THE DEFINITION OF THE INSTANTANEOUS ELASTICITY MODULUS OF CONCRETE IN THE DYNAMIC CALCULATION OF STRUCTURES

A. Capecchi[1], G. Conti[1] and A. Tafanelli[1]

Summary

The determination of the elasticity modulus of concrete is effected by quasi static testing under compression and the value thus obtained is called "static modulus". Some standards also allow for testing by stationary wave excitation on specimen to obtain a "dynamic modulus". Ultrasonic wave propagation is also used for the same purpose although it is not included in standardized methodologies.

The Authors then critically discuss the testing methods together with backing theory; they believe that the viscosity of the concrete and hence the duration of the test are the most important parameter for determination of the modulus whereas the non homogeneity and non continuity are thought to be disturbance factors. They suggest the utilization of fast compression tests or dynamic methods to achieve an univocal determination of an instantaneous modulus as well as the determination of the modulus decay curve versus time, according to design requirements.

Definition of the Modulus of Elasticity

The following relations defining Young's modulus are valid for the elastic medium:

$$E = \frac{\sigma_{i(i)}}{\varepsilon_{i(i)}} \qquad (1)$$

$$E = \rho c_0^2 \qquad (2)$$

$$E = \rho c_1^2 \frac{(1+\nu)(1-2\nu)}{(1-\nu)} \qquad (3)$$

$$E = 2\rho c_2^2 (1+\nu) \qquad (4)$$

$\rho$ = density

$c_0$ = bar velocity

$c_1$ = bulk velocity

$c_2$ = distorsional wave velocity

$\nu$ = Poisson's ratio

$G$ = Shear modulus

obtained from

$$G = \rho c_2^2 \qquad (5)$$

$$G = \frac{E}{2(1+\nu)} \qquad (6)$$

Relation (1) defines the modulus in the static range; (2) relates it to the bar

(1) Assistant Professors "Istituto di Scienza delle Costruzioni" University of Genoa (Italy)

velocity of a longitudinal wave inherent to the first vibration mode of the cylinder having an infinite length and $\emptyset/\lambda = 0$ and, with fair approximation, also of a cylinder having a finite length; (3) refers to the propagation velocity of a longitudinal wave in an unbounded medium; (4) relates it to a shear wave also in the same unbounded medium.
Since the solid is cylindrical, it should be noted that the velocity is not constant but may vary according to the dispersion curve versus the $\emptyset/\lambda$ parameter; $c_0$ is the maximum value of this curve if the excited mode is actually the first. If higher modes are excited, the propagation velocity may reach the bulk velocity value, i.e. the velocity of a longitudinal wave in an unbounded medium.
The velocity $c_1$ does therefore not depend on the wave length and on the frequency in the unbounded medium , whereas in bounded media, the velocity $c_1$ is depending on high frequency waves with values of $\emptyset/\lambda$ higher than 5, since modes to which the propagation velocity $c_1$ correspond, are also excited in these conditions. The motivations stated for $c_1$ are also holding true for $c_2$ in the unbounded medium. However, in the bounded medium, distinction must be made between torsional and flexural waves. In the former case there will be no dispersion for the first mode, so that $c_2$ will be constant, while in the latter mode, the existing dispersion curve will make it almost impossible to determine $c_2$ .
It must therefore be inferred that the excitation of bounded specimens must be performed with continuous longitudinal waves at the lowest possible frequency to determine $c_0$ or with torsional waves to determine $c_2$ and in such case the frequency will not affect the result.
From this theoretical preamble it can be inferred that Young's modulus value in an elastic medium is univocally determined by static and dynamic stresses. But in the latter case the excitation mode must be known and the $\emptyset/\lambda$ ratio must be almost 0. For an elastic medium, the definition of a static modulus differing from a dynamic modulus is therefore without meaning.
In the case of pulse propagation, a spectrum analysis shows an energy distribution over a proad frequency range. If the medium under examination is unbounded, the wave front will be single since the propagation velocity remains constant versus frequency; whereas in the case of transversally bounded specimen, a typical wave guide propagation will result in which different velocity values will correspond to each excited mode for each $\emptyset/\lambda$ ratio. For each such mode there will be velocity variation versus the $\emptyset/\lambda$ ratio. As the pulse is propagated in the medium, distortion occurs caused by the different velocities of the various components. The fastest of these, at high frequency, will form the wave front, and will travel at bulk velocity. Therefore, in case of pulse excitation of a cylinder, the value measured at first arrival will be the bulk velocity. This propagation will be treated hereinafter in a real damping material.

## Damping in concrete

When analyzing a solid, the rheological model of which is containing viscous damping elements, the complex modulus is defined

$$E(i\omega) = E_1 + i E_2$$

the imaginary part of which is always a function of damping, whereas the real part may be a function or not, according to the chosen model.
If this solid is dynamically stressed, it will be possible univocally to define a dynamic modulus coinciding with the part $E_1$ only if the latter is assumed to be conservative. In this case, the static stress/strain relationship is influenced by the viscous element and hence it should be possible to determine the purely elastic part coinciding with the conservative part, i.e. with the dynamic modulus, in case of instantaneous stress variation.
When assuming in first approximation Hansen's model (fig. 1) for concrete, it will be noticed that for almost static stress variations achieved in very short time, the typically elastic component due to the series connected conservative-elastic element must be evidenced.

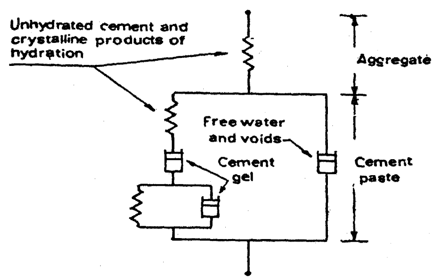


Fig. 1

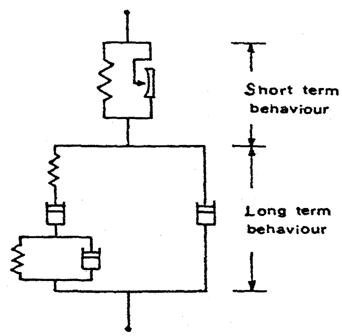


Fig. 2

In concrete, Authors think that distinction must be made between short-term and long-term behaviour due to the influence of the various damping factors in the actual rheological model. It has already been demonstrated [1] that the damping of vibrations in concrete is not due to viscosity but rather to solid-friction whereas the viscous behaviour due to long lasting phenomena is unquestionable. It may therefore be assumed that there are damping elements of different characteristics in the rheological model earmarking the above explained phenomena (fig. 2).
It should be stressed that the overall effect of the damping on the complex concrete modulus, measured at about 5 kHz, should not exceed 0,5 ‰ [2] so that there will be no difficulty to identify the conservative part of the complexe modulus with the instantaneous modulus as measured dynamically or in fast quasi-static tests.
When considering pulse propagation in a cylinder of a material showing damping some unreliability will be found in the determination of the first arrival and this may considerably alter the precision of the method.
Finally it should be pointed out that the measurement of the propagation velocity $c_2$ of shear waves by pulse methods in bounded and unbounded media involves

a further difficulty for the determination of the first-arrival due to the fact that the dilatational waves, always associated, are arriving before the shear waves; so that the latter are superimposed on existing disturbances and accurate measurements become almost impossible.

Discussion of the results

This research has the aim to measure the modulus of elasticity in loaded concrete during the very fast loading and unloading phases and to make a comparison between the value thus obtained and the value calculated with (2) in resonance tests. It will thus be possible to determine an instantaneous modulus of elasticity to be used in the calculation of dynamically stressed structures. The value obtainable by standard tests, i.e. with a duration of 10-15 minutes is not deemed suitable for the design purposes since it cannot be used for dynamic stresses and also for permanent loads. The latter will require the determination of modulus decay curve versus time as normalized in long-time creep tests for steel to be employed in prestressed concrete.
The values of the modulus that can be inferred from (3) and (4) in case of pulse propagation, were found to be anomalous since they refer to a monotonic propagation following a preferential path due to the non homogeneous nature of the material. Therefore they are unable to reflect the average behaviour of the material through which the pulses are passing.
The test findings show that in an aluminium cylinder of 100 mm Ø and 400 mm length, the compression tests performed at standard time, have led to the determination of an elasticity modulus identical to he modulus measured during the longitudinal resonance test for $\text{Ø}/\lambda = 0.125$. This corroborates the fact that the elasticity modulus statically measured in a virtually non viscous material is the same as the dynamically measured value and that the theoretical relations of LOVE are also valid for specimen of this size.
To investigate a non homogeneous but also not viscous material as an intermediate solution between metals and concrete, a test specimen of 50 mm Ø and 200 mm length was fabricated with irregularly shaped brass pieces having a maximum dimension of 6 mm embedded in a tin-lead alloy. The findings obtained with this specimen show a perfect agreement between the modulus measured under compression test and the resonance test.
Working finally on concrete specimens, compression tests were carried out during about 8 msec. in the unloading stage and about 70 msec. during the loading stage. A differentiated behaviour of the concrete was evidenced probably due to the microcracking of the specimen during testing, caused both by the quality of the concrete and by its curing conditions. This phenomenon is further raised in poor concrete or may be due to air curing that considerably reduces the failure stress and modulus of the concrete in question. If the concrete is not much cracked, especially if water-cured, the value of the instantaneous modulus will be indeed almost the same both by resonance or fast quasi static tests. Considerable cracking is less affecting the resonance tests and will therefore give higher instantaneous modulus values by about 7%.
It was thus observed that discontinuity is one of the causes of anomaly which cannot be anticipated and quantified at theoretical level. However it is deemed that this phenomenon will be negligeable if the concrete is always cured in wa-

ter.
Since the stress level in the resonance tests is always extremely limited, the outcome cannot be used for dynamic calculations because the concrete, stressed under working conditions, would be more affected by its cracking conditions.
We must therefore conclude that the value obtained during fast unloading under compression test is the most reliable value in all cases. However, since the value obtainable by resonance tests is easier to determine, the latter may be used whenever the state of microcracking is limited.
As to the results that can be inferred from ultrasonic wave propagation, the related difficulties have already been outlined. In particular, the utilization of relation (3) will give rise to uncertainties in the determination of $c_1$ due to damping and measuring errors because of the non-homogeneity of the medium and difficulties in obtaining Poisson's ratio.
The latter difficulty might be overcome by measuring at the same time $c_1$ and $c_2$ , which in the praxis is impossible because $c_2$ cannot be exactly determined and also by the fact that both values are inexact since the wave front is traveling along preferential paths.
We must therefore conclude that ultrasonic wave propagation in such an non homogeneous material will not provide reliable indications for the determination of the modulus.
As to Poisson's ratio for concrete, it should be pointed out that the difference between $c_1$ and $c_2$ will not exceed 5% because of its low value so that it will be difficult to establish whether the measured velocity is bar or bulk unlike other materials, such as lead, in which the authors were able to measure the $a_1/c_0$ ratio at 1.81 corresponding to $\nu$ = approx. 0.43.

REFERENCES

[1] POZZO E.: "Rheology of Reinforced Concrete Dynamically Stressed", Research Report n. 16, Ist. Scienza delle Costr., University, Cagliari,1974

[2] JONES R.: "Les Essais non destructifs des Betons"Editions, pag.33, Eyrolles Paris, 1967

[3] EVANS R.H.: "Effect of rate of loading on some mechanical properties of concrete". Mech. Prop. of non metallic Brittle Materials, London, Butterworth, 1958

[4] LETHERSICH W., PELZER H.: "The Measurement of the Coefficient of Internal Friction of Solid Rods by a Resonance Method". Brith. Journ. of Appl. Phys, n. 1, 1950

[5] BLAND D.R., LEE E.H.: "Calculation of the Complex Modulus of linear viscoelastic Materials from vibrating Reed Measurements". Jour. of App. Phys. 0.26 n. 12, 1955

[6] CAPECCHI A., TAFANELLI A.: "Modulo elastico istantaneo del calcestruzzo, determinazione dinamica e quasi statica", Atti VI Convegno Nazionale AIAS, Brescia, 1978, pp. 373-384.

[7] CAPECCHI A., CONTI G., TAFANELLI A.: "Fenomeni elastici nel calcestruzzo", Atti VII Convegno Nazionale AIAS, Cagliari, 1979, pp. 7.19-7.27.