

MATHEMATICAL MODEL FORMULATION OF A TWO-STOUREY STEEL FRAME  
STRUCTURE USING PARAMETRE SYSTEM IDENTIFICATION AND  
SHAKING TABLE EXPERIMENTS

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SUMMARY

In this paper, parameter system identification is used to formulate a realistic nonlinear mathematical model to represent the seismic behaviour of a two storey steel frame structure. The form of the model is a second order nonlinear differential equation with linear viscous damping and Ramberg-Osgood type hysteresis. The damping coefficient, elastic stiffness matrix and the two parameters in the hysteretic model are established.

The suggested mathematical model of the two storey frame represents six degree of freedom system which is transferred to two degree by static condensation method. The test of the model is done on programmed single component shaking table for an earthquake input. The selection of the earthquake input was done on the base of the sensitivity of the response of the structure to the earthquake motion. The most sensitive response is obtained for Parkfield Earthquake N65W June, 1966.

The application of the parametrical identification process, makes possible the formulation of a mathematical model which gives responses showing correlation with the experimental results providing the condition to obtain such an error in difference of the responses which is equally dependent on all of the selected parameters. Beside that, this approach enables simultaneous investigation of the influence of a considerable number of parameters which gives the possibility of better understanding of the real essence of the dynamic behaviour of structures affected by an earthquake.

In the considered two-storey steel frame, nonlinear mathematical model has been constructed assuming that the damping consists of viscous damping and damping due to energy absorption, by the hysteretic behaviour of the material. The non-linear dependence has been defined by Ramberg-Osgood curves which have priority over the choice of the shape of the nonlinear model in this case. The first step in the investigation of the dynamic behaviour of the frame was the construction of a model with six parameters. The results of this analysis are presented in this paper.

TEST STRUCTURE AND TEST RESULTS

The test structure, consisting of two parallel single-bay, two storey, moment-resistant steel frames, is shown in Fig. 1. Only the essential geometric property data concerning the test structure are reported here. The two frames, marked as F (front frame) and B (back frame), were disconnected for 120 cm. They were connected at the floor levels by fixed cross beams and bracing angles, thus, the effect of a floor diaphragm rigid in its own plane was simulated. Cross bracing systems were provided in the orthogonal plane to resist motions transverse to the excitation axis. As it is shown in Fig. 1 the storey heights are 140 cm.

The frames were fabricated from standard rolled shapes of I8 sections, having yield strength of 2400 kg/cm<sup>2</sup>. The same sections were used for beams as well as for columns. All joints between girders and columns are welded.

In order to provide a fundamental period of vibration in longitudinal direction in the range appropriate to actual steel frame buildings, and to apply a gravity load to the girders, blocks of concrete weighing about 500 kg per floor, were added to the structure. Estimated weight of the steel struc-

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ture is about 550 kg.

The instrumentation scheme of the front frame is shown in Fig. 2. The number of instrumentation points is limited by the number of available channels of data acquisition system. Horizontal absolute accelerations and displacements were measured in the direction of the table motion for both floors as well as for shaking table using accelerometers and potentiometers as it is indicated in Fig. 2. Four channels of data acquisition system were used for joint rotation measurement. This was done by supporting a reference (instrumentation) bar at the centers of two panel zones and measuring the rotation of the columns face with reference to this bar. The instrumentation is so built that it does not accept any moment, shear or axial deformation due to the motion of the frame structure. The rest of 20 channels from the DAS system were used for strain measurement in order to get a picture for stress-strain distribution on the frame structure due to earthquake motion. The used strain gages were post yield type.

The structure, with its weights in place, was subjected to many types of table motions applied with progressively increased intensity. The aim of all these runs was to find the earthquake motion to which the frame structure is the most sensitive. All these runs were done in linear range and only the last run for the selected earthquake was done in a non-linear range. In table 1 are presented some of the runs with their characteristics.

TABLE 1

Run	Earthquake	Span	Shaking table displacement		Max. acc. of the second floor (g)
			max (cm)	min (cm)	
1	El Centro 1970	250	2.60	2.90	0.384
2	El Centro 1970	500	5.19	5.67	0.843
3	Monte Negro EQ May, 1979	250	3.04	1.61	0.323
4	Monte Negro EQ May, 1979	500	5.80	3.16	0.602
5	Parkfield N65E 1966	250	2.42	2.77	0.964
6	Parkfield Mod. artificial	100	1.14	0.99	0.228
7	Parkfield Mod. artificial	250	2.49	2.98	1.12
8	Parkfield Mod. t = 9/10	250	2.39	2.91	1.15
9	-	500	4.69	5.77	2.40
10	-	850	9.40	7.03	3.10

All runs shown in Table 1 are linear except run 10 which is non-linear. Mathematical model formulation is done for that run.

FORM OF THE MATHEMATICAL MODEL

The first step in system identification is determination of the form of the model and isolation of the unknown parameters. For the existing two storey structure subjected to earthquake motion, the following set of second order differential equations is used:

$$\underline{U} \ddot{\underline{x}} + \underline{C} \dot{\underline{x}} + \underline{P}(\underline{x}) \underline{I} = - \underline{M} \underline{I} \ddot{\underline{x}}_g \dots\dots\dots (1)$$

In this equation  $\underline{M}$  is the mass matrix taken to be constant,  $\underline{C}$  is damping matrix and  $\underline{P}(\underline{x})$  is non-linear force matrix calculated from Ramberg-Osgood relationship.  $\underline{I}$  is identity vector, and  $\ddot{\underline{x}}$  and  $\dot{\underline{x}}$  are corresponding acceleration and velocity vectors.

For the considered frame, a system with 6 degree of freedom, two translations and four rotations, has been taken into account. Ignoring the rotation moments in the nodes, the model has been condensed in a system with only two degree of freedom (translation at first and second floor). In this way, the stiffness matrix becomes of the order 2X2, while the damping matrix is assumed to be proportional to the stiffness matrix, namely:

$$\underline{C} = \gamma \underline{K} \dots\dots\dots (2)$$

The matrix of the floor structure forces, is defined by means of Ramberg-Osgood law for ascending or descending curve using the step-by-step technique for solving the equation (1). Then, the increase of the elastic forces within the structure for the dependance defined by the Ramberg-Osgood equation, can be determined by two terms of the Tailor's series developed near the point  $x_i$ . The incremental form of the equation (1) from which dynamic response is calculated using constant acceleration method, has the following form:

$$\underline{M} \Delta \ddot{\underline{x}}_i + \underline{C} \Delta \dot{\underline{x}}_i + \underline{TS}_i \Delta \underline{x}_i = - \underline{M} \underline{I} \Delta \ddot{\underline{x}}_{gi} \dots\dots\dots (3)$$

$\underline{TS}$  represents tangent stiffness matrix at time  $t_i$ , calculated from Ramberg-Osgood curves.

The final step in this section is the isolation of the vector of unknown parameters. For the model used here, six parameters are selected: damping ratio  $\gamma$ , stiffness parameters  $K11, K12 = K21$  and  $K22, A$  and  $R$ . So, the vector of the isolated parameters is:

$$\underline{\beta} = [\gamma, K11, K12, K22, A, R] \dots\dots\dots (4)$$

The second phase in system identification is selection of a criterion function by means of which the "goodness of fit" of the model responses to the actual system responses can be evaluated, when both model and system are forced by the same inputs. For the purpose of this analysis, the criterion function is an integral mean squared error function that includes errors in time histories of displacements at both levels.

$$J(\underline{\beta}, T) = \int_0^T [\underline{x}(\underline{\beta}, T) - \underline{y}(t)]^T [\underline{x}(\underline{\beta}, T) - \underline{y}(t)] dt \dots(5)$$

where  $\underline{x}(\underline{\beta}, T)$  are response quantities calculated from the model using parameters  $\underline{\beta}$  and excitation  $\ddot{\underline{x}}_g(t)$  and  $\underline{y}(t)$  are measured response quantities from the structure subjected to the same excitation and  $T$  is a time interval properly selected.

The third phase of this process was selection of an algorithm for adjustment of the parameters in such a way that the differences between modal and system responses are minimized. Defining the vector of parameters  $\underline{\beta}_{i-1}$ ,

the next step of  $\beta_i$  will be an improved version of the parameters which will give smaller value of  $J$ . The modified Gauss-Newton method used here is derived by expanding the error function in a Taylor's series about the previous point  $\beta_{i-1}$  and retaining only the first three terms. Setting the gradient of the error with respect to  $\beta_i$  equal to zero, we get:

$$\beta_i = \beta_{i-1} - \alpha \overline{AH}^{-1}(\beta_{i-1}, T) \cdot \overline{\Delta J}(\beta_{i-1}, T) \dots\dots\dots (6)$$

where  $\overline{AH}$  is the Hessian matrix,  $\overline{\Delta J}(\beta_{i-1}, T)$  is the gradient vector and  $\alpha$  is a positive scalar which ensures that the error function is decreased in each iteration. The step size of  $\alpha$  in this analysis is calculated from quadratic extrapolation.

For the identification process in this analysis, a special computer program has been developed for PDP 11/45 DEC computer.

#### MODEL FORMULATION

For the formulation of a non-linear mathematical model, the obtained experimental results during run 10 (see the table) have been used. One of the first steps in using parametrical system identification program is the selection of the initial vector values of the parametre. In this case, the initial values are estimated on the basis of approximate analysis by which the initial values of vector  $\beta$  have been obtained.

$$\beta = \{0.0008, 1.50, 2.00, 4.00, 0.1, 6.0\}$$

The selected time of integration is  $T = 5.0$  sec. Using the iterative procedure according to equation 6, the finite vector of parameters is defined as follows:

$$\beta = \{0.000432, 1.37, 1.87, 4.51, 0.0126, 11.18\}$$

with program stopping tolerance of 1% for criteion function . The proportions used in the analysis are the following: ton, centimeter and second. The stiffness parametres represent initial tangent stiffness, while the damping matrix represens viscous damping.

For the last established parameters, the response structure has been calculated. The comparison between experimental and calculated responses is presented in Fig.3 and Fig. 4 respectively for acceleration and displacement responses at both floors.

#### CONCLUSIONS

The results presented in this paper are to be the first step in the investigations performed on the considered frame. In the following investigations based upon these results and applying the iterative procedure of integration, the parametres which define the model with 6 degree of freedom, will be determined. During the selection of the frame structure, the main point was the simulation of such frame motions which will cause considerable elasto-plastic deformations. However, this condition was only partially obtained due to the defects of the seismic shaking table.

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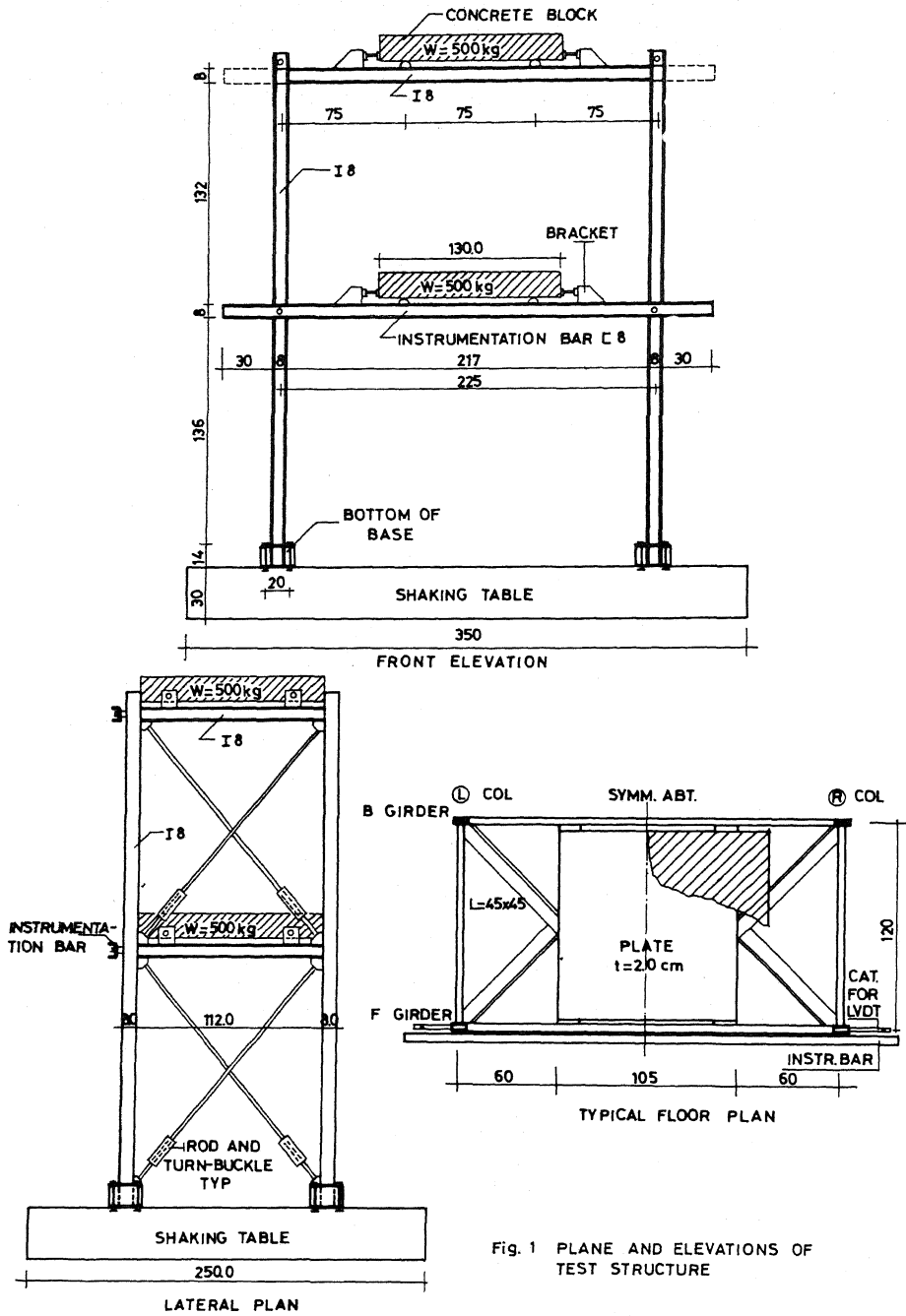


Fig. 1 PLANE AND ELEVATIONS OF TEST STRUCTURE

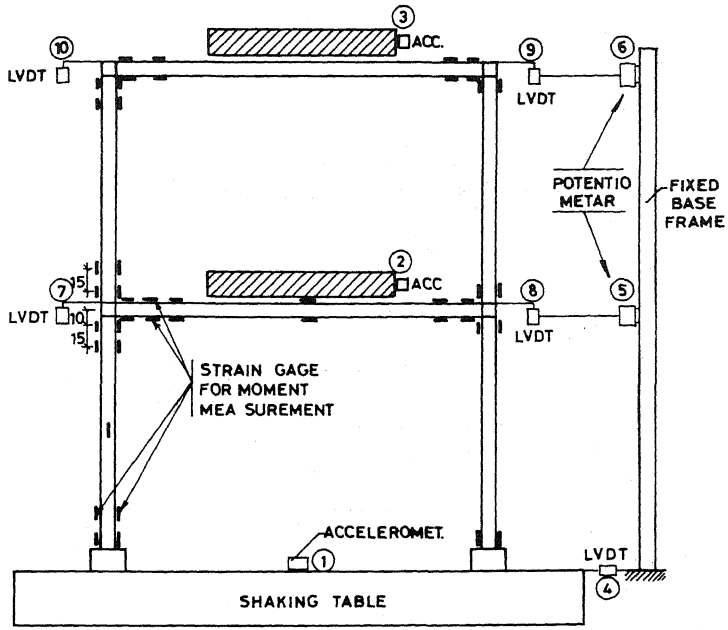
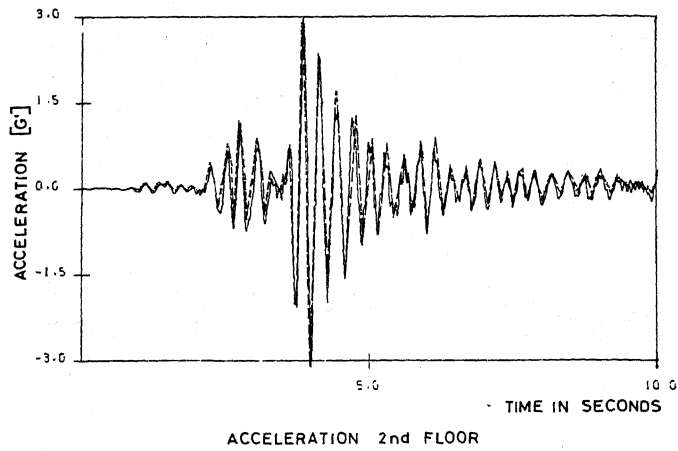


Fig. 2 SCHEM INSTRUMENTATION OF FRAME (F)



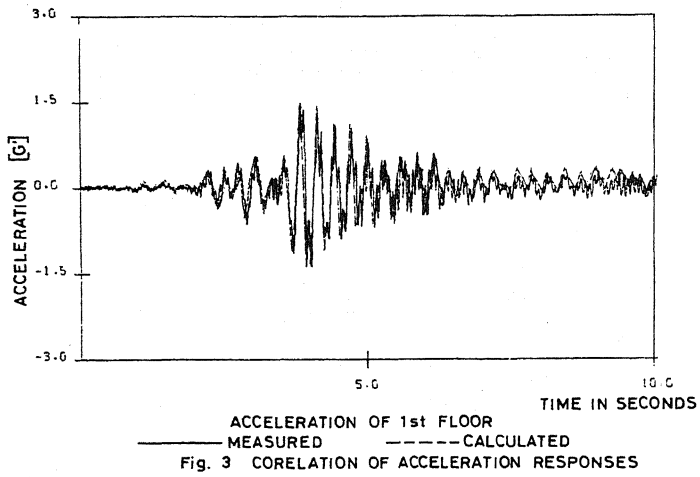


Fig. 3 CORRELATION OF ACCELERATION RESPONSES

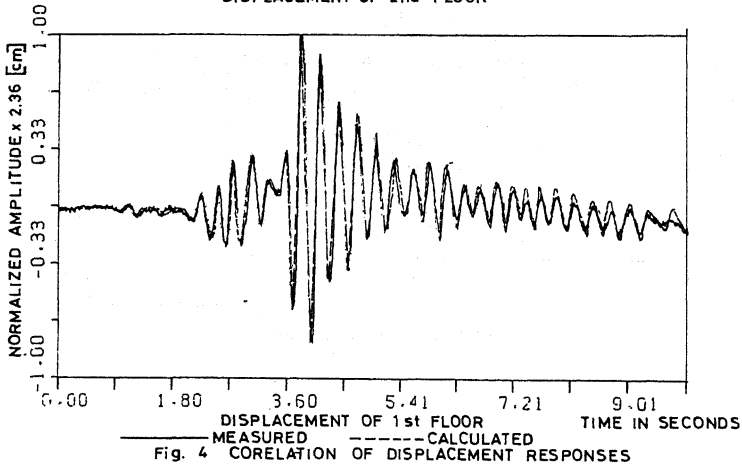
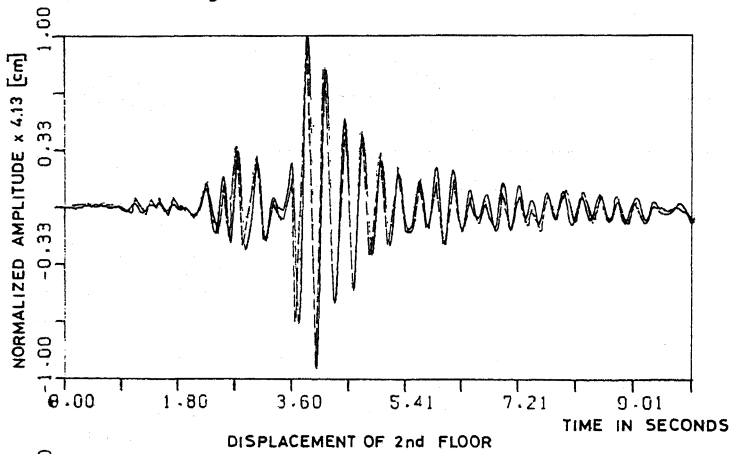


Fig. 4 CORRELATION OF DISPLACEMENT RESPONSES