A PRELIMINARY STUDY ON A MULTI-VARIATE SYSTEM IDENTIFICATION TECHNIQUE FOR BUILDING SEISMIC RECORDS

by

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ABSTRACT

A general multiple-input multi-variate system identification technique is used for building seismic records. The minimization of error operates in frequency domain and as many modes as necessary can be treated. As an example, two orthogonal horizontal basement motions of a building are used as system input and the changing behavior of the building is studied through a time-window analysis. Two roof top records are used as separate output motions. Use of two horizontal motions can better explain the motion at top. But the results are not conclusive because the building studied is symmetrical and the orthogonal input effect is least expected. The approach presented would be more useful for unsymmetrical building layout.

INTRODUCTION

Building seismic records are usually obtained at basement level, top level, and, sometimes, mid-height of a building. Often the top level records are examined and building behavior during earthquake is inferred accordingly(1,2). Such an approach implies that the seismic inputs at the basement level are white noises or nearly so. Therefore the top level records contain essentially only the building characteristics, such as natural frequencies and damping ratios. The changing nature of a building during an earthquake can also be studied with this approach by examining successive segments, or time windows, of the top level records (2). A more elaborate approach is to take two records, one at the top and one at the basement, and to study the building behavior with the two records as an input-output pair (3-9). Thus, the white noise assumption is no longer needed. The changing behavior can also be studied with the time window technique(6,7). Alternatively, continuous nonlinear models can be used(8,9).

In this study, top level records are considered again as building system output, but at least two records at the basement level are used as input. These records are available because even a single strong motion instrument produces three mutually perpendicular records. The output record results mainly from the input record of the same direction for a building with a symmetrical layout. However, as complete symmetry is only an ideal condition, the top level modal motion of a building is

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often not a simple one-directional translation or torsion. Thus, conversely, the basement record in an orthogonal direction should also play a part in the making of the top level record. The purpose of this study is to examine this orthogonal input effect with the help of a multi-variate system identification technique.

BUILDING SYSTEM MODELLING AND IDENTIFICATION

Assuming a linear elastic behavior for the building under consideration, the modal equation of motion of the building under the action of basement acceleration $\ddot{V}_{gx}(t)$ and $\ddot{V}_{gy}(t)$ can be written as

$$\ddot{U}_m+2\xi_m\omega_m\dot{U}_m+\omega_m^2U_m=-\sum_{ij}(\phi_{mi}M_{ij}I_{xj})\ddot{V}_{gx}(t)-\sum_{ij}(\phi_{mi}M_{ij}I_{yj})\ddot{V}_{gy}(t)$$

$$m=1,2,\cdots,n \qquad (1)$$

where U_m is the m-th generalized relative displacement in a n-DOF system, ξ_m and ω_m are damping ratio and vibrational frequency, ϕ is the mode shape normalized with respect to the mass matrix M, and I is a vector containing only unity and zero with unity terms corresponding to displacements in x or y direction as denoted in its subscription. Denoting the x and y direction recorded top level accelerations by $\ddot{V}_x(t)$ and $\ddot{V}_y(t)$, respectively, the following equation is obtained

$$\ddot{V}_x(t)=\sum_m(\phi_{mx}\ddot{U}_m)+\ddot{V}_{gx}(t)$$

$$\ddot{V}_y(t)=\sum_m(\phi_{my}\ddot{U}_m)+\ddot{V}_{gy}(t) \qquad (2)$$

where, without loss of generality, ϕ_{mx} and ϕ_{my} denote the m-th mode contribution at the top level to V_x and V_y, respectively.

It is clear from Eq.1, that the solution for U_m, or $\ddot{U}_m$, can be written as Duhamel integrals in $\ddot{V}_{gx}(t)$ and $\ddot{V}_{gy}(t)$, plus terms containing initial conditions of U_m and $\dot{U}_m$. Writing every term in the frequency domain and using the same notation for accelerations, we obtain

$$V_x(f)=-\sum_m(\phi_{mx}H_m(f)\Sigma(\phi_{mi}M_{ij}I_{xj})V_{gx}(f)+\phi_{mx}H_m(f)\sum_{ij}(\phi_{mi}M_{ij}I_{yj})$$

$$V_{gy}(f))+\sum_m(\phi_{mx}A_mC_m(f)+\phi_{mx}B_mS_m(f))+V_{gx}(f) \qquad (3)$$

with a similar equation for $V_y(f)$. In the above equation, H_m is the acceleration frequency response function of the m-th modal equation, A_m and B_m contain initial conditions, modal frequency and damping, and C_m and S_m are known functions of modal frequency and damping. From Eq.3, we can lump the coefficients together to get

$$V_x(f)=\sum_m(\alpha_{mxx}H_m(f)V_{gx}(f)+\alpha_{mxy}H_m(f)V_{gy}(f)+\beta_{mx}C_m(f)+\gamma_{mx}S_m(f))$$
$$+V_{gx}(f) \quad (4)$$

with a similar equation for $V_y(f)$, containing α_{myx}, α_{myy}, β_{my} and γ_{my}.

In these equations, only two translational motions at the basement level are taken as input. It is clear however, that other motions, such as rocking or torsion, can also be considered. The inclusion of initial condition terms also permit the application of these equations to a time-window type analysis of earthquake records. Therefore, a nonlinear system can be studied as an equivalent linear system with changing parameters.

For each mode considered in Eq.4, a total of six unknown parameters are to be determined. They are the modal frequency ω, damping ratio ξ and the four parameters of α, β, and γ. The α's are associated with the so call participation factors and the β and γ are related to modal initial displacements and initial velocity. Strictly speaking, the modal initial conditions are not independent parameters. They should be superposed to form the actual recorded initial displacement and velocity, which are known quantities, obtainable by integrating the corrected acceleration record. In other words, these modal initial conditions are constrained to the actual initial conditions. The reason that these conditions are to be determined for each time segment separately, instead of taken from the modal end conditions of the last time segment is that each segment is only approximately modelled by an equivalent linear system and use of the calculated end conditions would lead to accumulated errors in the following segments. In the present study the initial conditions are taken to be independent parameters so that the use of a more sophisticated and less efficient constrained minimization scheme is avoided. The calculated modal initial conditions are superposed afterwards, however, to check with the actual initial values.

Identification of these parameters are accomplished by minimizing the error between the measure $V_x(f)$ and calculated $V_x(f)$ with $V_{gx}(f)$ and $V_{gy}(f)$ as known input. Since $V_x^x(f)$ is complex-valued, the error is defined as the integration of the square of the absolute value of the difference over the frequency range of the record. A gradient method(10) is used to search for the minimum with the gradient vector estimated by a simple finite difference formula.

AN EXAMPLE AND DISCUSSION

A 12- story steel-frame bank building in Taipei is chosen for this study. The layout of the building as shown in Fig.1, is almost symmetric. On September 2, 1978, strong motion instruments installed near the geometric center at the roof top and basement were triggered by an earthquake of magnitude 6.6, located about 80Km away. Portion of the horizontal components of the motion recorded are shown in Fig.2. The Fourier amplitude spectra of the roof top acceleration for different time segments are shown in Fig.3. The shifting frequencies indicate changing building behavior and, therefore, a time window approach is warranted. In the following, three slightly overlapping 5-second segments are investigated.

For the first segment, three modes are considered. Since the initial conditions can be assumed to be zero, a total of twelve parameters are determined. For the next two segments, the third mode is not included because its contribution is negligible. The number of unknown parameters is also twelve in these cases. Only one single output at the roof top is considered at a time, but two horizontal motions at the basement are used as input. The error function can be easily modified to consider multiple output at once. For the present study, however, separate evaluation is used.

The parameters obtained are listed in Table 1. The first mode is mainly a translational mode in y-direction and the second mode is mainly translational in x-direction. However, they do have components in the other direction, otherwise there should be no orthogonal effects. The third one is a torsional mode. Previous ambient(11) and man-excited(12) studies resulted in different values for frequency as shown in Table 2. The changing nature is more apparent if the frequencies and damping ratios are plotted against a background of changing magnitude of relative displacement at roof top(Fig. 4). It is unfortunate that separate evaluation of the x and y -direction records does not yield identical values for the modal parameters. The discrepency is smaller in frequency than in damping.

The relative importance of the two orthogonal basement motions can be visualized by plotting the ratios $\alpha_{mxy}/\alpha_{mxx}$ and $\alpha_{myx}/\alpha_{myy}$ as in Fig.5. The erratic change of sign and value is yet to be explained, but the fact that the contribution from the other direction is not to be neglected should be noted. If, indeed, only a single input is considered, the same system identification procedure would yield the results listed in Table 3, in which the corresponding frequency and damping values of Table 1 are also listed for convenience in comparison. As far as parameter identification is concerned, the difference seems to be small if only the V_y results for the first mode and the V_x results for the second mode are considered.

To see how well the roof top measured accelerations are represented by the two orthogonal horizontal basement motions, the measured and calculated amplitude spectra for V_y are shown in Fig.6. Also shown are the time domain results. While good agreement is observed, it is suspected that other input motions such as rocking and torsion may be needed to fully explain the motion at top.

In summary, a general multiple-input multi-variate system identification technique is used for the study of building seismic records. An example is given for a symmetric building, where orthogonal input effect is least expected. The orthogonal effect is noticeable, but its significance unclear. For unsymmetrical buildings, more meaningful results may be obtained. Also, use of only two horizontal motions at the basement level may not be sufficient in predicting the behavior of a building during earthquake.

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Table 1. Parameter determined using V_x and V_y

V_x		0-5 sec	4-9 sec	8-13 sec	V_y		0-5 sec	4-9 sec	8-13 sec
first mode	f	-	0.765	-	first mode	f	0.692	0.698	0.720
	ξ	-	2.750	-		ξ	6.650	7.450	10.800
	α_{xx}	-	0.394	-		α_{yy}	1.522	1.240	2.574
	α_{xy}	-	- 0.088	-		α_{yx}	- 0.347	- 0.368	- 0.059
	y_0	-	- 0.749	-		y_0	0.374	0.662	0.028
	$\dot{y}_0$	-	- 0.499	-		$\dot{y}_0$	1.987	- 1.296	- 0.806
second mode	f	0.831	0.821	0.781	second mode	f	0.790	0.836	0.898
	ξ	4.500	9.200	7.200		ξ	3.900	8.400	8.750
	α_{xx}	1.622	1.442	- 0.286		α_{yy}	- 0.244	0.293	- 0.286
	α_{xy}	0.603	0.167	- 0.530		α_{yx}	0.053	- 0.319	- 0.482
	y_0	- 0.166	4.051	- 1.276		y_0	0.144	- 0.893	- 0.115
	$\dot{y}_0$	- 1.498	- 9.185	-12.931		$\dot{y}_0$	- 0.777	0.518	0.259
third mode	f	-	-	-	third mode	f	2.157	-	-
	ξ	-	-	-		ξ	4.750	-	-
	α_{xx}	-	-	-		α_{yy}	- 0.713	-	-
	α_{xy}	-	-	-		α_{yx}	0.000	-	-

y_0: Initial displacement(cm) $\dot{y}_0$: Initial velocity(cm/sec)

Table 2. Frequency previously found(Hz)

Mode	Ambient	Man Excited
1	0.91	0.89
2	1.01	0.99
3	1.17	1.16

Table 3. Comparison of frequency(Hz) and Damping ratio(%)

V_x			0-5 sec	4-9 sec	8-13 sec	V_y			0-5 sec	4-9 sec	8-13 sec
first mode	f_1	Si.*	-	0.752	-	first mode	f_1	Si.	0.653	0.702	0.720
		Tw.**	-	0.765	-			Tw.	0.692	0.698	0.720
	ξ_1	Si.	-	3.230	-		ξ_1	Si.	6.450	4.980	9.350
		Tw.	-	2.750	-			Tw.	6.650	7.450	10.800
second mode	f_2	Si.	0.816	0.840	0.780	second mode	f_2	Si.	0.810	0.806	0.850
		Tw.	0.831	0.821	0.781			Tw.	0.790	0.836	0.898
	ξ_2	Si.	4.250	8.350	6.950		ξ_2	Si.	4.800	8.050	8.750
		Tw.	4.500	9.200	7.200			Tw.	3.900	8.400	8.750

* Single-Input ** Two-Input

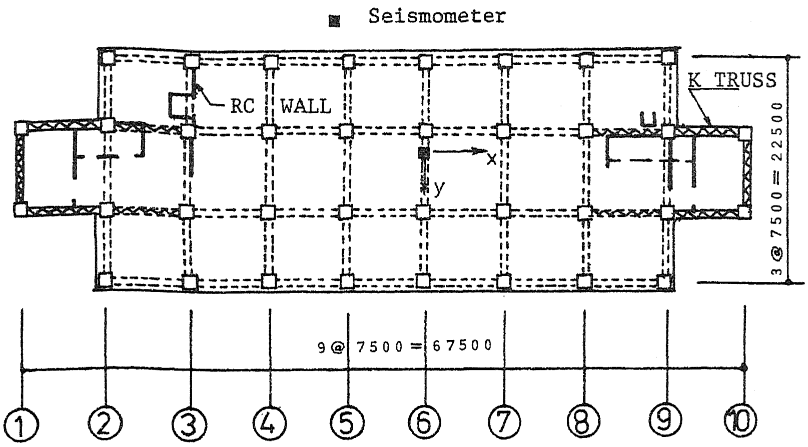

Figure 1. Plane view of a bank building in Taipei

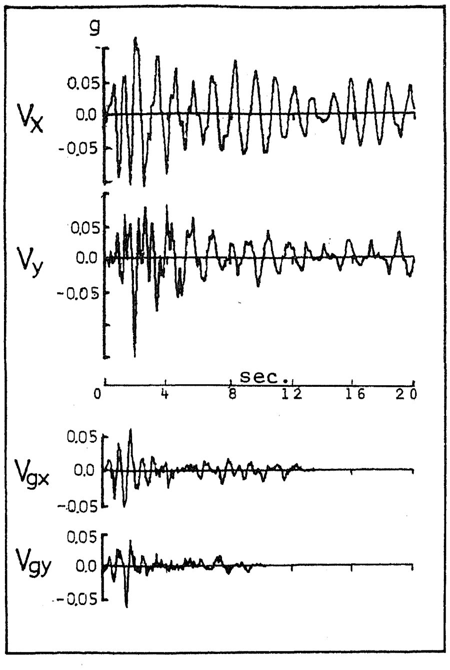

Figure 2. Recorded acceleration of the building

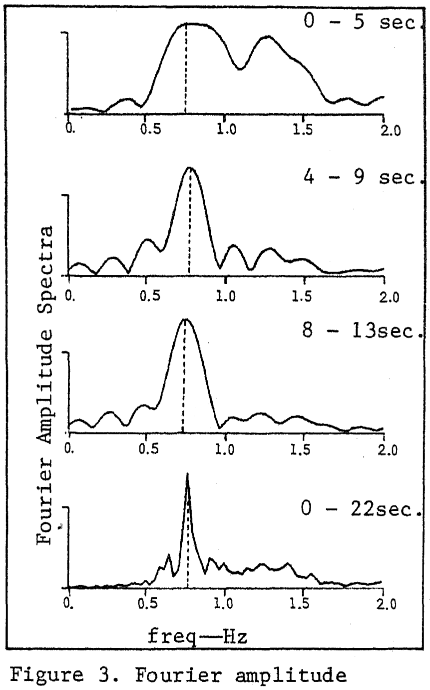

Figure 3. Fourier amplitude spectra of V_x

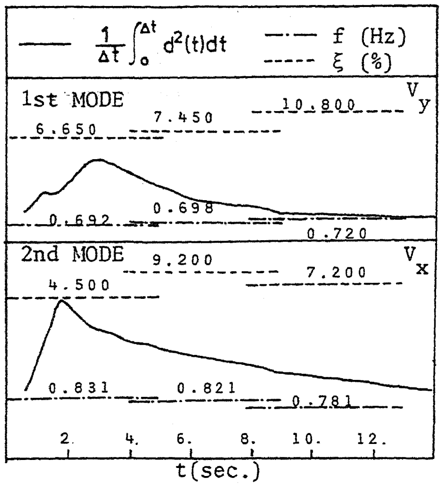

Figure 4. Changing relative displacement and behavior

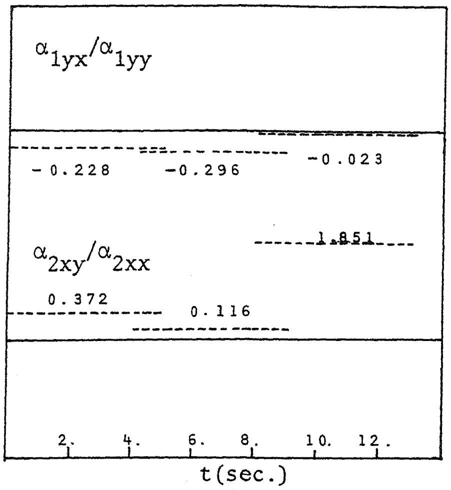

Figure 5. Ratio of participation factors

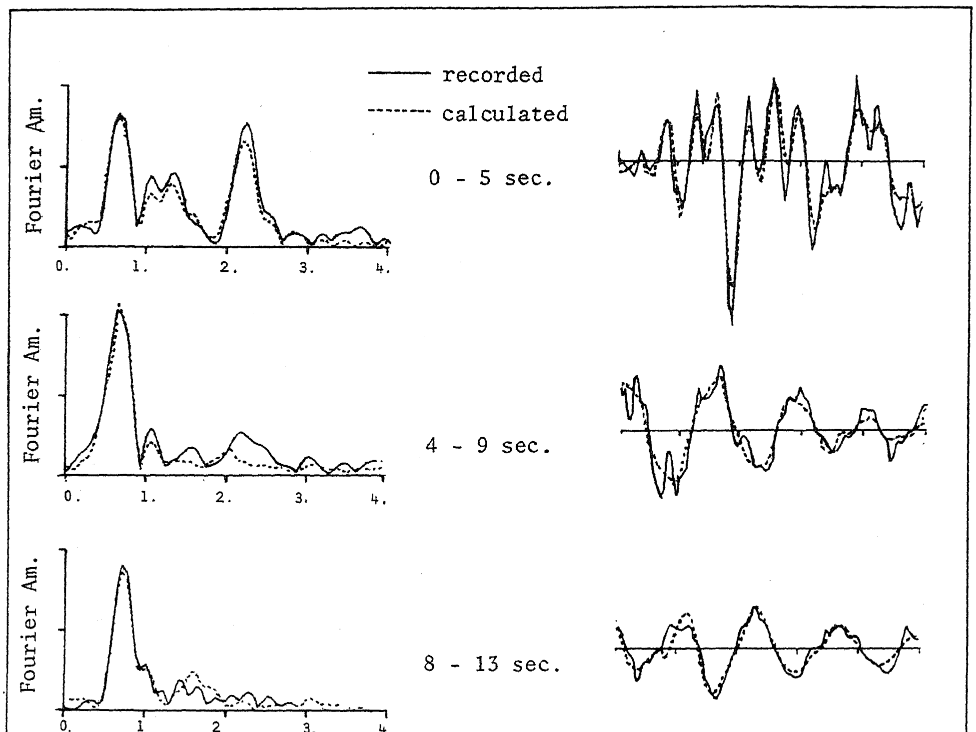

Figure 6. Recorded and calculated amplitude spectra and time history of V_y