ON THE PROBLEM OF THE DAM-FOUNDATION-RESERVOIR INTERACTION DURING EARTHQUAKES

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SUMMARY

The paper deals with the interaction problem for the media involved in the earthquake analysis of gravity dams. Firstly the substructure investigation is considered and special attention is paid to the dam-reservoir interaction. A more correct scheme is given for obtaining the frequency equation. Then the same problem is considered by taking into account the dam-reservoir-foundation interaction with dynamic relaxation method. Some improvement in formulating the initial conditions is achieved.

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The earthquake analysis of a gravity dam taking into account its interaction with the foundation and the reservoir may be performed using one of the following two approaches:

a/By solving the dam-foundation and dam-reservoir contact problems separately. The foundation is considered as an elastic halfspace and an analytic solution is sought, the dam being investigated numerically. Then the separate solutions are coupled to form the general solution of the problem. This approach has been developed in [1-5] and is referred to as "substructure analysis".

b/The problem may be solved simultaneously for all three media involved by discretizing both the governing equations and the boundary conditions. A numerical method may be applied known as "dynamic relaxation method" which requires space and time discretization. It has been suggested in [6-8] mainly for static problems and developed in [9] for dynamic investigation of gravity dam under earthquake loading.

Each of these two approaches has advantages and disadvantages. The aim of this paper is to analyze some points of the solutions in order to suggest some improvements.

1. Solution of the Hydroelastic Problem with Fourier Series

In the substructure analysis the elastic dam-reservoir interaction problem is considered to be of basic importance.

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The solution of this problem with Fourier series is suggested as early as 1935 [10] and developed in [11]. Unfortunately in these publications there are some details which are not correct.

Let us consider the equation

$$D\frac{\partial^4 w}{\partial y^4}+\rho S\frac{\partial^2 w}{\partial t^2}=p(0,y,t)\,, \qquad (1)$$

which governs the free bending vibrations of a dam with constant cross section. Here $w(y,t)$ stands for horizontal displacement at the level y in moment t (Fig.1), ρ and S are the density of the dam material and its cross section area respectively, and $p(0,y,t)$ is the hydrodynamic pressure on the dam.

The solution of Eq.1 may be written in the form

$$w(y,t)=\sin\omega t\Big[A_1\sin\beta y+A_2\cos\beta y+A_3\,\mathrm{sh}\,\beta y+ + A_4\,\mathrm{ch}\,\beta y-\rho_0\omega\sum_{m=1,3,\dots}^{\infty}\frac{C_m Y_m}{D\alpha_m^4-\omega^2\rho S}\,, \quad \beta^4=\omega^2\rho S/D$$

$$Y_m=\cos\alpha_m y \qquad\qquad \alpha_m=m\pi/2H. \qquad (2)$$

The velocity potential Φ has been found as follows [10] :

$$\Phi(x,y,t)=\cos\omega t\sum_{m=1,3,\dots}^{\infty}C_m.e^{-\alpha_m x}.Y_m(y). \qquad (3)$$

Now equating the dam and water velocities along the contact line

$$\left.\frac{\partial\Phi}{\partial x}\right|_{x=0}=\frac{\partial w(y,t)}{\partial t} \qquad (4)$$

leads to the equation

$$-\frac{1}{\omega}\sum_{m=1,3,\dots}^{\infty}\alpha_m C_m Y_m=A_1\sin\beta y+A_2\cos\beta y+A_3\,\mathrm{sh}\,\beta y+ + A_4\,\mathrm{ch}\,\beta y-\rho_0\omega\sum_{m=1,3,\dots}^{\infty}\frac{C_m Y_m}{D\alpha_m^4-\omega^2\rho S}. \qquad (5)$$

Similar condition but with some errors is given both in [10] and [11] .

Further analysis is sketched in [11] /according to the idea in [10] / as follows. The partial integrals $\sin\beta y$, $\cos\beta y$, $\mathrm{sh}\,\beta y$, $\mathrm{ch}\,\beta y$ are to be expanded in Fourier series on the system of orthogonal functions $\sin\alpha_m y$ $(m=1,3,\dots)$
and then, after expressing the C_m constants as linear combinations of A_k , the frequency equation is obtained. The following improvement may be made in this approach.

It is known that the expansion of an arbitrary continuous function $f(x)$ in sine series in the $(0,L)$ interval equals its odd extension in the $(-L,0)$ interval. In the ends of the $(0,L)$ interval the series will give zero values. Accordingly, if the same function would be expanded in cosine series this would equal its even extension in the $(-L,0)$ interval and in the end points of the base interval $(0,L)$ the series would converge to the values $f(+0)$, $f(L-0)$.

Now it is clear that for non-zero values of the function in the ends of the $(0,L)$ interval the sine series should not be used. Just the same case arises in the considered hydroelastic problem where the partial integrals $\cos\beta y$, $ch\beta y$ differ from zero for $y=0$. Owing to this reason the expanding in sine series probably would influence the accuracy of the frequency equation.

A better approach involves expanding in cosine series, looking out as follows. For the partial integrals we have

$$\cos\beta y = \sum_m a_m \cos\alpha_m y \,, \quad \sin\beta y = \sum_m b_m \cos\alpha_m y \tag{6}$$

$$a_m = \frac{1}{2H}\left[\frac{\sin(\alpha_m+\beta)2H}{\alpha_m+\beta} + \frac{\sin(\alpha_m-\beta)2H}{\alpha_m-\beta}\right] \tag{7}$$

$$b_m = \frac{1}{2H}\left[\frac{\cos(\alpha_m-\beta)2H}{\alpha_m-\beta} - \frac{\cos(\alpha_m+\beta)2H}{\alpha_m+\beta} - \frac{2\beta}{\alpha_m^2-\beta^2}\right] \tag{8}$$

$$ch\beta y = \sum_m c_m \cos\alpha_m y \,, \quad c_m = -\frac{\beta\, sh2\beta H}{H(\alpha_m^2+\beta^2)} \,, \tag{9}$$

$$sh\beta y = \sum_m d_m \cos\alpha_m y \,, \quad d_m = -\frac{\beta}{H}\cdot\frac{ch2\beta H+1}{\alpha_m^2+\beta^2} \,. \tag{10}$$

Substituting Eqs.6 ÷ 10 into Eq.5 now yields

$$-\frac{1}{\omega}\sum_m \alpha_m \cdot C_m \cos\alpha_m y = -\rho_c\omega \sum_m \frac{C_m \cos\alpha_m y}{D\alpha_m^4 - \omega^2\rho S} +$$

$$+\sum_m \left(A_1 b_m + A_2 a_m + A_4 c_m + A_3 d_m\right)\cos\alpha_m y. \qquad (11)$$

It is not difficult to express C_m from this equation in terms of A_1, A_2, A_3, A_4 for each m. This is especially convenient when solving a given numerical example because then the coefficients a_m, b_m, c_m, d_m are definite numbers for each m. Further the solution follows the well known way - the Eq.2 contains only the A_k constants and the boundary conditions /which are homogenous/ lead to the frequency equation.

It may be expected that the improvements suggested here will give better accuracy in the evaluation of the frequencies of the dam-reservoir system.

2. On the Solution of the Contact Problem for the Dam-Foundation-Reservoir System with Dynamic Relaxation Method

It is shown in [12] how the dynamic relaxation method should be applied to this problem. The equations of the motion /in tensor notation/

$$\rho\ddot{u}_i + k\dot{u}_i = T_{ij,j} \qquad (12)$$

should be solved, u_i being the components of the displacement vector, k - the damping coefficient, and T_{ij} - the stress tensor.

Two models for boundary conditions are suggested in [12]. The first one is a travelling wave of the type

$$u(x,t) = u_g\left(t - \frac{x}{v_t}\right) \qquad (13)$$

for the displacements in x-direction (Fig.2), where v_t is the travelling wave velocity and u_g is the horizontal displacement seismogramm. Boundary of this type may be introduced along the horizontal bottom edge when there exists a real physical boundary.

The second type boundary conditions are the so called "energy-transmitting" boundaries introduced by Lysmer and Kuhlemeyer. They may be used both along the horizontal bottom edges and the vertical boundary lines. In the latter case they are to be written as follows:

$$\sigma_n = -a\rho c_p \cdot \dot{u}_n, \quad \tau_n = -b\rho c_s \dot{u}_t, \qquad (14)$$

where a and b are constants depending on the frequency and depth, and c_p and c_s stand for velocities of P- and S waves respectively.

The solution of the considered boundary problem with the dynamic relaxation method is a numerical solution in finite differencies. The space and time discretization

corner point A of the foundation block because its volume is conventional and as a matter of fact the point under it will influence its motion.

In order to go this way it is necessary to know the incident angle α. Keeping in mind the situation of possible foci for a given site it may be stated that this angle is more or less known. However, the influence of the dam itself on the wave propagation in its vicinity is not taken into account. To do this one would require solution of the contact wave problem which is very difficult.

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