IDENTIFICATION OF PARAMETERS IN AN ANALYTICAL MODEL FOR REINFORCED CONCRETE SUBJECTED TO CYCLIC LOADS

by John F. Stanton[I]

SUMMARY

A layered model for reinforced concrete inflexure is developed using system identification. Physical reasoning is used to develop the form of the model, and then the numerical coefficients are adjusted automatically so that the model predicts as closely as possible the results of physical experiments. The accuracy of the predictions is influenced strongly by the constitutive law chosen for the steel reinforcing, for which a new formulation was developed. It is particularly successful in reproducing the nonlinear behavior of reinforcing steel subjected to large inelastic strain reversals.

INTRODUCTION

The behavior of reinforced concrete structures subjected to seismic forces is not easily described in analytical terms. Current design philosophy accepts that it is uneconomical and often impossible to design a structure which will respond elastically to a severe earthquake, and some inelastic action is implicitly permitted. Our confidence in a structure's ability to remain standing under a maximum credible earthquake, albeit with cosmetic damage, depends on our ability to predict its dynamic response. The last quarter of a century has seen dramatic developments in computing capability and matrix analysis, and obtaining the dynamic response of a linear system to a given input is now a routine matter. But with nonlinear systems, two major problems still persist. The first concerns the numerical procedures needed to solve the nonlinear equations of motion. The second problem, addressed in this paper, is the question of choosing a nonlinear model. The linear constitutive model is unique in form, but an infinite number of nonlinear ones exist and one must be chosen from among them. The choice is clearly critical to the faithfulness with which physical behavior can be predicted by analysis.

Furthermore, in the case of reinforced concrete, the nonlinearities are not simple to reproduce. It is a composite of two materials, each of which exhibits nonlinear behavior which is history-dependent, and a third source of nonlinearity is provided by possible slip at the interface.

Field experience and laboratory experiments have provided much qualitative information about the inelastic behavior of reinforced concrete, but analytical models have met with only mixed success in describing the more important features of response. This paper describes a rational effort to derive a mathematical model from a series of experiments. The research program was concerned only with flexural behavior, partly because it dominates the behavior of frames and partly because to do otherwise would introduce extra difficulties which might jeopardize the chances of achieving even a modest objective. The particular behavior for which an analytical model was sought was the moment curvature relationship for a doubly reinforced concrete

---

I Assistant Professor, Civil Engineering Department, University of Washington, Seattle, Washington.

beam of arbitrary cross-section. The experimental results on which it was based were those of Ma, Bertero and Popov (1).

SYSTEM IDENTIFICATION

System Identification is used to develop the best possible model from experimental data. In its simplest form the technique consists of constructing an analytical model whose framework is fixed but whose coefficients are free to be chosen arbitrarily. Next, an objective criterion function is selected which describes the difference between the response predicted by the model and that observed during the experiments. The maximum absolute error or the integral squared error are common choices. Numerical optimization techniques are then used to minimize the criterion function by suitable adjustment of the coefficients (or parameters) in the model.

The method has a number of attractions. First, the use of an objecttive criterion function removes from the comparison process any subjectivity in assessing which set of parameters gives the best set of results. Second, automation makes the adjustment feasible, which it would not be if the computations had to be done by hand, particularly if the model contained more than a few parameters, or if extensive data were used in the comparison. These advantages permit a completely objective appraisal of the model form. If the optimum values are found for the coefficients, any discrepancies between predictions and observed physical behavior must be due to inadequacies in the model form. If they are serious, the model form must be changed. This decision cannot be made without the availability of the optimum parameter set, which is supplied by the identification process.

Once a model form has been chosen, the identification procedure rests heavily on numerical techniques. Their efficiency is central to the tractability of the whole operation, particularly if the analytical model is complex and the number of degrees of freedom is large, in which case the computational effort becomes considerable. A least squares error is a common choice of objective cost function, partly because it provides a convenient measure of average error, but also because it permits the use of a particularly efficient group of optimization methods. This study used the least squares error, but not the efficient Gauss-Newton optimization scheme because it requires the use of derivatives of the error with respect to each parameter. For any but the simplest models, analytical expressions for the derivatives are usually cumbersome or not available, and extensive reprogramming is needed if the model form is changed. System identification was regarded here as a tool for improving the model so from the outset, changes in the model's form were regarded as inevitable. Thus, a routine was used (2) which required an expression only for the value of the error function, and not its derivatives. By this means the programming effort was vastly reduced at the expense of a modest increase in computer time.

CHOICE OF A MODEL FOR REINFORCED CONCRETE

A literature survey shows that most of the better models can be classified as either one- or two-material models.

The one-material models treat reinforced concrete as a homogeneous material whose properties depend on the load history, and they link section forces (e.g., moments) directly to section deformations (e.g., rotations) without concerning themselves with stresses. Of them the Takeda model (3) is perhaps the most widely used, and an extended version of it is to be found in the nonlinear analysis program DRAIN 2D (4). The response which it predicts compares moderately well with experiment, but it was considered a poor candidate for identification because its accuracy appears to depend more on a rather complicated decision making logic (to determine the basis of calculation for evaluating the next tangent stiffness) than on the values of its numerical coefficients.

The two material models treat concrete and steel as independent materials, each with its own constitutive law relating stress to strain. The member cross-section is divided into layers in each of which the stress is considered to be uniform. For any given rotation, a neutral axis is assumed, the corresponding strain field calculated, the stress in each layer computed from the appropriate constitutive law and then integrated over the cross-section to give a moment and an axial force. The process is repeated with a different assumed neutral axis position until the calculated axial force converges to the value of the applied force within some pre-selected tolerance.

The two-material, layered approach was chosen in this study, largely because of its greater potential accuracy. Constitutive models were then sought for each individual material.

Many concrete layers are needed, and so to keep the costs to a reasonable level, an economical constitutive model is important. Fortunately, the moment curvature relation appears not to be sensitive to the accuracy of the concrete stress-strain law used, which can therefore be simple. The reason for this is that when the beam undergoes large inelastic reversals of curvature, it cracks clear through the depth, and the two concrete faces lose contact for much of each load cycle. Hognestadt's parabola, extended to account for hoop confinement, was chosen as an envelope and suitable curves were selected to describe unloading and reloading. Nine coefficients were left free to be chosen by the identification process, which meant that the basic form could adapt to a wide variety of specific shapes.

The model for the reinforcing steel was more demanding. Previous investigators have reported that its accuracy is crucial to the successful modeling of inelastic bending, and physical reasoning supports this contention for a beam cracked from top to bottom. A uniaxial constitutive law therefore had to be found for reinforcing steel which is accurate over a wide range of strains (from zero to about 50 times the yield strain), both for monotonic and reversed loading. Two features which must be represented are strain hardening (permitting stresses above the yield stress) and the Bauschinger Effect (or loss of sharp yield on load reversal). Constitutive laws which do not include them (such as an elastic-perfectly-plastic one) are unable to reproduce the behavior observed during large inelastic load reversals. The task is made a little easier by observing that all load reversals are represented by branch curves in σ-ε space, asymptotic to an envelope curve. This dynamic envelope coincides closely with the shape of the monotonic curve, although its position may be shifted from that displayed by virgin material. A number of analytical expressions have been proposed to describe this behavior. The most successful appears to be a combination of a Ramberg-Osgood curve and straight line segments (1, 5). Those authors found that the combination worked well with their data. However, it is mathematically somewhat inelegant. The curve is defined piece-wise, with a different equation for each segment, and when the branch curve meets the envelope, their slopes are different. The Ramberg-Osgood equation is also nonlinear and has to be solved iteratively, which increases the solution effort. Last, its coefficients do not bear a one-to-one correspondence with individual geometric features of the stress-strain curve, and this mars the numerical conditioning of the identification process. These difficulties were considered serious enough to look for a new model. Menegotto and Pinto's (6) equation describes a rounded transition curve from one straight line asymptote to another. It is:

$$\sigma^* = b\varepsilon^* + \frac{(1-b)\varepsilon^*}{(1+\varepsilon^{*R})^{1/R}} \tag{1}$$

where $\sigma^* = \dfrac{|\sigma-\sigma_{rev}|}{\sigma_o}$; $\varepsilon^* = \dfrac{E_o(\varepsilon-\varepsilon_{rev})}{\sigma_o}$; $b = \dfrac{E_\infty}{E_o}$

The asymptotes are defined as:

$$\varepsilon^* = \sigma^*, \qquad \varepsilon^* << 1 \tag{2}$$

$$\varepsilon^* = (1-b) + b\sigma^*, \qquad \varepsilon^* >> 1 \tag{3}$$

Equation 1 overcomes all the difficulties displayed by the Ramberg-Osgood one but the straight line asymptote (equation 3) represents poorly the monotonic strain hardening curve. By allowing b and R to vary in a specific manner, it was modified to make the branch curves asymptotic to an arbitrary envelope curve, defined by specifying coordinates along it and interpolating between them with cubic splines. Fig. 1 shows the match between predicted and experimentally observed curves. It is very close. The best match was achieved by adding a small stress-shift to the monotonic curve to obtain the dynamic envelope.

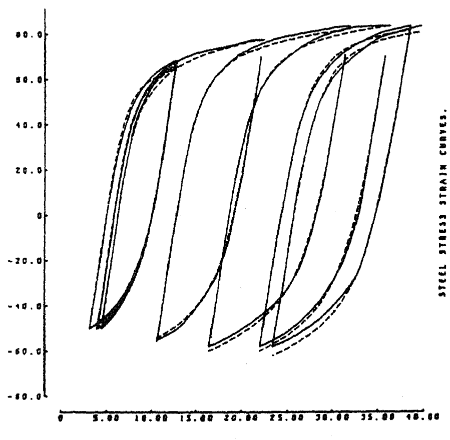

Fig. 1 Steel Stress-Strain Law

Once the constitutive models were established, they were incorporated into a global model which relates section moments to curvature. The method used is based on equilibrium and the assumption that plane sections remain plane. Initially, perfect bond was assumed between the concrete and steel. The experiments which the model was made to match were run between predetermined displacement limits, so the analytical model was constructed to predict moments for a given curvature history. A neutral axis position was guessed, which with the given curvature defined the strains in each material layer. The stress at the center of each layer was calculated from the appropriate nonlinear law, and it was assumed to be constant throughout the thickness of the layer. Section moment and axial force were obtained by summation over the cross-section. A new neutral axis position was then taken and the process continued until the axial force converged to zero within an acceptable tolerance.

An unexpected difficulty arose. The relationship between axial force and neutral axis position was far from smooth, which complicated the search for the correct neutral axis position. (In fact the strain at the section centroid was used rather than the position of the neutral axis, thus avoiding numerical problems when the curvature is small.) The relationship shows ripples superimposed on a smooth curve (see Fig. 2) whose magnitude depends on the size of the concrete layers.

At least 25 layers were needed to ensure that the relationship was smooth enough to be single-valued. The centroidal strain was then found by a combination of bisection and parabolic interpolation using the best features of each method.

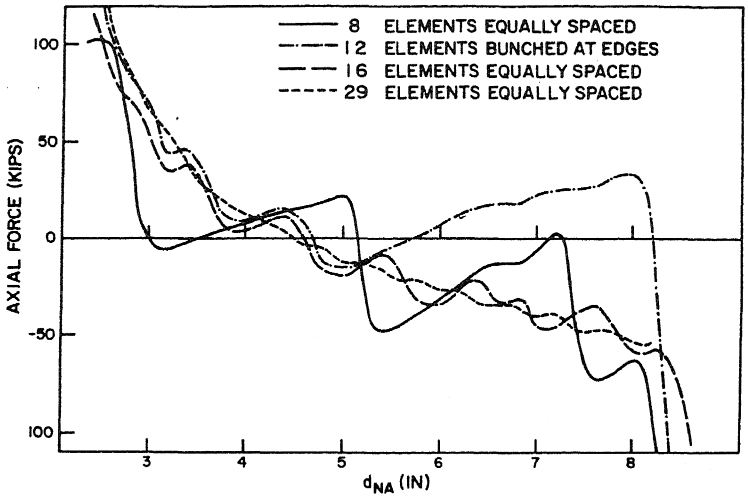

Fig. 2 Axial Force vs. Neutral Axis Location

IDENTIFICATION OF PARAMETERS

The steel model continued 11 parameters and the concrete, 9, all of which were free to be identified. This was considered too large a number to identify at one time. First, the cost of identification rises rapidly with the number of parameters. Second, most minimization algorithms find only a local minimum on the error surface and other techniques such as physical reasoning and restarting the search elsewhere must be used to ensure that the point located is indeed the global minimum. The most serious difficulty, however, is associated with the physical interpretation of the parameters. Suppose that one free parameter represents the initial stiffness of the steel and another, the effect of confinement on the concrete strength. It is possible that identification with all parameters free would lead to an optimum set which contained values for these two parameters which are physically unreasonable. From a mathematical point of view, the problem has been solved when the optimum values are known, but an engineer would have little use for the results. It was felt that a better solution would be obtained by identifying smaller sub-sets of parameters, one at a time. Thus the parameters in the steel model were found first by identification using the results of axial tests on reinforcing bars. They were then fixed and the parameters in the concrete model were found from the moment-curvature data. This procedure tacitly assumes that the constitutive behavior of the steel is independent of its embedment in the concrete. Although there is no proof, the assumption seems reasonable.

As anticipated, the moment curvature relationship proved insensitive to most of the parameters in the concrete model, but three aspects of the physical behavior appear to merit consideration. The concrete in the beam core is thought by many investigators to be capable of carrying some stress even at very large strains. It was modeled here as a constant stress ($a\ f'_c$) for all strains greater than some strain (ε_a). The values of a and ε_a were found to influence the value of the error function. The behavior of the cracks in the concrete during reloading was also important. The best match was obtained by assuming that some rubble fell into the cracks so that a gradual build-up of stress occurred as the crack closed. Thus, compressive reloading of the concrete was modeled as gradual rather than sudden. Last, the behavior of the concrete cover influences the overall accuracy. It is, however, hard to identify because the functional form of its stress-strain relation is not known, and in particular, the strain at which the cover spalls off. Most authors who have given thought to the matter have made different assumptions and there is, as yet, no consensus. In this study, the concrete core and cover were assumed to carry the same stress ($a\ f'_c$) at large strains, although the strain at which it was first attained differed in the two regions.

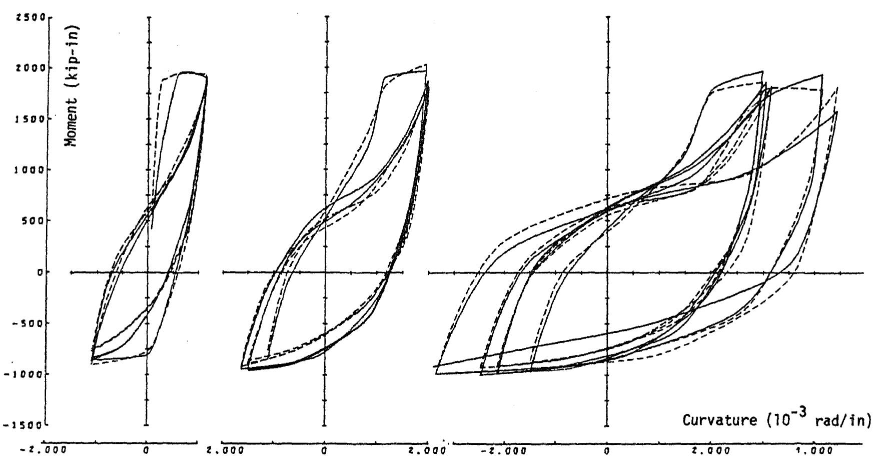

Fig. 3 Moment Curvature Relations, Experimental Results and Analytical Predictions. (Loop groups separated for clarity.)

Typical results of moment curvature identification can be seen in Fig. 3 in which the solid lines represent test results, and the dashed lines, analytical predictions. Shorter load histories are easier to match because those parameters in the individual constitutive models which describe stiffness decay are called into play to a lesser extent.

BOND-SLIP MODEL

Although the predictions were good in most cases, the root mean square difference between predicted and measured moments was generally 5-10% of the maximum moment. Since this is greater than the random error expected from physical sources (experimental errors, draftsmen, digitization, etc.) the global model was judged to be less than perfect. The assumption of perfect bond was selected as the most likely source of error, and a bond-slip relation was introduced to relate the bar strain to the concrete strain at the same level. (They are identical when bond is perfect.) This would reduce the bar strains and forces, and so, the moments for which the predictions were generally too large. The model describes the relation between bar stress and slip, and is based on reversed loading pull-out experiments by Viwathanatepa (7). It consists of a region of free slip at constant but low load, then a nonlinear increase in stress followed by a steep linear unloading segment. The length of the free slip and the stiffness of the loading curve both vary with the load history. The model contains 11 parameters in all, and initial values were obtained by identification against data from some of Viwathanatepa's specimens.

Introduction of bond-slip was expected to make only a modest difference to the moment-curvature prediction, because in the reduction of the experimental data the bar slip through the anchorage block had already been subtracted from the total movement. Thus, only the bar slip in the beam itself remained and this was expected to be small.

Bond-slip was included in the global model and the optimum parameters were identified for four different sets of experimental results. For the two beams with short load histories, for which the predictions were already good, little change occurred. In fact the r.m.s. error increased by 2% in one case, although the worst individual error was reduced by 30% as compared to the runs with perfect bond. The two beams with longer load histories showed improvements of about 50% and 20% in r.m.s. error. Although the results improved, the introduction of bond-slip created numerical problems and roughly doubled the computer time required. In this instance, therefore, it was felt to be more effort than it was worth. However, in other situations where rotations due to bar slip cannot be subtracted from the total deformation, then it must be included because it contributes significantly to the member rotations.

CONCLUSIONS

A number of conclusions can be drawn from this study:

1. System identification is a powerful tool for developing mathematical models of material or member behavior. Particularly important is the ability which it affords for appraising the model's functional form.

2. The constitutive law chosen for steel is particularly critical in modeling reinforced concrete. The new law developed was successful in reproducing the complicated behavior at modest cost. It has also predicted successfully the results of other axial bar tests (8).

3. The global model works best in situations where its assumptions (of low shear stress and reinforcement which does not buckle) are not violated. Even for beams with high shear it predicted the maximum moments well, although the "pinching" in mid-cycle was not well reproduced.

4. The layered model used here is able to reproduce with remarkable accuracy complicated physical behavior. However it is expensive to use and unsuitable for present-day design purposes. As computer capabilities improve, this situation is expected to change.

5. The study showed clearly the dilemma posed by the physical significance of the parameters. To identify the values of the parameters in such a way as to maintain their physical meaning was considered more important than obtaining the smallest possible error value.

ACKNOWLEDGMENTS

The support of the National Science Foundation, under Grant No. ENV 76-04262-A02, is gratefully acknowledged.

REFERENCES

1. Ma, S.Y.M., Bertero, V. V. and Popov, E. P., "Experimental and Analytical Studies on the Hysteretic Behavior of Reinforced Concrete Rectangular and T-Beams," Report No. EERC 76-2, Earthquake Engineering Research Center, Univ. of California, Berkeley, California, 1976.
2. Brent, R., Algorithms for Minimization Without Derivatives, Prentice-Hall, 1973.
3. Takeda, T., Sozen, M. and Nielsen, N., "Reinforced Concrete Response to Simulated Earthquakes," Journal of the Structural Division, ASCE, Dec., 1970.
4. Kanaan, A. and Powell, G. H., "DRAIN-2D - A General Purpose Computer Program for Dynamic Analysis of Inelastic Plane Structures," Report No. EERC 73-22, Earthquake Engineering Research Center, Univ. of California, Berkeley, California, 1973.
5. Park, R., "Theorization of Structural Behavior with a View to Defining Resistance and Ultimate Deformability," Bulletin of the New Zealand Society for Earthquake Engineering, Vol. 6, No. 2, June 1973.
6. Menegotto, M. and Pinto, P., "Method of Analysis for Cyclically Loaded Reinforced Concrete Plane Frames Including Changes in Geometry and Non-elastic Behavior of Elements Under Combined Normal Force and Bending," IABSE Symposium on the Resistance and Ultimate Deformability of Structures Acted on by Well-Defined Repeated Loads, Lisbon, 1973.
7. Viwathanatepa, S., "Deterioration of Bond in Reinforced Concrete Under Generalized Loading," Ph.D. Dissertation, Univ. of California, Berkeley, California (in preparation).
8. Su, T. L., Hawkins, N. M. and Stanton, J. F., "Reinforcing Bars Under Inelastic Cyclic Reversed Loadings," SM 79-2, Dept. of Civil Engineering, Univ. of Washington, Seattle, Washington, 1979.