HYDRODYNAMIC INTERACTION OF ELASTIC STRUCTURES

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SUMMARY

The interaction of elastic cylinders in a fluid layer has been investigated with applications to the dynamic analysis of large, fluid surrounded structures such as piers, towers, and platforms. The response of such structures to earthquake excitations has been derived via classical beam theory and assuming the fluid pressure to be governed by the equations of potential flow. It is shown that the fluid interaction is important to the beam response for excitation frequencies higher than the first fundamental frequency of the system and that the difference in the natural frequencies of a fluid interacting and a non-interacting system is dependent on the geometry and the direction of excitation

DISCUSSION OF THE PROBLEM AND RESULTS

The dynamic interaction of elastic structures with a fluid medium is quite complex but of great importance to the proper design of seafloor-mounted structures. One mechanism which is present to some degree in the response of close proximity, multiple structure interactions is due to the pressure loads on one structure caused by the deformations of another. This problem is examined in part in this paper by evaluating the dynamic response of multiple, elastic, vertical beams standing in a fluid layer. Investigations of the response of a single elastic beam in a fluid layer have shown that the hydrodynamic effects significantly lower the natural frequencies of the beam if it is assumed that the mode shapes are approximately unchanged [1,2]. The beams are rigidly attached to a base which is given a unit harmonic acceleration (for purposes of a steady state response analysis), and they are modeled as cylindrical Euler beams where $y_{jk}(z,t)$ represents the bending displacement on the j-th beam in an array due to a base excitation in the k direction (k=1,2,; see Fig. 1). The fluid is modeled by potential flow theory and thus it is sufficient to formulate the interaction from the pressure field, $p(r,\theta,z,t)$. Solving the boundary value problem associated with the steady state response equations, the pressure distribution on a rigid cylinder due to the motion of another cylinder in the array can be described by a series expansion of Hankel functions which represent radiated and incident cylindrical waves. The fluid pressure is integrated to yield the force distribution on a beam. The Euler beam equation with this hydrodynamically imposed force distribution is

$$EI \frac{\partial^4 y_{jk}}{\partial^4 z} + M \frac{\partial^2 y_{jk}}{\partial t^2} = \sum_{n=0}^{\infty} \phi_n^{jk} \cos\alpha_n z e^{-i\omega t} \quad , \qquad (1)$$

where the right hand side of Eq. 1 is the series expansion for the hydrodynamic force in terms of a known coefficient $\phi_n{}^{jk}$, E is the modulus of

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elasticity, I is the moment of inertia, and M is the mass per length of the beam. The coefficient ϕ_n^{jk} was found from the interaction analysis for an array of rigid beams, expressing the compliances for the response of one beam's motion through an expansion of the eigenfunctions of the fluid layer.

The qualitative behavior of the dynamic solution depends on the relative stiffness of the beams compared to the fluid and upon the ratio of the fundamental frequency of the fluid layer to the first natural frequency of the beam, ω_f/ω_b. Solutions at frequencies higher than ω_f admit radially traveling waves, while at lower frequencies, the solution is expressible by exponentially decaying (in the radial direction) standing waves.

The frequency response of a two-cylinder array is shown in Fig. 2 for various values of EI (in lb sec^2 ft^5). Varying EI corresponds to changing the relative flexibility of the beams. The free beam response is for a beam in vacuo, and L=1 corresponds to the direction in line with the beams' centerlines while L=2 is perpendicular to that. For this figure, ω_f/ω_b = 0.48. The effect of the fluid interaction tends to lower the natural frequency of the beam system, particularly for the more flexible systems, Figs. 2a and 2b, where the system displays multiple resonant peaks below the free beam natural frequency. For all but the most flexible systems, Fig. 2a, the first mode shape did not change significantly with the fluid interaction; however, the second mode was strongly influenced by the fluid and multiple beam interaction for all responses in Fig. 2. At sufficiently high frequencies such that $\omega > \omega_f$, the radiation damping of the pressure waves contributes to attenuating any resonance. The response is significantly dependent on the multiple beam interaction at frequencies higher than the free beam natural frequency for the more flexible systems in Figs. 2a and 2b. The fluid interaction response shown in Fig. 2a is more strongly damped at the fundamental resonance frequency than are the responses for the beams with higher or lower stiffnesses shown in the other plots. Detailed analysis of the responses for a range of ω_f/ω_b indicates that this attenuation of the resonant response occurs when the second natural frequency of the fluid interacting system is close to the fundamental frequency of the free beam.

The frequency responses for various equally spaced, three-beam geometries are shown in Fig. 3, where k corresponds to the beam number, ℓ to the excitation direction, r_0 is the beam radius, and h is the height. Only two response curves are shown in Fig. 3d. It is seen that the response is more dependent on the geometry and excitation direction than it was for the two-cylinder response. The response in directions perpendicular to the excitation direction is small (except at the resonant frequencies) and thus the part of the pressure field due to the multiple interaction is small compared to the pressure field due to the immediate interaction between a single beam and the fluid. As seen from the figures, however, the multiple beam interaction is significant enough to influence the response in line with the excitation. The primary effect of different excitation directions on the response is to slightly shift the frequency of a response peak and/or to amplify or attenuate the response at the peaks. More details on this study can be found in [3].

REFERENCES

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FIGURES

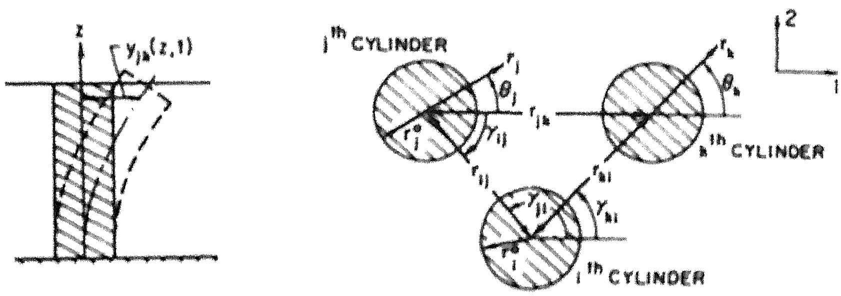

Fig. 1: Left: Definition of the Displacement, $y_{jk}(z,t)$

Right: Polar Coordinate System for the Array

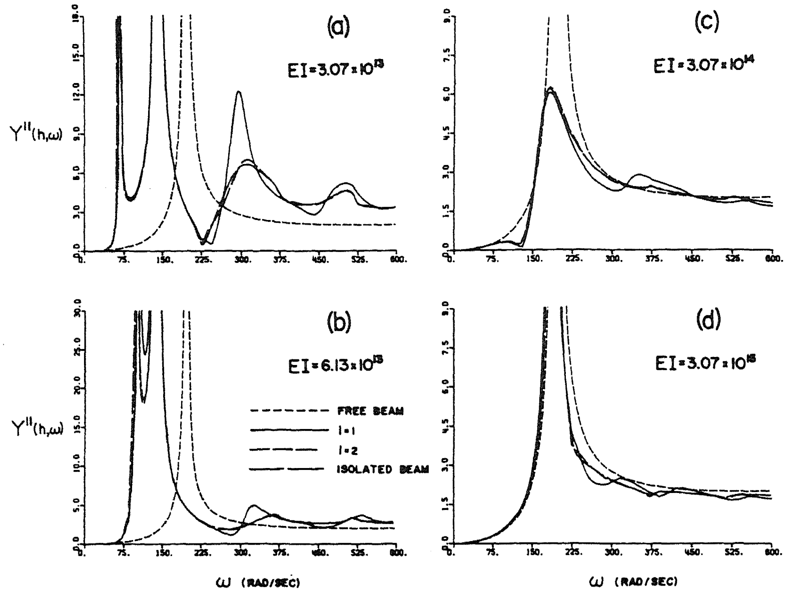

Fig. 2: The Response at the Top of a Beam for a Two-Cylinder Array with $M/EI = 3.26 \times 10^{-12}/sec^2\ ft^4$

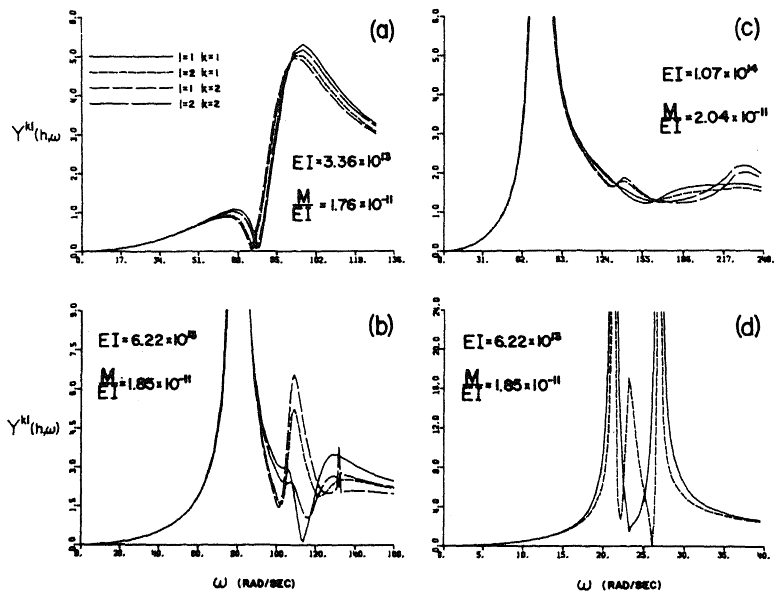

Fig. 3: The Frequency Response for Several Three-Beam Arrays ($r_0/h = 0.2$ for a, b, and c; $r_0/h = 0.1$ for d)