EVALUATION OF NONLINEAR STRUCTURAL RESPONSE TO SEISMIC EXCITATIONS BY SYSTEM IDENTIFICATION

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SUMMARY

The evaluation of the response of a one story steel frame with partitions to earthquake type excitations is discussed. Modeling of the structure is attempted by means of system identification techniques. Alternatives are sought for a model, suitable in representing nonlinear inelastic response. Conclusions are formed concerning the applicability and the limitations of the Kelvin model as the analogy of a single degree of freedom oscillating structure and the Gauss-Newton numerical procedure as a means of performing system identification of such a model.

INTRODUCTION

The Earthquake Simulator of the Earthquake Engineering Research Center at the University of California, Berkeley provides the opportunity to investigate structural behaviour under earthquake type excitations. During the last few years experiments have been performed on the shaking table of the simulator with one and multistory steel frames, reinforced concrete frames, masonry buildings etc.

A large variety of structural responses have been obtained, ranging from linear elastic to nonlinear inelastic ones. The evaluation of this experimental data involves a search for meaningful parameters characterizing the structural behaviour and linking it to an idealized model with analogous performance. System identification techniques are helpful in this process.

SYSTEM IDENTIFICATION

The identification of a system with a given parametric model usually involves the minimization of an error function in search of the optimal parameters. Experimental data and a numerical procedure are required. The modified Gauss-Newton method has been applied in the studies of one (1) and three (2) story steel frames. The following error functions are considered:

$$J_1(\bar{a},T) = \int_{T_1}^{T_2} (\ddot{y}(\bar{a},t) - \ddot{x}(t))^2 dt \qquad (a)$$

$$J_2(\bar{a},T) = \int_{T_1}^{T_2} (y(\bar{a},t) - x(t))^2 dt \qquad (b) \qquad (1)$$

$$J_3(\bar{a},T) = J_1(\bar{a},T) + b\, J_2(\bar{a},T) \qquad (c)$$

where x and $\ddot{x}$ are measured structural displacements and accelerations,
y and $\ddot{y}$ are simulated values obtained from the model,
$\bar{a}$ is the parameter vector and b is a weighting factor.
Error J_3 is eventually abandoned since it implies a relationship between displacements and accelerations, which is the subject of the modeling.

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Reported in (3) and (4) are some of the results of applying the above procedure to data obtained during the tests of a one story steel frame with various infill partitions. The primary concern in this case was with the increased complexity of the behaviour expected of the partitioned frame. The deteriorating stiffness of the partitions and their buffeting against the frame for instance would require adequate energy dissipation devices in the model. Modeling the yield of the bare frame alone falls short of simulating nonlinear displacements as pointed out in (1), where Eq. 2 is applied.

$$C\,\dot{x} + P(x) = -\,M\,\ddot{x}_{abs} \tag{2}$$

where P(x) is the chosen load-displacement function, M is the concentrated mass, C is the viscous damping and $\ddot{x}_{abs}$ is the absolute acceleration.

A preliminary inspection of the test data usually consists of obtaining 'pseudo' load-displacement relationships by neglecting the velocity term in Eq. 2. Fig. 1 illustrates a typical response of a partitioned frame. Another estimate of the structural behaviour is provided by Eq. 2 in the form:

$$\begin{aligned} C_i\dot{x}_i + K_i x_i &= -\,M\,\ddot{x}_{abs,i} \\ C_i\dot{x}_{i+1} + K_i x_{i+1} &= -\,M\,\ddot{x}_{abs,i+1} \end{aligned} \tag{3}$$

where K is the structural stiffness and i,i+1 refer to consequtive timesteps in the test history, which is usually recorded at frequency of 100 Hertz.

System (3) is solved for C and K. The result corresponding to the relationship of Fig. 1 is shown on Fig. 2. It is noted that changes of K occur systematically and are always accompanied by changes in C.

In order to accomodate this behaviour, the identification procedure described in (3) and (4) is carried out over individual cycles of motion, allowing for a change of C and K at given points during the cycle. For the test of Fig. 1 the resulting parameters are shown on Fig. 3 along with a comparison between the measured and simulated acceleration. The identification uses error J_1. Average values in agreement with the results of elastic analysis are discernible but the abrupt changes of both parameters at points of motion reversal persist. This can not be solely attributed to data noise. A closer examination of the initial model is indicated.

MODELING OF THE SDOF STRUCTURE

The modeling of a system by numerical identification of parameters serves two purposes:

Quantitatively, the values of the parameters are obtained.

Qualitatively, if a good fit is demonstrated between measured and simulated responses, the model assumption is substantiated.

Eqs. 1,2 and 3 are based on the Kelvin model of Table 1, Column 2, which is the standard assumption for a single degree of freedom structure. Nonlinearity is built in the system by replacing K with P(x) in Eq. 2. Two objections to such a model are raised:

Changes in the stiffness do not lead to changes in the damping.

No provision is made for permanent eccentricities due to inelastic behaviour.

Viscous damping can not be discreditted as a means of representing the global effects causing decay of oscillatory motion. On the other hand yield in structures involves a different type of energy dissipation. Hysteretic damping has been suggested in view of the generally hysteretic shape of non-linear load-dispacement relationships. Coulomb damping, which is a function of the displacement rather than the velocity can account for friction.

Rather than speculate on the nature of the damping independent of the model configuration, the present study reexamines other possible analogies available from viscoelasticity (5) as shown in Table 1. Column 1 contains the basic alternative to the Kelvin solid, i.e. the Maxwell fluid. If such a model is to be considered for structural purposes, the damping acquires a new significance. Instead of being too small and hence possibly negligible, it becomes too large to affect the linear oscillations of the spring element. Only if the spring's capacity is exceeded does it come into effect, resulting in permanent displacements.

Such reasoning is not entirely foreign to solids. Yielding of metals has been modeled as a slip between molecules already stressed to their elastic limit. It appears that while the paralel action of the damping and the spring elements of a Kelvin model is typical of elastic oscillations, during yield their position may be in sequence. Columns 3 and 4 of Table 1 show two of the possible Kelvin-Maxwell combinations allowing the oscillator to display the flexible behaviour suggested by the tests. The model of Column 3 can be used as follows: with C_2 tending to infinity it is reduced to a Kelvin model with an increased stiffness; K_2 tending to infinity produces another Kelvin model with a higher damping. Such behaviour has been observed during the test of the relatively flexible frame with a stiffer partition. In this case the bond between frame and partition can be modeled as a damper of the C_2 type. Internal modeling of the partition would be required for it's nonlinearities.

The model of Column 4 also has it's applications. The yield in an internally statically indeterminate structure (such as the partitioned frame) corresponds to the action of an internally positioned damper, such as C_2 of Column 3. The forming of yielding areas and the ensuing permanent eccentricities of the bare frame however are more accurately represented by an external damper, such as C_2 of Column 4. The above considerations can be summed up in the following conclusions.

CONCLUSIONS

The Kelvin model on which the standard SDOF equation of motion is based fails to represent structural yield regardless of the provisions made for a nonlinear spring behaviour. Also required is a damping device in sequence, rather than in paralel with the spring. The damper itself need not be necessarily a viscous one.

Qualitative modeling of the parameters requires a numerical procedure which, unlike the Gauss-Newton method would allow parameters to vanish. Identification would then consist not only of establishing the values of the parameters, but also of determining the number of the significant ones. As in viscoelasticity (5), the general model would be of the following form:

$$M\,(\ddot{x}_{abs} + p_1\dddot{x}_{abs} + \ldots) = q_0 x + q_1\dot{x} + \ldots \qquad (4)$$

In general the parameters of a system obtained by identification, such as p_i and q_i of Eq. 4 need not necessarily have physical significance. In the case of Eq. 4 however, they do. The model is generated on the basis of certain physical analogies and the physical compatibility of the results has to be maintained. Furthermore, the physical significance of a parameter may vary, depending on the configuration of the model as can be seen by a comparison between Columns 3 and 4 of Table 1. It has been noted that a structural response containing local nonlinear behaviour can be roughly approximated by a linear model. Once the parameters of such a model are determined, it is still of interest to identify the elastic and the damping characteristics of which they consist in order to judge the form of the model and to draw conclusions on its subsequent performance.

The need for more information on structural motion, particularly the velocity and the rate of change of the acceleration is stressed. The concept of parsimony which requires that the model parameters should be necessary as well as sufficient extends to the identification procedure as well. The search for the significant parameters should not be encumbered with irrelevant data. The models of Table 1 with four parameters, one of which is allowed to vanish limit the requirements for data on the motion to the above.

ACKNOWLEDGMENTS

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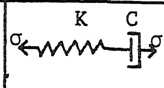
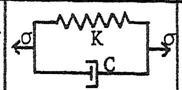
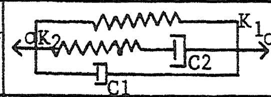
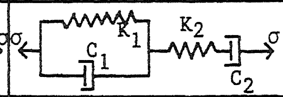
Table 1. Models of an Oscillating System.

	1	2	3	4
Model				
Viscoelastic Equation	$\sigma + \frac{C}{K}\dot{\sigma} = C\dot{\varepsilon}$	$\sigma = K\varepsilon + C\dot{\varepsilon}$	$\sigma + \frac{C_2}{K_2}\dot{\sigma} = K_1\varepsilon + (C_1 + C_2(1+\frac{K_1}{K_2}))\dot{\varepsilon} + \frac{C_1C_2}{K_2}\ddot{\varepsilon}$	$\sigma + (\frac{C_1}{K_1} + \frac{C_2}{K_2} + \frac{C_2}{K_1})\dot{\sigma} + \frac{C_1C_2}{K_1K_2}\ddot{\sigma} = C_2\dot{\varepsilon} + \frac{C_1C_2}{K_1}\ddot{\varepsilon}$
Structural Analogy	$-M\ddot{x}_{abs} - M\frac{C}{K}\dddot{x}_{abs} = C\dot{x}$	$-M\ddot{x}_{abs} = Kx + C\dot{x}$	$-M\ddot{x}_{abs} - \frac{C_2}{K_2}\dddot{x}_{abs} = K_1x + (C_1 + C_2(1+\frac{K_1}{K_2}))\dot{x} + \frac{C_1C_2}{K_2}\ddot{x}$	$-M\ddot{x}_{abs} - M(\frac{C_1}{K_1} + \frac{C_2}{K_2} + \frac{C_2}{K_1})\dddot{x}_{abs} - M\frac{C_1C_2}{K_1K_2}x^{iv}_{abs} = C_2\dot{x} + \frac{C_1C_2}{K_1}\ddot{x}$
Reduced Order	$A - M\ddot{x}_{abs} = K(x + \frac{M}{C}\dot{x}_{abs})$	-	-	$A - M\frac{K_1}{C_2}\dot{x}_{abs} - M\frac{C_1}{K_2}\dddot{x}_{abs} - M(1 + \frac{C_1}{C_2} + \frac{K_1}{K_2})\ddot{x}_{abs} = K_1x + C_1\dot{x}$
$K_2 \to \infty$	-	-	$-M\ddot{x}_{abs} = K_1x + (C_1+C_2)\dot{x}$	$A - M\frac{K_1}{C_2}\dot{x}_{abs} - M(1 + \frac{C_1}{C_2})\ddot{x}_{abs} = K_1x + C_1\dot{x}$
$C_2 \to \infty$	-	-	$A - M\ddot{x}_{abs} = (K_1 + K_2)x + C_1\dot{x}$	$A - M(1 + \frac{K_1}{K_2})\ddot{x}_{abs} - M\frac{C_1}{K_2}\dddot{x}_{abs} = K_1x + C_1\dot{x}$
$K_2, C_2 \to \infty$	-	-	Rigid Body	$A - M\ddot{x}_{abs} = K_1x + C_1\dot{x}$

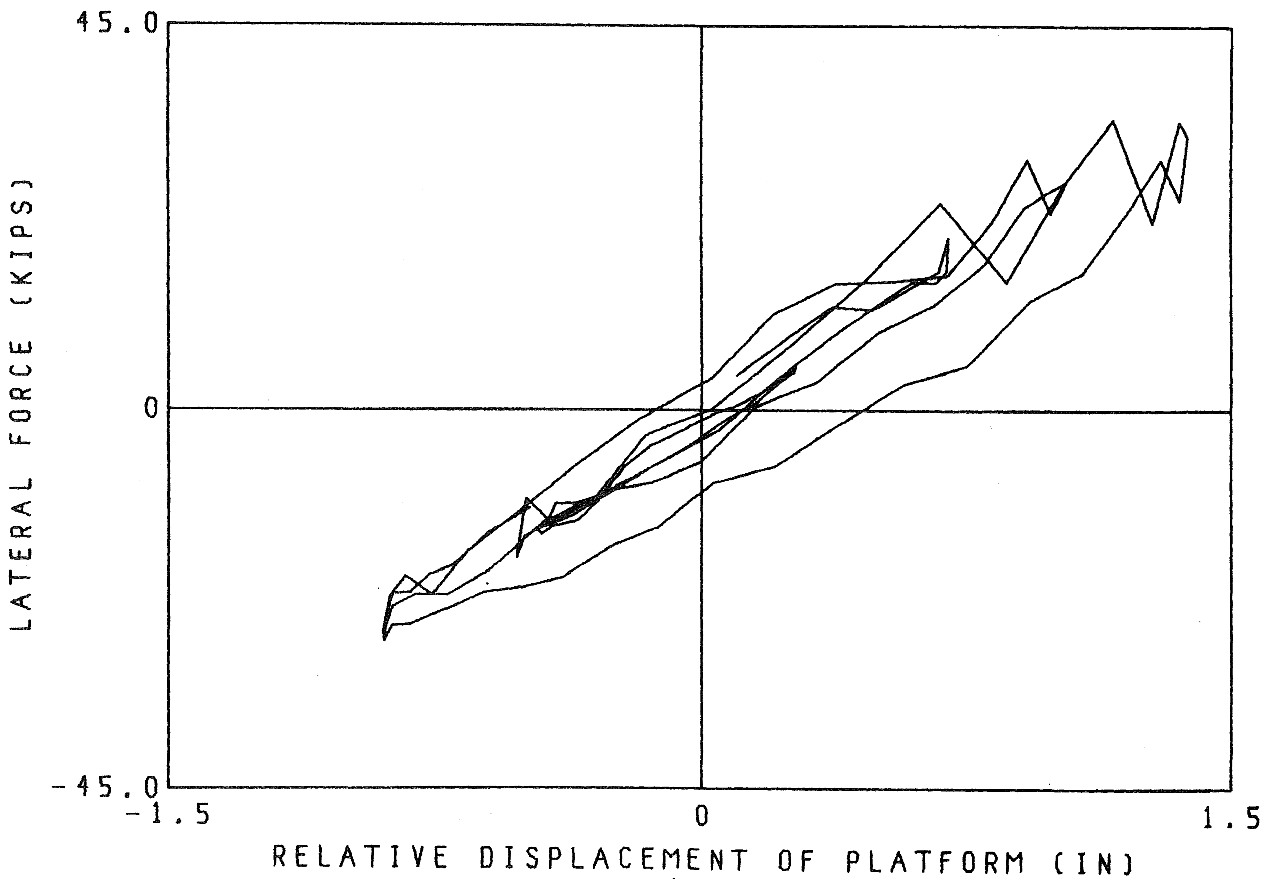


Figure 1. 'Pseudo' Response of a Partitioned Single Story Steel Frame

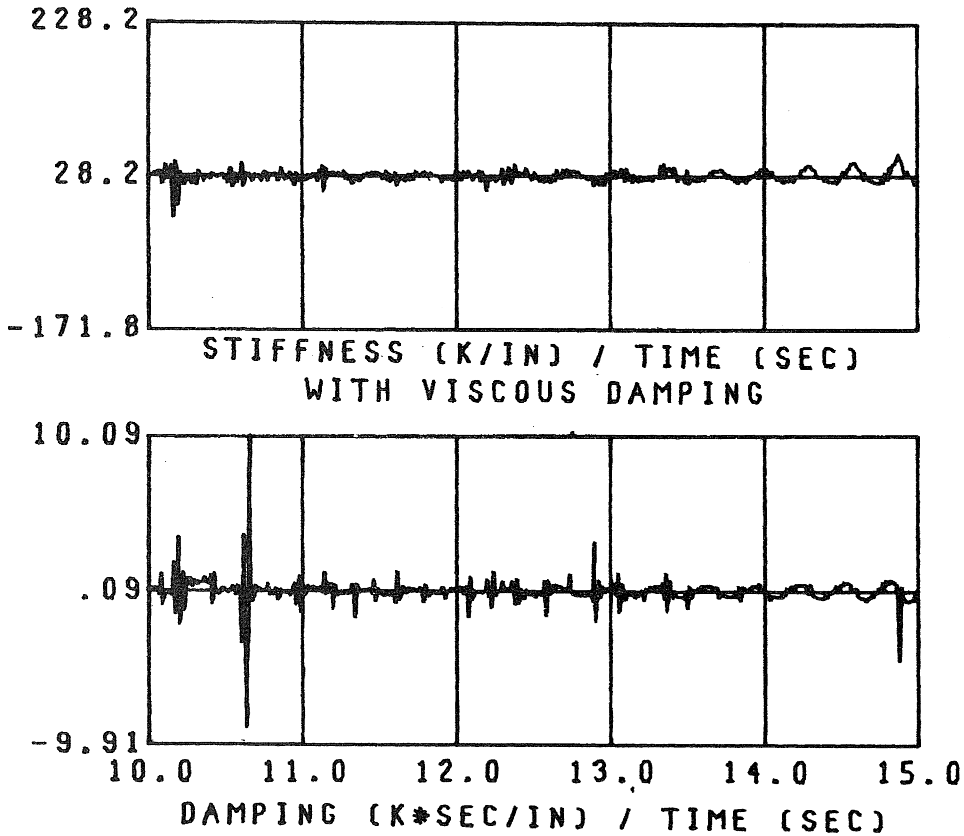


Figure 2. C_i and K_i Directly Obtained From the Experiment of Figure 1.

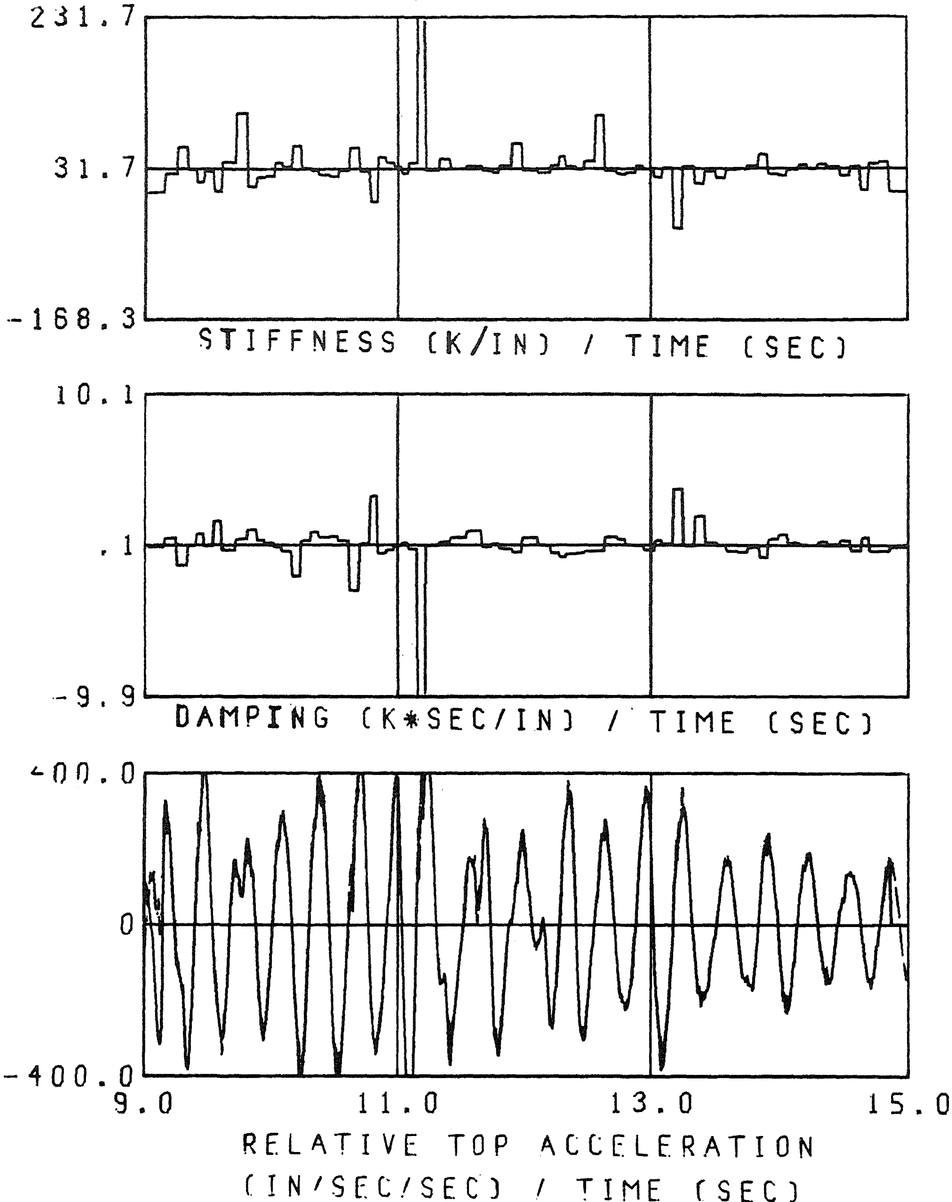


Figure 3. $C_1^i, C_2^i, K_1^i, K_2^i$ obtained by Identification Over Individual Cycles of Motion (i).

- - - measured acceleration

_____ simulated acceleration