displacement (K_2 = 0.) (b) Probability Density of Displacement (K_2/K_1 = 0.1)

Fig. 6 Comparisons of Displacements for Various Duration Times (White Noise, T=0.5 sec)



(a) Probability Density of Story Displacement ($K_2=0.$)   (b) Probability Density of Story Displacement ($K_2/K_1=0.1$)

Fig. 7 Comparisons of Story Displacements for Three-Degree-of-Freedom Systems (Stationary Wave, T=0.5 sec)



(a) Probability Density of Displacement ($K_2=0.$)   (b) Probability Density of Displacement ($K_2/K_1=0.1$)

Fig. 8 Comparisons between the Real Earthquake Responses and the Analytical Results (El Centro 40 NS, T=0.5 sec)