DECAY IN SEISMIC VIBRATIONS OF ACTUAL TALL BUILDINGS
(The Case of the San Fernando Earthquake of 1971)

By
Kiyoshi Kanai(I) and Katsuyoshi Yamabe(II)

SYNOPSIS

This paper discusses the multiple reflection problem of seismic waves in buildings. The results of the present investigation tell us that the nature of decay in seismic vibrations of actual buildings is caused mostly by seismic energy dissipating into the ground.

In the present investigation, the multiple reflection phenomena of seismic waves in buildings is dealt with more quantitatively by means of various aspects of the strong motion accelerograms obtained from 27 reinforced concrete (RC) buildings and 13 steel frame (S) buildings during the San Fernando Earthquake of February 9, 1971[1].

The theoretical results of the multiple reflection phenomena of waves in an elastic body adopted here is as follows[2]:

$$\frac{1}{2}\{ u_{z=H}(\tau + \frac{H}{V}) + u_{z=H}(\tau - \frac{H}{V}) \} = u_{z=0}(t) \quad \text{-------}(1)$$

in which, z=H and z=0 express the top and the bottom of the body, τ and t represent the time of the wave forms at z=H and z=0, respectively, and V is the transmission velocity of waves in the body. The values of H/V are estimated to be one fourth the peak periods of Fourier's spectra from the accelerograms obtained at the top floor of the buildings.

As an example, one case of comparisons between Fourier's spectra of the accelerograms recorded at the bottom floor of the buildings and the theoretical results calculated by Eq. (1) is shown in Fig.1. It will be seen in Fig.1 that, the agreement of the recorded values and the theoretical ones in the buildings is well beyond expectations.

In Fig.2, the ordinate represents the ratio of number of coincidences of the period corresponding to each peak (circles in Fig.1) of Fourier's spectra of strong motion accelerograms to that of the calculated seismic motion against the total number of the peaks at the bottom floor of the buildings and the abscissa represents the predominant periods of the buildings during the earthquake.

Fig.2 tells us that the smaller the height of the tall buildings the better the appearance of the multiple reflection phenomena of waves in them.

The relation between maximum acceleration and the number of floors concerning RC buildings of 14 stories or higher is shown in Fig.3.

The distribution of the ratio of maximum acceleration at the top floor ($\alpha_{T,max}$) and the middle floor ($\alpha_{M,max}$) to that at the bottom floor ($\alpha_{B,max}$) concerning RC buildings of 14 stories or higher is shown in Fig.4.

In Fig.5, the abscissa represents the ratio of the duration between the time of the initial wave and that of the wave of maximum acceleration in the accelerograms obtained at the top floor of the buildings to the predominant periods of the buildings during the earthquake and the ordinate represents maximum acceleration obtained at the top floor of the buildings. As the values of the abscissa in Fig.5 may be considered as the function of the number of multiple reflections of the waves in the buildings, Fig.5 tells

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(I) Consultant, College of Industrial Technology, Nihon University, Japan
(II) Lecturer, ditto

us that maximum acceleration of earthquake motions in a building is influenced considerably by the multiple reflection phenomena of waves in it.

Consequently, it should be borne in mind that, as a matter of fact, the vibration problem of a building should be treated as a multiple reflection phenomena of waves in that building, and the building alone cannot be regarded as a conservative system, but the vibration in that building should be considered as taking place in a dissipative system.

It is an equally important fact that, as neither solid viscosity nor other features such as attenuation in the building has been assumed from the start of mathematical treatment, the decaying character of oscillation in the present problem depends entirely upon the fact that partial emission of the waves into the ground during their multiple reflection gradually diminishes their energy.

Finally, we shall try to get an empirical formula for estimating the damping coefficient, h, in designing earthquake-proof buildings by the trial and error method.

As a first step, we deal with the comparison between the damping coefficient of the theoretical results based on the idea of the multiple reflection phenomena of waves in a surface stratum and that of the observational results by means of the autocorrelation function concerning the strong motion accelerograms obtained at the top floor of the buildings.

The theoretical formula$^{3)}$ of the damping coefficient, h, adopted here is expressed as follows:

$$h = \frac{1}{\pi}\log_e\left|\frac{1+\alpha}{1-\alpha}\right| , \quad \alpha = \frac{\rho_1 V_1}{\rho_2 V_2} \qquad \text{-------(2),(2')}$$

where ρ_1, ρ_2 and V_1, V_2 are densities in t/m^3 and velocities of S-waves in m/sec of the surface stratum and the semi-infinite lower medium, respectively. If we assume ρ_1=W/HA, V_1=4H/T, Eq. (2) may be rewritten as follows:

$$h_1 = C_1\log_e\left|\frac{1+\alpha'}{1-\alpha'}\right| , \quad \alpha' = \frac{4W}{\rho_2 V_2 TA} \qquad \text{-------(3),(3')}$$

in which H, W and A are the height in m, the total weight in tons and the sectional area in m^2 of the buildings, respectively, and T is the predominant period in sec of the buildings during the earthquake and C_1 is some kind of constant.

Using Eq. (3), the values of C_1 are calculated for each building in which the strong motion accelerograms were obtained at the time of the San Fernando Earthquake of 1971.

The average value of C_1 becomes 0.41. That is, an empirical formula of the damping coefficient of buildings obtained here becomes as follows:

$$h_1 = 0.41\times\log_e\left|\frac{1+\alpha'}{1-\alpha'}\right| , \quad \alpha' = \frac{4W}{\rho_2 V_2 TA} \qquad \text{------(4),(4')}$$

The relation between the values calculated by Eq. (4) and the damping coefficient values recorded in the buildings is shown in Fig.6.

Next, referring to the second term of the right hand side of Eq. (3), nine types of empirical formulae$^{4)}$ will be assumed as the idea of the trial and error method. The empirical formulae assumed here are as follows:

$$\frac{W}{\rho_2 V_2 TA} , \quad \frac{W}{\rho_2 V_2 TH\sqrt{A}} , \quad \frac{W}{\rho_2 V_2^2 T^2 H} , \quad \frac{W}{\rho_2 V_2^2 T^2 \sqrt{A}} , \quad \frac{W}{\rho_2 V_2^2 T^2 H^2} ,$$

$$\frac{W}{\rho_2 V_2^2 T^2 H\sqrt{A}} , \quad \frac{WH}{\rho_2 V_2 TA^{3/2}} , \quad \frac{W}{\rho_2 V_2 TAB} , \quad \frac{W}{\rho_2 V_2 TA^{3/2}} ,$$

where B is the length in m parallel to the vibration direction of the buildings.

Using nine types of Eqs., the same kind of relation represented in Fig.6 is obtained, and also, the results of them tell us that the empirical formula of (4) is most suitable for the present problem.

On the other hand, the predominant period in sec of the buildings during earthquake is almost equal to the natural period in sec estimated in designing the buildings against medium earthquake (70~110 gal).

CONCLUSION

Generally speaking, the conclusion of the present paper consists of two parts.

(1) The results of the various aspects of the studies on strong motion accelerograms obtained in the buildings during the San Fernando Earthquake tell us that the decaying character of oscillation of buildings during an earthquake depends entirely upon the fact that partial emission of waves into the ground during their multiple reflection gradually diminishes their energy.

(2) An empirical formula to estimate the damping coefficient, h, in designing earthquake-proof buildings is obtained.

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FIGURE CAPTIONS

Fig.1. Tishman Plaza, N90E, T=1.24sec. Comparison between Fourier's spectra of accelerograms recorded at bottom floor of the buildings and theoretical results calculated by Eq. (1).

Fig.2. Abscissa; Predominant periods of the buildings during the earthquake: Ordinate; Ratio of number of coincidences of periods corresponding to each peak (circles in Fig.1) of Fourier's spectra of strong motion accelerograms to that of calculated seismic motion against total number of peaks at bottom floor of the buildings.

Fig.3. R.C. buildings of $14 \geq$ stories. Abscissa; Maximum acceleration: Ordinate; Number of floor.

Fig.4. R.C. buildings of $14 \geq$ stories. Ratio of maximum acceleration at top floor ($\alpha_{T,max}$) and middle floor ($\alpha_{M,max}$) to that at bottom floor ($\alpha_{B,max}$).

Fig.5. Abscissa; Ratio of duration between time of initial wave and that of wave of maximum acceleration in accelerograms obtained at top floor of the buildings to predominant periods of the buildings during the earthquake: Ordinate; Maximum acceleration obtained at top floor of the buildings.

Fig.6. Abscissa; Values calculated by Eq. (4): Ordinate; Recorded values of damping coefficient of the buildings.

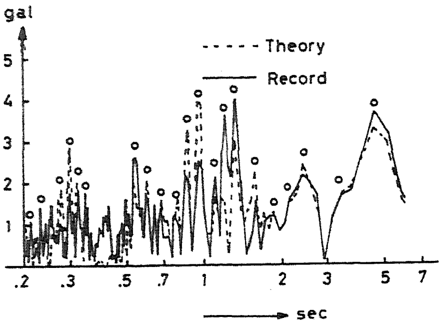

Fig. 1.

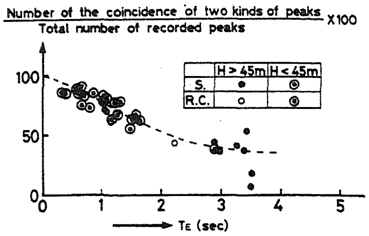

Fig. 2.

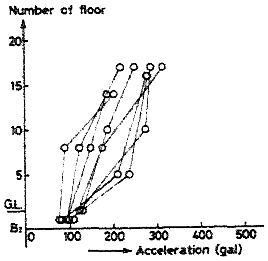

Fig. 3.

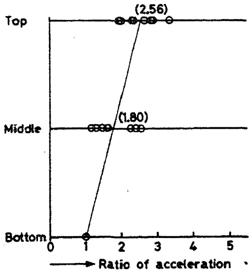

Fig. 4.

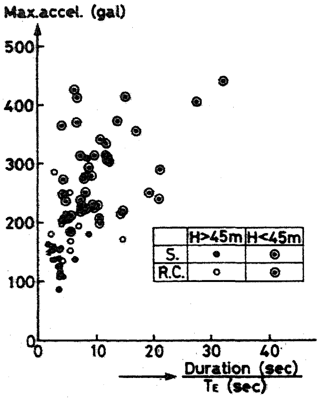

Fig. 5.

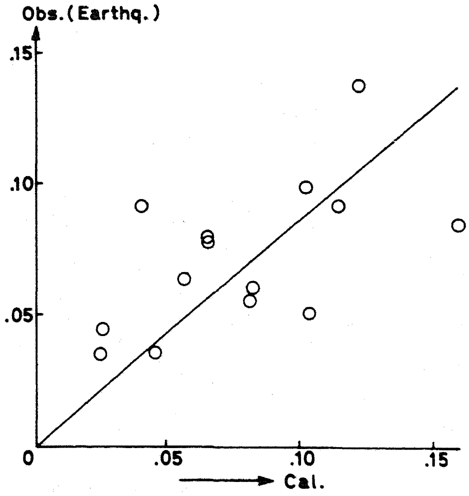

Fig. 6.