APPLICATIONS OF SYSTEM IDENTIFICATION TECHNIQUES TO RECORDED EARTHQUAKE RESPONSES

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SUMMARY

Two systematic techniques are used to determine linear models of multi-storey structures from their earthquake excitation and response. The periods, dampings and participation factors of the dominant modes are estimated by least-squares matching of either the time histories or the Fourier transforms of the model and recorded responses. The optimal versions of simple time-invariant linear models are found to provide good reproductions of measured responses of buildings up to the onset of structural damage. In addition, the variation of the equivalent linear parameters due to nonlinear behaviour is studied by determining optimal linear models for short segments of the response.

INTRODUCTION

Strong-motion earthquake records provide an opportunity to evaluate the ability of simple linear models to represent the structural behaviour of tall buildings at amplitudes of motion relevant to earthquake-resistant design. It is also of considerable importance to estimate the equivalent linear parameters directly from earthquake records in view of the often large discrepancies between the modal periods used in design analyses and those measured in low-amplitude vibration tests, and because equivalent viscous damping factors cannot be synthesized satisfactorily from the properties of the structural components.

Transfer function approaches and trial-and-error parameter-adjustment methods have been found generally unsatisfactory for determining linear models of structures from records of their earthquake excitation and response. Improved accuracy has been achieved by two output-error techniques developed recently to provide optimal estimates of the parameters by achieving least-squares matches of the recorded and model responses in the time and frequency domains (1, 2, 3, 4).

THE IDENTIFICATION TECHNIQUES

Rigid-base, planar, linear models possessing classical normal modes, as commonly used in design, are considered. The planar assumption can be relaxed to allow both components of the horizontal base motion to contribute to the model response in a given direction, permitting better modelling of modes whose dominant motion is torsional.

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Beck (1) has shown by considerations of identifiability and accuracy that the appropriate parameters to estimate for linear models of structures from earthquake records are certain parameters, given below, of the dominant modes in the measured response rather than the elements of the damping and stiffness matrices. The limited number of locations within a structure for which response records are usually available and the presence of model and measurement error prevent reliable estimation of the matrix elements.

The equations of motion for the response of an R degree-of-freedom model to a base acceleration $\ddot{z}$ may be written in terms of the uncoupled modal equations. For the rth mode relative displacement at position p, x_{pr},

$$\ddot{x}_{pr} + 2\zeta_r w_r \dot{x}_{pr} + w_r^2 x_{pr} = - c_{pr}\ddot{z} \qquad (1)$$

The total response at p for the displacement relative to the base, x_p, is the sum of the modal contributions

$$x_p(t) = \sum_{r=1}^{R} x_{pr}(t) \qquad (2)$$

The parameters of the rth mode are ζ_r, the fraction of critical viscous damping, the modal frequency w_r, and the effective participation factor at position p, c_{pr}, which is independent of the normalization of the mode shape $\underline{\phi}_r$,

$$c_{pr} = \phi_{pr} \; [\underline{\phi}_r^T M \underline{1}] \; / \; [\underline{\phi}_r^T M \underline{\phi}_r] \qquad (3)$$

Here, every component of the column vector $\underline{1}$ is unity, M is the mass matrix, and ϕ_{pr} is the value of the rth mode shape at p.

The equations can be transformed to the frequency domain by taking Fourier transforms, calculated for the measured records by using an FFT algorithm which produces the complex-valued transform at N equally spaced frequencies w_n ($=2\pi n/T = n\Delta w$, $n=0,\ldots N-1$) from 2N equally spaced samples of a record of length T from t_i to t_f. Letting $A_{pT}(w)$ and $Z_T(w)$ denote the transforms of the absolute accelerations at p and the base respectively, the input-output relation becomes, for the sampled frequencies,

$$A_{pT}(w) = \left[1 + \sum_{r=1}^{R} \frac{w^2(w_r^2-w^2) - iw^3\, 2\zeta_r w_r}{(w_r^2-w^2)^2 + w^2(2\zeta_r w_r)^2} c_{pr} \right] Z_T(w)$$

$$+ \sum_{r=1}^{R} \frac{w_r^2(w_r^2-w^2) + w^2(2\zeta_r w_r)^2 - iw^3 2\zeta_r w_r}{(w_r^2-w^2)^2 + w^2(2\zeta_r w_r)^2} v_{pr}$$

$$+ \sum_{r=1}^{R} \frac{2\zeta_r w_r^3 w^2 + iww_r^2(w_r^2-w^2)}{(w_r^2-w^2)^2 + w^2(2\zeta_r w_r)^2} d_{pr} \qquad (4)$$

where $d_{pr} = x_{pr}(t_f) - x_{pr}(t_i)$, $v_{pr} = \dot{x}_{pr}(t_f) - \dot{x}_{pr}(t_i)$.

Beck's method (1) performs a minimization of the time-domain measure-of-fit J_1 with respect to the parameters of R modes (w_r, ζ_r, c_{pr}, $x_{pr}(t_i)$, $\dot{x}_{pr}(t_i)$, $r = 1, \ldots, R$), where

$$J_1 = V_1 \int_{t_i}^{t_f} (x_o - x_p)^2 dt + V_2 \int_{t_i}^{t_f} (v_o - \dot{x}_p)^2 dt + V_3 \int_{t_i}^{t_f} (a_o - \ddot{x}_p)^2 dt \qquad (5)$$

The normalising factors V_i are zero if the corresponding quantities are not used in the identification. Also, x_o, v_o and a_o are the relative displacement, velocity and acceleration histories derived from the acceleration records in the base and at position p in the structure, and x_p, $\dot{x}_p$ and $\ddot{x}_p$ are the corresponding quantities calculated for the model, using as input the measured base acceleration $\ddot{z}$ (Eq.1 and 2).

McVerry (2) achieves a frequency-domain least-squares fit of the Fourier transform of the acceleration response over a selected frequency band, minimizing J_2 with respect to the modal parameters (w_r, ζ_r, c_{pr}, d_{pr}, v_{pr}, $r=1,\ldots,R$), where

$$J_2 = \sum_{k=k_{min}}^{k_{max}} | A(k\Delta w) - A_{pT}(k\Delta w)|^2 \Big/ \sum_{k=k_{min}}^{k_{max}} |A(k\Delta w)|^2 \qquad (6)$$

Here, $A(k\Delta w)$ is the finite Fourier transform of the recorded acceleration at p, while A_{pT} is the corresponding model response from Eq. 4.

With both methods, it is possible either to determine a single linear model appropriate for the entire response, or, by considering a series of models for short segments of the records, to study the changes of the effective linear parameters due to nonlinear behaviour.

APPLICATIONS TO RESPONSE RECORDS

The identification studies have so far considered 10 structures of various types over a range of responses classified into 4 categories (Table 1). Class A response was of small amplitude (less than 0.1g), with the effective fundamental periods close to those determined in vibration tests. Class B, with peak responses around 0.2g, was typical of that of many of the instrumented buildings which suffered no damage during the 1971 San Fernando earthquake, California. The measured response was reproduced very well by time-invariant linear models, but with effective periods typically 50% larger than in vibration tests. Nonlinear behaviour in class C response was reflected in large changes in the effective linear parameters during the motion, with time-invariant models successfully reproducing the strongest portion of the response. The final category, class D, was for those buildings which suffered minor structural damage and represents the limit of applicability of linear models to seismic response.

Records from the nine-storey, steel-framed JPL Building 180 illustrate the dependence of the effective linear models on the amplitude of excitation. Estimates of the modal parameters identified from the longitudinal (S82E) roof and basement records of the 1968 Borrego Mountain (peak response 3.1%g), 1970 Lytle Creek (3.7%g) and 1971 San Fernando (37%g) earthquakes are given in Table 2.

The matches of the Borrego Mountain records were excellent, as were the Lytle Creek matches apart from high frequency content in the measured response caused by the contribution of more modes than were included in the models. The modal periods in even these low-amplitude earthquake responses were 10-30% longer than those measured in man-excited tests on the completion of the structure. However, the periods identified from the records of the Borrego Mountain earthquake were very close to those of Wood's full-composite model (5), synthesized using the assumption that there was no cracking of the concrete encasing the steel columns. The identified fundamental mode dampings of 3-5% were higher by factors of five to eight than the value of 0.6% in both directions measured in forced vibration tests.

For the San Fernando data, the fundamental periods in the longitudinal and transverse directions lengthened by about 40-60% compared to the vibration test values. The second mode periods were lengthened by more than 30%. The periods were similar to those calculated from synthesized models which assumed cracking of the concrete in the tension zones of the columns (5). The match of the longitudinal roof acceleration by a two-mode model estimated from the velocity record is shown in Fig.1. Although a time-invariant linear model with only two modes for each direction gave satisfactory reproduction of the roof response, identification using short segments of the San Fernando data revealed significant variations with time of the effective linear parameters (Fig.2). Some of the irregular temporal variation of the damping and participation factors shown in Fig. 2 is likely to be due to the interaction observed during their estimation, as discussed below. It was also found that the probable cause of a double peak in the Fourier amplitude spectrum of the roof response was a temporal variation of the period of the second mode rather than torsional response.

Several difficulties were encountered in the identification of linear models from the longitudinal San Fernando response. Interaction occurred between the estimates of the damping and participation factors because of the greater sensitivity of the modal response to their ratio than to their individual values. Also, surprisingly, interaction occurred between the estimates of the periods and participation factors. The parameter estimates also depended on which response quantity (acceleration, velocity or displacement) was matched, which was attributed to the nonlinear behaviour of the structure. The estimates of the parameters of the third mode, which contributed little to the roof response, were questionable, apparently being strongly influenced by noise and model error.

The 16-storey, steel-framed KB Valley Centre is an example where systematic identification successfully resolved difficulties which were encountered in an earlier trial-and-error study. A high value of 20% obtained by Gates (6) for the damping of the fundamental transverse mode was probably caused by difficulties in matching the response using an inaccurate period estimate of 3.0 seconds. The systematic frequency-domain technique produced a period of 3.3 seconds and dampings of 8.6% and 8.8% for the fundamental mode from the roof and 9th floor records.

The records of three buildings which suffered appreciable, although repairable, structural damage during the San Fernando earthquake were studied using the frequency-domain technique. Time-invariant linear models were unable to reproduce the entire response history for this type of structural behaviour, although segment-by-segment analysis produced some good matches and showed that the response was characterized by great period lengthening and high damping (Table 1). For example, for the longitudinal (N11E) component of the roof response of the Bank of California building, a 12-storey reinforced-concrete structure, considerably different parameter values were required for the two segments of the record for which the matches of the velocity histories are shown in Fig.3.

CONCLUSIONS

The systematic identification studies have shown that optimal time-invariant linear models with only a few modes provide good characterizations of the responses of tall buildings up to the onset of structural damage. Improved matches of the recorded responses and much more reliable estimates of the modal parameters are obtained compared with those given by trial-and-error or transfer-function methods.

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TABLE 1: SUMMARY OF IDENTIFICATION STUDIES

Structure and Component		Stories	Construction	Miles From Pacoima Dam*	Max. Acc. (%g) Ground	Response	Fundamental Period (Sec) Pre-Eq.	Post-Eq.	Eq.	Eq/Pre-Eq.	1st Mode Damping (%) Eq.	Tests	Response Type
JPL 180 - Borrego Mountain	S82E	9	Steel Frame	15	0.7	3.1	0.91		1.09	1.2	2.9		A
	S08W				0.7	2.3	0.88		1.15	1.3	2.7		
- Lytle Creek	S82E				1.5	2.5	0.91		1.02	1.12	4.7		A
	S08W				2.4	3.7	0.88		1.13	1.3	3.5		
- San Fernando	S82E				21	37	0.91	1.01	1.26	1.4	3.8		C
	S08W				14	21	0.88	1.16	1.42	1.6	6.4		
Millikan - Lytle Creek	NS	9	R.C. Shearwall	19	1.9	5.4	0.53		0.52	0.98	2.9	1-2	A
	EW				1.9	3.5	0.69		0.71	1.03	2.2	0.7-1.7	
- San Fernando	NS				20	31	0.53	0.54	0.62	1.17	6.4		C
	EW				18	34	0.69	0.79	0.98	1.4	7.0		
1900 Avenue of Stars	N44E	27	Steel Frame	20	8	14	3.3	3.6	4.37	1.3	4.4		A-B
	S46E				8	11	3.3	3.6	4.24	1.3	2.2		
Union Bank	S38W	39	Steel Frame	21	12	12	3.11	3.7	4.63	1.5	4.9	1.5	B-C
	N52W				15	20	3.53	4.1	4.71	1.3	4.1	1.7'	
KB Valley Center	S09W	18	Steel Frame	14	13	22	2.18	2.37	3.30	1.5	8.6		B
	S81E				14	22	1.94	2.27	3.05	1.6	6.3		
Kajima	N36E	15	Steel Frame	21	9	19	1.32	2.10	2.84	2.2	3.8		B
	N54W				12	17	1.88	2.15	2.77	1.5	3.6		
Sheraton-Universal	NS	19	R.C. Frame	15	16	11	1.22	1.4	1.98	1.6	7.3		B
	EW				15	18	1.26	1.5	2.24	1.8	6.2		
Holiday Inn - Marengo	S52W	7	R.C. Frame	22	13	41	0.49	0.63	1.17	2.4	5.0(?)		D
	N38W				12	23	0.53	0.64	1.06	2.0	17.8		
Holiday Inn - Orion	NS	7	R.C. Frame	8	25	38	0.48	0.68	1.42	3.0	19.2		D
	EW				13	31	0.52	0.72	1.20	2.3	17.3		
Bank of California	N11E	12	R.C. Frame	14	22	28	(0.85)	1.70	2.35	2.8	12.1		D
	N79W				15	24	(1.33)	1.60	3.01	2.3	10.0		

*Taken as the center of energy release in the San Fernando earthquake.

Response types:
- A - Small amplitude, excellent matches, close to vibration test periods.
- B - Moderate amplitude, good to excellent matches, periods changed from vibration tests.
- C - Variation of linear parameters with time, strongest response matched by time-invariant models
- D - Minor structural damage, limit of linear models.

(a) Lytle Creek (0-20.48s)		
Mode	1	2
Period (sec)	1.02	0.32
Damping (%)	4.7	5.4
Participation Factor	1.33	-0.38
(b) Borrego Mountain (0-20.48s)		
Mode	1	2
Period (sec)	1.09	0.36
Damping (%)	2.9	5.1
Participation Factor	1.25	-0.38

(c) San Fernando (0-40.96s)		
1	2	3
1.26	0.38	0.30
3.8	5.3	12
1.44	-0.24	-0.41
(d) San Fernando (0-20.0s)		
1	2	3
1.25	0.38	0.30
4.2	5.3	12
1.49	-0.24	-0.40

Table 2. Parameters estimated for JPL Building 180 from longitudinal (S82E) earthquake records by matching the roof acceleration in the frequency domain (a, b, c) and time domain (d).

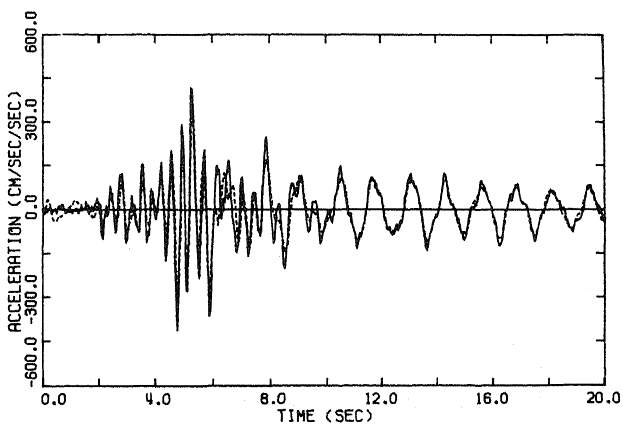

Figure 1. Relative acceleration (——) at the roof of JPL Building 180 and calculated acceleration (---) of two-mode model determined by matching velocity history, S82E component, San Fernando earthquake.

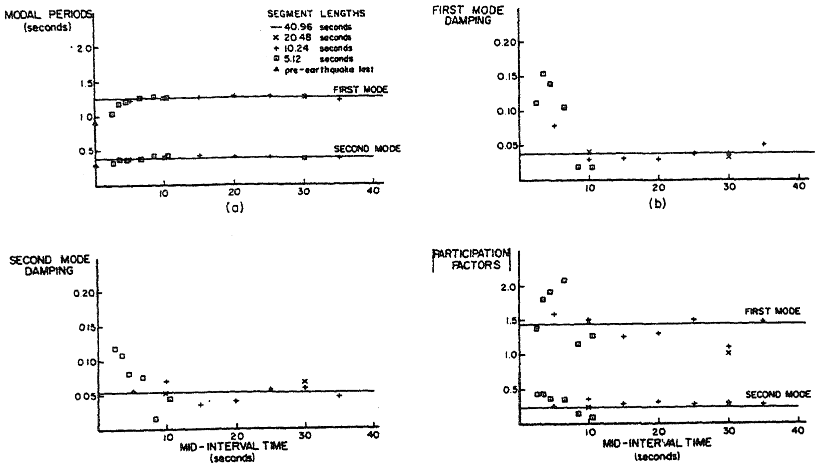

Figure 2. Time variation of modal parameters for JPL Building 180 identified from segments of the San Fernando S82E records.

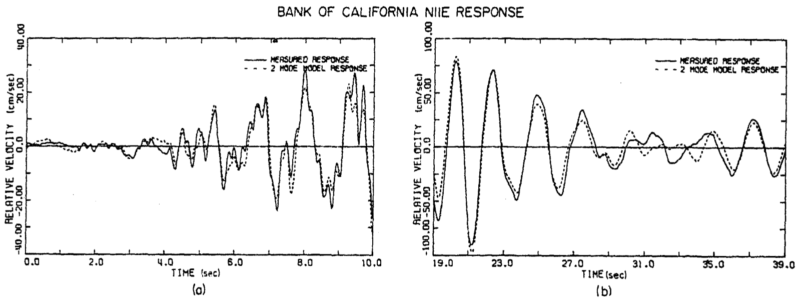

Figure 3. Velocity matches achieved by optimal two-mode models identified in the frequency-domain from the Bank of California roof response, longitudinal (N11E) component, San Fernando earthquake.

(a) 0-10 second segment:

$T_1 = 1.47s$	$\zeta_1 = 8.0\%$	$c_1 = 1.5$
$T_2 = 0.47s$	$\zeta_2 = 4.9\%$	$c_2 = -0.5$

(b) 19-39 second segment:

$T_1 = 2.35s$	$\zeta_1 = 12\%$	$c_1 = 1.5$
$T_2 = 0.82s$	$\zeta_2 = 48\%(?)$	$c_2 = -0.8(?)$