LINEAR MATHEMATICAL MODELS TO PREDICT THE SEISMIC RESPONSE OF A THREE-STORY STEEL FRAME

by

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SUMMARY

Mathematical models are constructed to predict the linear seismic response of a three-story steel frame using system identification. The experimental data used are from tests performed on a shaking table by Clough and Tang. The models accommodate both floor translations and joint rotations and they predict the time histories of these responses with exceptional accuracy.

INTRODUCTION

In this study we construct mathematical models, using system identification, to predict the linear seismic response of a three-story steel frame. As the models are linear, many will no doubt feel that models presently in use are adequate. We show in what follows that this is not so.

In earlier studies using system identification for constructing mathematical models to predict seismic response, such as McNiven and Matzen [1], we have argued that the major value of system identification is derived from the fact that using it enables one to appraise the form of the model. This is not its value here. In this study we have accepted the usual form for a set of simultaneous, linear differential equations. System identification has been invaluable, however, in arriving at sets of parameters which, when introduced into a set of equations, give models that predict accurately the seismic response of the frame.

There is no such thing as a single mathematical model for a physical frame. There are large numbers of models of different orders of complexity. The job of the person constructing the models is to ensure that each model of a particular order is the best possible of all models of that order. The order of a model, reflected in the number of parameters involved, derives from the number of degrees of freedom which the model accommodates. In this paper we construct only an eight parameter model accommodating six degrees of freedom. A nine parameter model is constructed only to gain physical insight into the behavior of the frame.

System identification needs experimental response data. We are fortunate in having an excellent set of data from experiments performed in 1975 on the shaking table at the Earthquake Engineering Research Center of the University of California, Berkeley, and reported by Clough and Tang [2]. The frame, the experiments performed on it, and the test results are described briefly in the paper.

The eight and nine parameter models predict response quantities that match all of the experimental responses with exceptional accuracy. We learn

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what it is about the engineering of the frame that must be understood in formulating the models and what constitutes complete data for a model of a particular order. We find that each response quantity in that data must impose a constraint on the structure independent of all of the others.

In the interest of brevity, we do not describe system identification as we used it, nor the problems encountered in the numerical analysis. We refer for a detailed account to a report by Kaya and McNiven [4].

THE TEST STRUCTURE

Details of the test structure are shown in Fig. 1. It is sufficient here to note that the frame is forced in a direction parallel to its major

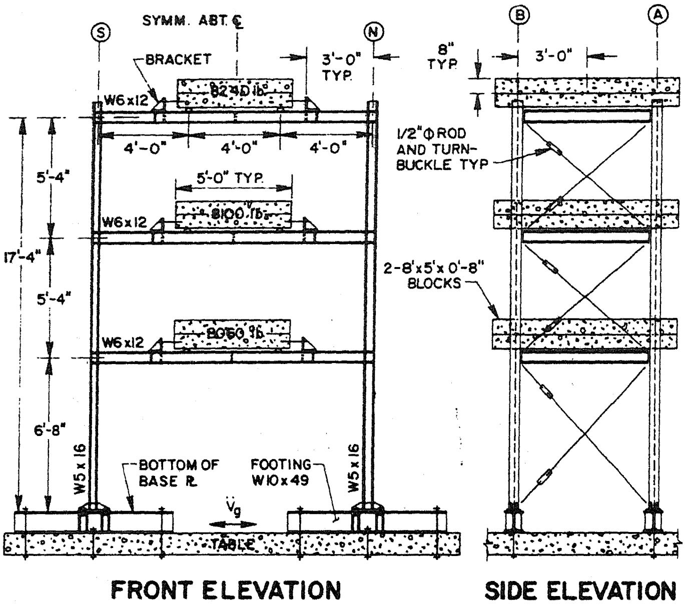

FIGURE 1: ELEVATIONS OF TEST STRUCTURE

axis and that response data are the time histories of the floor accelerations, displacements and strains close to the ends of the members.

AN EIGHT PARAMETER MODEL

The form of the model consists of the usual set of equations appropriate to a multistory frame:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{I\}\ddot{u}_g \qquad (1)$$
$$\{\dot{u}\}_o = \{u\}_o = \{0\}$$

where [M] is the mass matrix and [C] and [K] are the damping and stiffness matrices for the structure. {I} is a unit vector; $\ddot{u}_g$ is the base acceleration and {u}, {u} and $\{\ddot{u}\}$ are vectors for relative nodal displacement, velocity and acceleration, respectively.

The mass of the structure is assumed to be lumped at the nodal points, so that [M] is a diagonal matrix, taken here to be constant. The damping is taken in the form

$$[C] = A_o[M] + A_1[K].$$

We assume a stiffness matrix that will accommodate six degrees of freedom, three translational and three rotational. Examination of the frame leads us to anticipate that the joints will in fact rotate. As the frame is symmetrical, the joint rotations on each end of a girder will be the same.

Construction of the stiffness matrix begins by writing the complete 6 x 6 matrix.

$$\begin{Bmatrix} \overline{M} \\ \overline{P} \end{Bmatrix} = \begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix} \begin{Bmatrix} \overline{\omega} \\ \overline{\Delta} \end{Bmatrix} \tag{2}$$

where $\overline{M}$ and $\overline{P}$ are external joint moments and forces while $\overline{\omega}$ and $\overline{\Delta}$ are the rotations and displacements of the joints.

For our frame there are no external moments at the joints, so

$$\{\overline{M}\} = \{\overline{0}\}. \tag{3}$$

This circumstance allows for the condensation of the 6 x 6 matrix into a 3 x 3, giving the reduced equation

$$\{\overline{P}\} = [\overline{K}] \; \{\Delta\} \tag{4}$$

where $$[\overline{K}] = [-[K_{21}]\,[K_{11}]^{-1}[K_{12}] + [K_{22}]] \tag{5}$$

Resulting from the condensation is the equation

$$\{\overline{\omega}\} = -\,[K_{11}]^{-1}[K_{12}]\;\{\overline{\Delta}\}. \tag{6}$$

This relationship will turn out to be of major significance in what follows and we will return to it in the appropriate context.

The form of the model is now complete, but before continuing the system identification we must introduce eight parameters into the model and assume initial values of all of the quantities that are candidates for optimization. As the stiffness matrix is symmetrical, we assign one parameter to each independent element and evaluate each of the elements using Eq. (5) and the E, I and L for each member where L is the center to center distance. The stiffness matrix resulting is

$$[K] = \begin{bmatrix} \beta_1 \times 23.15 & -\beta_2 \times 33.10 & \beta_3 \times 11.52 \\ -\beta_2 \times 33.10 & \beta_4 \times 71.89 & -\beta_5 \times 49.64 \\ \beta_3 \times 11.52 & -\beta_5 \times 49.64 & \beta_6 \times 69.48 \end{bmatrix} \tag{7}$$

The remaining two parameters are associated with A_o and A_1 whose initial values we found appropriate for a five parameter model of the same frame. The damping components are

$$A_o = \beta_7 \times 0.2340 \text{ and } A_1 = \beta_8 \times 0.0003.$$

We begin with the parameter vector β_i = < 1 1 1 1 1 1 1 1 > so that our initial model corresponds to existing practice. The ability to predict response of this model can be seen in Fig. (2) designated "before optimization". Examination of the figure shows that it models the frame poorly.

The response quantities we use in the cost function are the floor accelerations and displacements of Frame II to the El Centro, Span 400. Even though the duration of the disturbance is about eleven seconds, we use only the first six seconds to reduce the cost of optimization. This first six seconds contains the "rich" part of the input and trial showed that the values of the parameters obtained using only the first six seconds change only slightly when the full duration is used.

For optimization of the parameters we use the Modified Gauss-Newton algorith which is described in [1].

The final values of the parameters are <1.147, 1.175, 1.080, 1.131, 1.024, .998, .575, .607>. The ability of the initial model and the model after optimization are shown in Fig. [2] comparing response time histories for the MEC 600. The improvement is significant and the predicted responses for the improved model matched the experimental so closely that at this stage we felt that the model was complete.

One thing, however, still bothered us. Examination of final values of the parameters shows that during optimization the stiffness parameters change by as much as 18% and the damping parameters even more. We wanted to know, if possible, the physical reason that would account for this significant adjustment. This uneasiness led to the formulation of a second eight parameter model.

SECOND EIGHT-PARAMETER MODEL

The first impulse in trying to account physically for the significant change in the parameters, was to try to trace backwards the relationship between the parameters and the individual members of the frame, where the physical significance would be found. It quickly became apparent that, because the condensation of the stiffness matrix cannot be reversed, physical insight would be gained only by constructing an entirely new model in which the parameters are introduced in association with each individual member of the frame.

The stiffness of each member is derived from its E, I and L and we felt that of the three, the L is the most vulnerable to change during optimization, so that we associated a new parameter δ_i ($i = 1$-6) with each L, one with each column and each girder. The 6 x 6 matrix was formulated as before, condensed as before giving a new 3 x 3 stiffness matrix in which the δ's appear in each element in a somewhat complicated way. The damping parameters were taken in the same form as the first eight parameter model with δ_7 and δ_8 replacing β_7 and β_8.

Using the floor accelerations and displacements in the cost function, optimization was begun with the new model, and to our surprise the iterative process did not converge. After a fruitless search for errors, we sought mathematical and physical reasons for the lack of convergence. We had

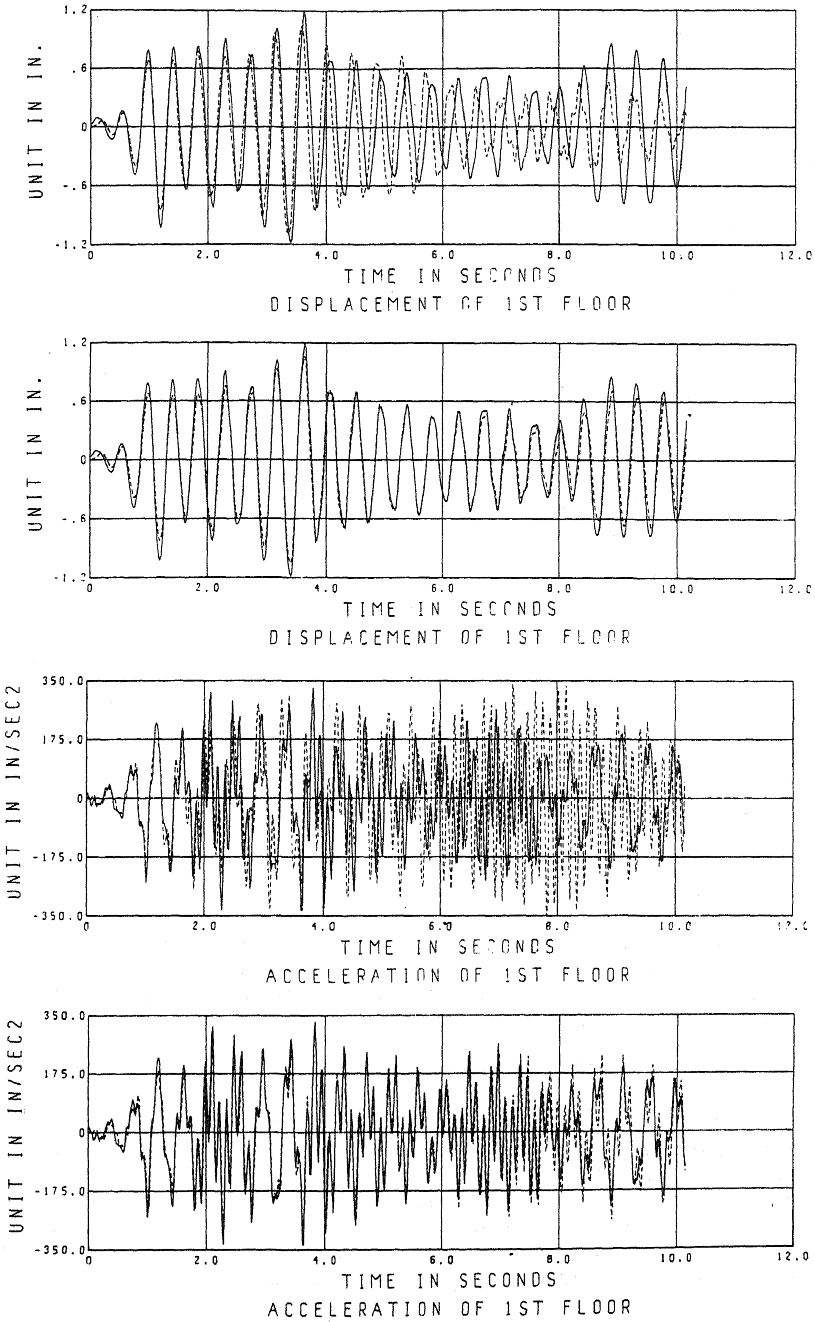

FIGURE 2: MEC 600-II COMPARISON OF MEASURED AND COMPUTED RESPONSE TIME HISTORIES BEFORE AND AFTER OPTIMIZATION OF PARAMETERS (FIRST EIGHT PARAMETER MODEL)

already noted for the first eight parameter model that the same set of values were obtained if in the cost function we used only the acceleration or only the displacements. We concluded from this that they are not independent response quantities perhaps accounting for the lack of convergence.

We recognized that we had strain gage readings on the columns and by using the moments derived from these and moment area we could calculate the joint rotation. Here we recall, however, that due to the absence of external joint moments, the joint rotations are linearly related to the floor displacements. This relationship depends on two assumptions, first that the deformations are elastic, which is applicable here, but also that there is perfect continuity between beams and columns, which might not be applicable to our frame. Feeling that this last point was worth pursuing, we introduced the three joint rotations and the three floor accelerations into the cost function. To our gratification convergence was as rapid as it was in the first eight parameter model.

With the new set of parameters, the second eight parameter model was complete and we were pleased that the model predicts very accurately both the floor translation time histories and the time histories of the joint rotations. This is displayed in Fig. [3].

However, as we were anxious to gain physical insight into the improvement and as the set of parameters gives an effective length for each of the members, it bore close scrutiny. As the effective lengths for the columns were about five percent less than the center to center distances we thought this was reasonable and accounted for the improvement in matching, but the effective lengths of the girders turned out to be about twenty percent longer than their geometric length which was not. We reasoned that the girders were trying to account for vertical motion that they were not in fact responsible for.

At this stage we benefited from work done by Tang [3] in formulating a model of the same frame. Tang found that his model was improved by accounting for the rigid body pitching of the shaking table during excitation. We decided therefore to formulate one more model including the possible pitching, not particularly to improve the ability to predict response time histories, but to see if the effective lengths of the girders would adjust to reasonable values.

THE NINE PARAMETER MODEL

Pitching of the shaking table is accounted for by introducing a ninth parameter δ_9 associated with the rigid body rotational stiffness (k) of the shaking table and letting $k = \delta_9 \times 100^k/\text{in.}$ and taking $\delta_9 = 1$ as the initial value. When the model was completed, it predicted responses little better than the final eight parameter model. However, it showed $\delta_9 = 1.845$ indicating that the table probably does pitch and it lowered the effective length parameters for the girders to about 1.10, a more sensible value.

Both the fact that joint rotations and floor translations are independent response quantities, and that the model is significantly improved by using effective rather than geometric member lengths, indicate that there is

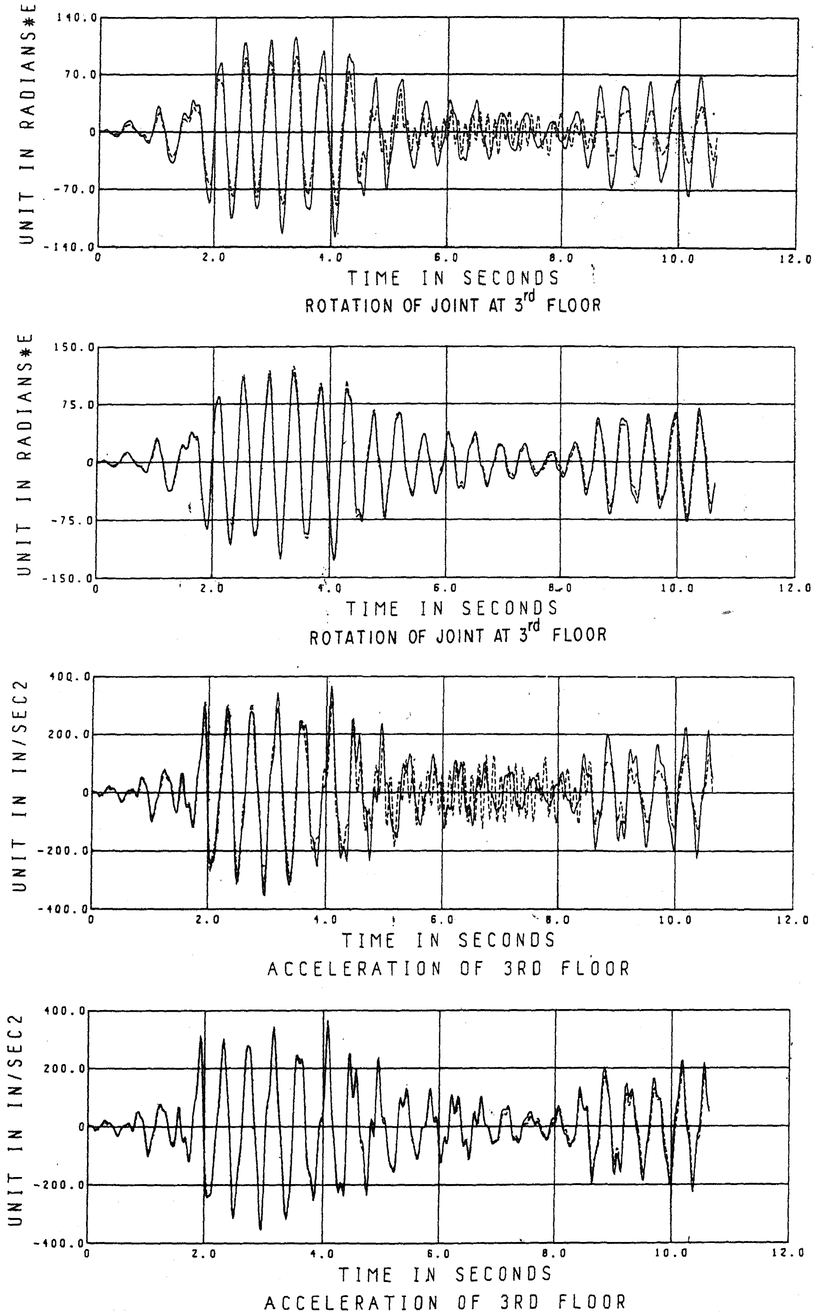

FIGURE 3: EC 400-II COMPARISON OF MEASURED AND COMPUTED RESPONSE TIME HISTORIES BEFORE AND AFTER OPTIMIZATION OF PARAMETERS (SECOND EIGHT PARAMETER MODEL)

not perfect continuity between columns and girders, that the panels at the joints do in fact deform.

We are fortunate in that the testing program allows us to substantiate this conclusion. When the frame was first tested linearly and the response quantities recorded, the joint panels were stiffened and the altered frame was tested and its response data recorded. We formulated nine parameter models based first on the response data of the first frame, and another based on the data from the stiffened frame. For the stiffened frame the effective length factors are closer to 1.0 than those of the first frame.

REFERENCES

1. Hugh D. McNiven and Vernon C. Matzen, "A Mathematical Model to Predict the Inelastic Response of a Steel Frame: Formulation of the Model," Earthquake Engineering and Structural Dynamics, Vol. 6, 189-202 (1978).

2. R. W. Clough and D. T. Tang, "Earthquake Simulator Study of a Steel Frame Structure, Vol. I: Experimental Results," Report No. EERC 75-6, Earthquake Engineering Research Center, University of California, Berkeley (1975).

3. D. T. Tang, "Earthquake Simulator Study of a Steel Frame Structure, Vol. II: Analytical Results," Report No. EERC 75-36, Earthquake Engineering Research Center, University of California, Berkeley (1975).

4. I. Kaya and H. D. McNiven, "Investigation of the Elastic Characteristics of a Three Story Steel Frame Using System Identification," Report No. EERC 78-24, Earthquake Engineering Research Center, University of California, Berkeley (1978).

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