A RELIABILITY THEORY FOR ASEISMIC DESIGN AND FOR PREDICTION OF THE SAFE LIFE OF INDIVIDUAL STRUCTURES

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Summary. A version of the reliability theory is developed with special application to the aseismic design of structures. Earthquakes are presented as a stream of random events, and the ground motion at a given site as a nonstationary random process. A probabilistic model for strong earthquakes occurence is developed taking into account the seismic history in neighbouring focal areas. Strong earthquakes are interpreted as results of the development of a slow random process including instrumentally measured precursors. The earthquake occurs when this process reaches a certain upper threshold which is a random value depending on the geological conditions in the focal area. Clustering of earthquakes is incorporated by conditional probabilities for corresponding lower thresholds.

Using the proposed model together with the methods of the reliability theory and the conventional models for dynamic response and damage accumulation under seismic input, a united approach to the aseismic design of structures is developed with applications both to the design and operational stages. In the latter case, observation datas concerning the operating structure are used. As the unified approach is too complicated for analytical realization, the Monte-Carlo procedure is applied beginning from the simulation of seismic activity up to the simulation of damage accumulation in the structure and the assessment of safety and reliability factors.

General concepts of the theory of reliability. Following mainly to references [1, 2] introduce the general concepts of the theory of reliability. Let consider a structure or a structural element under random loading (the term "structure" is used further). The state of the structure is described by the vector u which is an element of the state space U . Evolution of the structure in time t is described by the random vector process $u(t)$. Basic equation to evaluate the process $u(t)$ is the operator equation $Lu=q$ where $q(t)$ is the random vector process in the load space Q which includes environment of the structure. Choice of the spaces U and Q and the operator L is a problem of the mechanics of structures.

Operational and service requirements put restrictions on parameters of the structure and its behaviour. This properties are described by the vector v which is an element of the quality space V . A subset of states of the structures admissible from the point of view of operational requirements corresponds to the open region Ω in the quality space The boundary $\partial\Omega=\Gamma$ corresponds to limit states. The first excursion of the process $v(t)$ from the region Ω (the first positive crossing of the surface Γ) corresponds to the failure (Fig. 1).

The failure of a structure is a random event. The most important concept of the reliability theory is reliability function $P(t)$. This

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function is equal to the probability of non-failure operation of the structure during the time segment $[t_0, t]$:

$$P(t) = \underline{P}\{v(\tau) \in \Omega ; \tau \in [t_0, t]\} \tag{1}$$

Here $\underline{P}\{\cdot\}$ is the probability of a random event. The risk function $Q(t)$ and the failure rate $\lambda(t)$ are expressed by the reliability function $P(t)$:

$$Q(t) = 1 - P(t) \ , \ \lambda(t) = -\frac{P'(t)}{P(t)} . \tag{2}$$

This concept can be generalised to cases when repeated failures, repairs and recovers are included into consigeration.

The assessment of reliability factors requires taking into account random properties of structures. On the design stage, structure is to be considered as a randomly chosen element from the general sample of similar structures. Let properties of the structure to be characterized by the random vector r. The conditional reliability function for the structure with the given value r is

$$P(t|r) = \underline{P}\{v(\tau|r) \in \Omega(r) ; \tau \in [t_0, t]\} \tag{3}$$

where $v(t|r)$ is the conditional vector process. The unconditional (complete) reliability function is

$$P(t) = \int P(t|r) p_r(r) dr \tag{4}$$

where $p_r(r)$ is the corresponding probability density.

One of the applications of the reliability theory is prediction of the reliability and residual life factors for an individual structure on the operational stage. An apropriate procedure was proposed in [3] using current inspection data, load history records as well as apriori probabilistic information available on the design stage. The procedure consists of the estimation of the state of the structure at the given time moments and the stochastic extrapolation of the random vector processes upon the next time segments. Calculated probabilistic values are to be interpreted in the Bayessian sense as they correspond to an individual object.

The extrapolated quality process is $v(t|r, T_k)$ where the symbol T_k denotes all amount of datas concerning the given structure which were obtained from the inspection up to the last time moment t_k (Fig. 2). The value of the vector r is to be considered as a random one because only indirect methods of its estimation are available. The corresponding probability density $p_r(r|T_k)$ takes into account information collected up to the time t_k. The conditional aposteriori reliability function is

$$P(t|r, T_k) = \underline{P}\{v(\tau|r, T_k) \in \Omega(r) ; \tau \in (t_k, t]\} . \tag{5}$$

Further details may be found in [3] where an optimization approach is proposed to estimate the "best" moment for the next inspection, repair or stopping of service.

<u>A probabilistic model for strong earthquake occurence.</u> The existing methods for assessment of seismic risk [4, 5] are based on the macroseismic formulas obtained by statistical treatment of observation datas at the most interesting seismic regions. The Poisson stream of events is mostly accepted as a model for the sequence of strong earthquakes. The reason is partly in simplicity of the Poisson model and partly in lack of statistical information. Deficiencies of the Poisson model are well known. The main defect is a constitutional inability to take into account seismic history at a certain site. This defect seems to be especially serious if the problem of prediction of the reliability and safety factors for an individual operating structure is considered.

A modern approach has to incorporate both history effects and recent achievements on the field of earthquake prediction [6, 7]. Most of the proposed methods of prediction are based on measuring of precursors, among them mechanical parameters directly connected with geotectonic processes. These parametres usually are continuous and monotonously growing time functions if an interval between two earthquakes is considered. During earthquakes these parameters are subjected to discontinuities which sizes are in correlation with the magnitudes of the corresponding earthquakes. Hence, the natural approach to modelling of seismic activity considering seismic history and precursors is based on consideration of a piece-wise continuous processes. It is also natural that the threshold values of these processes have to be random ones with the probability distribution depending on geological conditions in the considered focal area. Clustering of earthquakes can also be taken into account. As the time between two neighbouring earthquakes relating to the same cluster is small in comparison with the expected time between two neighbouring clusters, a slow variation of the precursors during the clustering period may be neglected. These ideas lead to the model illustrated in Fig. 3. Here $s = s(t \mid T_k)$ is the precursor parameter process depending on the set of information available to the last observation moment t_k. It is assumed that this moment is after the last cluster; i.e. the function $s = s(t \mid T_k)$ is continuous in the interval (t_k, t). The upper threshold value of the parameter s is denoted s^*. It is assumed that the probability density $p_*(s^*)$ does not depend on history. A first drop corresponding to the first earthquake in the cluster is determined by the lower threshold s_1^*. It is assumed that s_1^* is determined by the conditional probability density $p_1(s_1^* \mid s^*)$. Second drop is determined by the conditional probability density $p_2(s_2^* \mid s^*)$, etc. The sequence of earthquakes terminates when the parameter s drops lower than a certain value s^{**}. This value can be a random one; in that case the conditional probability density $p_{**}(s^{**} \mid s^*)$ is to be given. After the termination of the cluster, growing of the parameter s proceeds till to the next cluster. In principle, each cluster can consist of a single earthquake only.

To present the proposed model in details, we need a set of additional assumptions. They include the following ones: the functional dependence $s = s(t \mid T_k)$ of the parameter s; a set of conditional probability densities $p_1(s_1^* \mid s^*)$, $p_2(s_2^* \mid s_1^*)$, ... as well as $p_{**}(s^{**} \mid s^*)$;

functional dependences of macroseismic parameters (such as the earthquake magnitude, the amount of released energy, etc.) on the drops $s^* - s_1^*$, $s_2^* - s_1^*$, ..., $s^{**} - s^*$. The central problem is to agree proposed analytical equations with generally acknowledged macroseismic formulas. Among them are the formulas [4]

$$N(M) = N_0 \exp[-\beta(M-M_0)], \quad E = E_0 \exp[\alpha(M-M_0)] \tag{6}$$

for the average number of earthquakes $N(M)$ with magnitudes equal or greater than M and the amount of released energy E.

A number of special cases were considered to make agreement between the proposed model and Eqs. 6. For example, the function $s = s(t \setminus T_k)$ was taken in the form

$$s = s_k + b(t-t_k)^{\mu}, \quad t \in (t_k, t_{k+1}), \ \mu > 0 \tag{7}$$

and the upper threshold value distribution was proposed to be the Weibull distribution

$$F_*(s^*) = 1 - \exp\left[-\left(\frac{s^* - s_0}{s_c}\right)^{\mu_s}\right], \quad s^* \geqslant s_0 \tag{8}$$

Here b, μ_s, s_c are positive constants, and s_0 is a nonnegative constant. Eq. 8 corresponds to ultimate strength distribution in the theory of brittle fracture [1] and therefore is in close relation with the considered problem. The mutual use of Eqs. 7 and 8 results in the probability distributions for time periods between two neighbouring clusters and for amount of released energy which are related to the Weibull distribution also. But to make agreement with Eqs. 6 we need to introduce a very special relation between the energy E and the drop of the parameter s

Another upper threshold value distribution was taken in the form

$$F_*(s^*) = 1 - \left(\frac{s_{th}}{s^* - s^{**}}\right)^{\mu_s}, \quad s^* \geqslant s^{**} + s_{th} \tag{9}$$

where s_{th} and μ_s are positive constants, and the lower threshold value s^{**} is assumed to be deterministic. This distribution seems to be at the sight artificial in comparison with the Weibull distribution. But in fact, because the stronger earthquake are selected, the distributions for macroseismic parameters, as a rule, have to be truncated ones. Eq. 9 together with Eq. 7 result in Eqs. 6 under the assumption that the precursor s is a characteristic strain or stress in the focal area and the released energy E is proportional to the power μ_E of the drop $s^* - s^{**}$. If $\mu = 1$ (the linear increasing of the precursor between earthquakes), $\mu_E = 2$ (the linear stress-strain relation); and $\mu_s = 1$, we obtain $\alpha = 4$ and $\beta = 2$.

The proposed model permits to estimate seismic risk factors with account to seismic history. For example, let consider a single earthquake and let its magnitude to be a continuous function $M = g(s^*, s^{**})$. The probability that an earthquake occurs at the time segment $(t_k, t]$ with the magnitude equal or greater than M is

$$Q(M; t|T_k) = \iint\limits_{\Lambda} p_{**}(s^{**}|s^*)\, p_*(s^*)\, ds^*\, ds^{**} \qquad (10)$$

Integration area is $\Lambda = \{s^*, s^{**} : s^* \leq s(t|T_k);\ g(s^*, s^{**}) \geq M\}$.

Monte-Carlo procedure for aseismic design. The sequence of ground motions at the given site C is a result of earthquakes occuring in the neighbouring focal areas $\mathfrak{F}_1$, $\mathfrak{F}_2$, . . . (Fig. 4). The sequence of earthquakes in each area is described by the model presented above. The size effect, i.e. dependence of the probability distribution on the volume of the area or on its surface can be included in Eqs. 8 and 9 in the same manner as in the theory of brittle fracture. Using known semi-empiric attenuation formulas [4,5,8], we can estimate the maximum ground accelerations at the site. There are analogous formulas for the duration of earthquakes, the dominant frequencies, etc.

The common analytical presentation of the ground accelerations [1, 9 - 11] is

$$a(t) = \sum_j A_j(t)\varphi_j(t), \quad t \geq 0. \qquad (11)$$

Here $A_j(t)$ are pseudoenvelopes, and $\varphi_j(t)$ are segments of stationary random processes with zero mathematical expectation. If the "rapid" time $t < 0$ we put $a(t) \equiv 0$. Parameters of the functions $A_j(t)$ and of the spectral densities of the processes $\varphi_j(t)$ are functions of macroseismic parameters for the considered site. Using the proposed model for strong earthquake occurence, macroseismic formulas and the nonstationary random process model for ground acceleration given by Eq. 11, we can in principle estimate reliability factors given by Eqs. 1- 5, such as the reliability function, the risk function, the failure rate, etc. Evaluation of structural response to a nonstationary random process is a complicated problem which can be solved analytically only under very simplified assumptions. For example, even for nonlinear one-degree-of-freedom systems numerical statistical simulation (the Monte-Carlo method) is the only way to get numerical results.

The general idea of the approach proposed below is to apply numerical statistical simulation through all stages beginning from simulation of seismic activity up to assessment of reliability and safety factors of a structure. The first step is to simulate the process $s(t|T_k)$ using known initial conditions and probability distributions for the threshold values s^*, s_1^*, etc. The second step is to simulate seismic inputs at the given site. The third step is to simulate structural response to seismic inputs, including damage accumulation due to comparatively weak earthquakes. Statistical scatter of structures and materials properties are to be taken into account here also. The last step consists in evaluation of the reliability parameters and their comparsion with its codified values. The complete program is in fact a multiple perfomance of expected seismic activity at the given region and of the corresponding behaviour of structures during their service life. It is essential that the program is applicable both for structural design against earthquakes and for prediction of the individual safe life of an operating structure.

The volume of computations grows if a high reliability level is required. Roughly speaking, the number of samples must be much greater that the inverse value of the codified risk level. To diminish the volume of computations, a semi-analytical approach is recommended. The Monte-Carlo method is applied only for estimation of structural risk under seismic inputs with given parameters. This conventional risk is comparatively high. Therefore, its estimation does not require too much computational work. The complete risk is to be evaluated analytically using formulas similar to Eq. 10.

As a model example for prediction of the individual safe life, the one-degree-of-freedom bilinear hysteresis system was considered. The residual displacement $v(t)$ was taken as the quality factor entering in Eqs. 1, etc. The initial value was taken $v_0 = 0{,}2$ at the last observation time $t_0 = 5$ years, and the critical value $v_* = 1$. The seismic activity region was divided into six equal areas with characteristic distances to the considered site $R_1 = 25$ km, $R_2 = 50$ km, $R_3 = R_4 = 100$ km, $R_5 = 150$ km; $R_6 = 200$ km. The seismic activity level was assumed the same for all areas. The linear law in Eq. 7 and the Weibull distribution given by Eq. 8 were assumed at $s_0 = 0$, $\mu_s = 2$. Clustering was not included into consideration. The reoccurence time for earthquakes with magnitudes $M \geqslant 5$ was assumed $T_c = 50$ years for all areas. The last occurence times were put $t_{-1} = 1, -10, -3, -15$ and 4 years correspondingly. Macroseismic formulas were taken from [4, 5, 8] . A special form of Eq. 11 with entering numeric datas was taken from [1, 9] .

In Fig. 5 a number of sample processes $v(t|T_0)$ are shown including the "worst" and the "best" ones chosen from 50 samples. The simulated aposteriori risk function $Q(t|T_0)$ is plotted here too.

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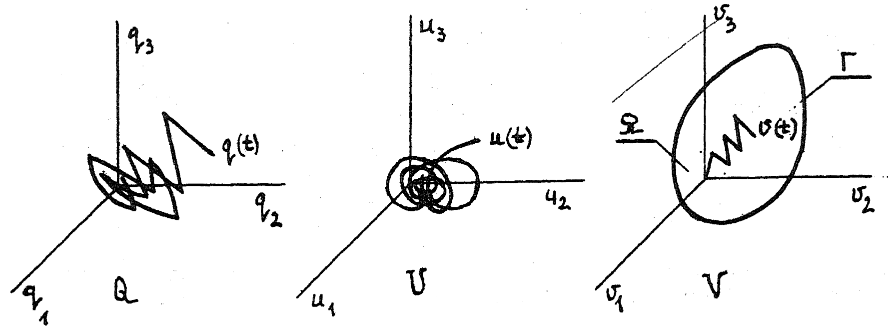

Fig. 1. Problem statement in the reliability theory.

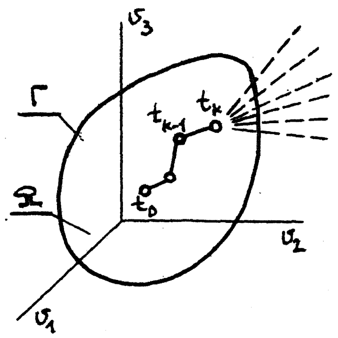

Fig. 2. Prediction of the individual safe life of a structure.

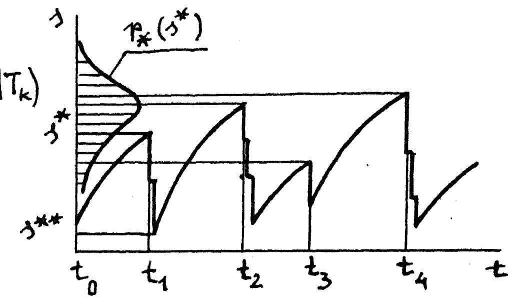

Fig. 3. The proposed probabilistic model for earthquakes occurence.

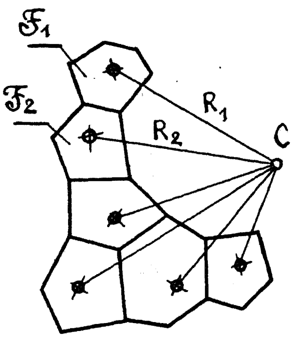

Fig. 4. Focal areas and the considered site.

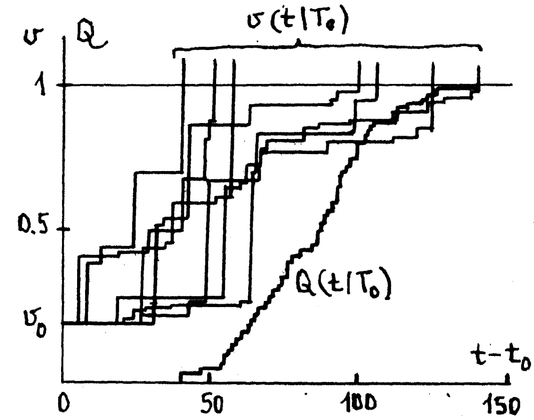

Fig. 5. Samples of aposteriori damage accumulation processes and the simulated aposteriori risk function.