

BRITTLE FAILURE CRITERIA OF REINFORCED CONCRETE  
SHORT COLUMNS CAUSED BY EARTHQUAKE LOADINGS

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SUMMARY

Based on the truss theory, the brittle failure criteria of reinforced concrete (RC) short columns are analysed. Evaluating the yield or failure condition of elemental materials at the ultimate states of truss bearing mechanisms, three equations of shear force are derived. Shear strength is given by the lowest value by the three equations, and the failure type of RC short column, if it is in brittle state or not, is judged by the corresponding equation.

Mainly the brittle properties of RC short columns are discussed for various values of the shear arm ratio, the amount of reinforcing, the magnitude of axial load and so on.

1. INTRODUCTION

For the aseismic design of reinforced concrete structures, one of the most important problems to be solved is, to prevent the columns to fail in brittle states caused by shear loadings.

In this report, brittle failures of short columns due to shear slip along diagonal cracks in core concrete are analysed and the practical design considerations for the aseismic design of RC short columns are introduced.

As the bases of this study, it is assumed that the ultimate states are reached when the shear load causes (1) shear slip along diagonal cracks in core concrete producing yield confining forces in tie reinforcing bars, (2) tensile yield in main bars and (3) compressive failure in concrete at the end portions. The equations which give the shear strengths of RC short columns are derived individually for each ultimate state, and the above mentioned brittle failure of type(1) is mainly discussed.

2. SHEAR BEARING MECHANISMS OF RC COLUMNS BY TRUSS ANALOGY

Based on the experimental studies so far<sup>1,2,3</sup>, and the analytical study of modified truss theory by Shibata et al.<sup>4</sup>, the following shear bearing mechanisms are composed.

(a) Under the perfect bond condition of main bars, whole shear load is resisted by beam mechanism. (Fig.1a)

(b) After bond failure, tensile stress in one end of a main bar is transferred to the opposit end and truss mechanism is composed. At this

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stage, bond failure shear load is resisted by beam mechanism, and the remaining shear load is resisted by truss mechanism. (Fig.1b)

(c) After shear slip in core concrete, a portion of the shear load which is resisted by truss mechanism is resisted by tie confining force. (Fig.1c)

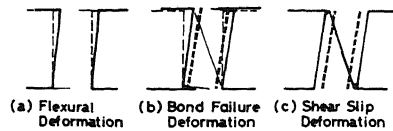


Fig. 1

The ultimate states are reached when the elemental materials failed or yielded as described in chapter 1, and it is considered that the brittle failure of columns is reached only when ties yielded at the stage "c".

### 3. SHEAR STRENGTH OF RC COLUMNS

Analysing the truss bearing mechanisms, equations for shear load under which RC columns should reach their ultimate states are derived. In the analysis, following assumptions are used.

- (1) Dowel actions in main bars are neglected.
- (2) In the confining effect of tie reinforcement, bond strength is neglected.
- (3) Deviations of equilibrium conditions caused by large deflections are neglected.
- (4) Effect of tie reinforcements are calculated by the product of the total sectional areas in the column length and its stresses, not withstanding the sorts of ties such as spiral, welded, anchored and so on.

#### 3.1 Shear Load of Tensile Yield in Tie Bar (failure in core portion).

$$Q_{uh} = Q_b + Q_i + Q_k \quad (1)$$

where

$$Q_b = f_a \psi j_i \quad (1.1)$$

$$Q_i = \frac{1}{2(a^2 + 16.18)} \left[ \left( 4.18a - \frac{18.53 + 2a^3}{a^2 + 1} \eta \right) + \sqrt{\left( 4.18a - \frac{18.53 + 2a^3}{a^2 + 1} \eta \right)^2 - 4(a^2 + 16.18)(\eta^2 - 4.18\eta - 1.39)} \right] \cdot \text{FebD} \quad (1.2)$$

$$Q_k = \frac{1}{a} \frac{a + \sqrt{0.758a^2 - 0.242}}{a^2 + 2 - a\sqrt{0.758a^2 - 0.242}} (T_y - aQ_b) \quad (1.3)$$

$$T_y = P_w \cdot f_y \cdot abD$$

### 3.2 Shear Load of Tensile Yield in Main Bar.

$$Q_{um} = \frac{2J}{D} \cdot \frac{A_t}{a} \sigma_{tb} + \frac{A_s}{a} (f_y + \sigma_{t1} - \sigma_{tb}) + \frac{N}{a} \quad (2)$$

where

$$\sigma_{tb} = \frac{f_a \psi \ell}{2A_t}, \quad \sigma_{t1} = \frac{E_t}{E_c} \cdot \frac{N}{A_e}$$

### 3.3 Shear Load of Compressive Failure in Concrete at End Portion.

$$Q_{uc} = f_a \psi \left\{ \left( 0.885 - \frac{20}{F_c - \sigma_{c1}} \right) - 0.56 \right\} d + \left\{ \left( 0.23 + \frac{40}{F_c - \sigma_{c1}} \right) - \frac{E_t}{E_c} \cdot P_t \cdot 1.11 \right\} \frac{F_c}{a} b d \quad (3)$$

where  $\sigma_{c1} = N/A_e$

\*Letter Symbols\*

- F<sub>c</sub> ; Compressive strength of concrete.
- F<sub>t</sub> ; Tensile strength of concrete.
- f<sub>y</sub> ; Yield strength of reinforcing bar.
- f<sub>a</sub> ; Bond strength of main bar.
- p<sub>w</sub> ; Tie ratio.
- P<sub>t</sub> ; Main bar ratio.
- N ; Axial load.
- A<sub>e</sub> ; Equivalent area of column section.
- A<sub>t</sub> ; Sectional area of main bar (tensile portion).
- A<sub>s</sub> ; Sectional area of main bar (total).
- E<sub>t</sub>/E<sub>c</sub> ; Ratio of elastic moduli of steel and concrete.
- b, D, d', b', d, l ; See "Fig.2".
- j ; Lever arm.
- a ; Ratio of column length to depth.
- ψ ; Perimeter of main bar in tension.

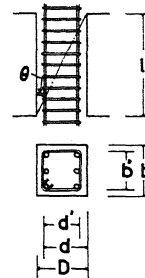


Fig. 2

### 3.4 Shear Strength and Failure Condition of RC Columns.

"Fig.3" shows the results of "Eq.1" ~ "Eq.3".

"Fig.3a" shows the relation between shear strength ( $Q_u/F_t \cdot b \cdot j$ ) and the shear arm ratio ( $l/D$ ) for one condition of main bar ratio ( $P_t=0.01$ ) and four axial loads. In this figure, it is noticeable that shorter columns are more probable to fail in brittle state, especially under large axial load conditions.

"Fig.3b" shows the relation between shear strength ( $Q_u/F_t \cdot b \cdot j$ ) and the axial load ( $N/F_t \cdot b \cdot D$ ) for two conditions of shear arm ratio and one main bar ratio. In this figure, shear strengths are increased as the axial loads are increased to a certain definite value. In this condition, the ultimate states are rather ductile, and shear strength is given by "Eq.2" or "Eq.3". In the larger axial load condition, shear strength is decreased as the axial load is increased. In this condition, the ultimate

states are rather brittle, and shear strength is given by "Eq.1".

#### 4. DISCUSSION AND CONCLUSION

It is author's opinion that the failures in the elemental materials of a composite member would only contribute to the change of bearing mechanisms, and the ultimate state of this member would be reached at the final bearing mechanisms after some steps of failure. The truss bearing mechanism of RC short columns is composed after first failure condition, i.e. the bond failure in main bars, and the ultimate states are analysed.

The verification of this analogy is not yet complete, but, as far as shear strengths and failure criteria are concerned, the former experimental studies by Higashi et al.<sup>5</sup> are not contradictory to this theory, and the lowest value by "Eq.1" ~ "Eq.3" trends to agree with the practical design equation by Arakawa.<sup>6</sup>

For aseismic design purposes, it is important to make columns as ductile as possible. The ultimate states caused by the yield or failure at the end portions should occur prior to the failures in core portion. For this purpose, "Eq.1" ~ "Eq.3" will be efficiently utilized.

#### REFERENCES

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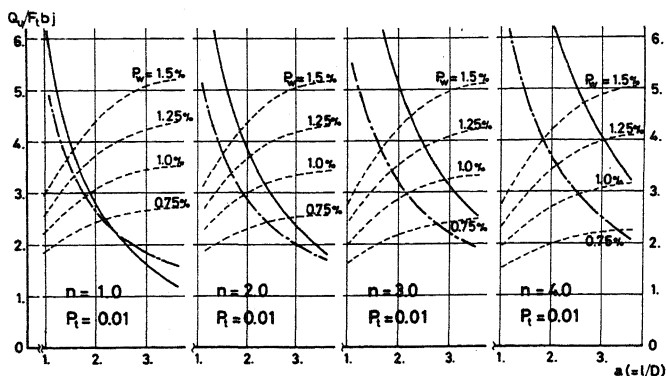


Fig.3a

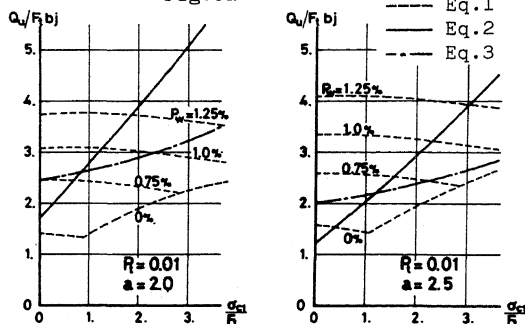


Fig.3b