DAMPING OF A SOIL-STRUCTURE SYSTEM

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SUMMARY

The influence of energy dissipation in the soil on the damping of a soil-structure system is studied. The model of the system is regarded as a lumped mass system with a foundation that can sway and rock in the soil. It is assumed that the damping mechanism for both soil and structure is of a hysteretic type and is included in the complex stiffnesses of the components. The participation of the components in the modal damping of the stiffness is illustrated graphically. A simple formula for evaluation of the damping of the first mode of vibration is proposed.

INTRODUCTION

Damping of a real structure can be significantly affected by the soil-structure interaction. The influence of the interaction is different for each mode of vibration. The energy dissipation, due to radiation and hysteretic action of the foundation in the soil, is different for swaying and rocking. This paper presents some results on the participation of the components in the overall damping of each mode of vibration of the soil-structure system, and proposes simple expressions for evaluation of the damping of the system. In the formulation of the model of the system hysteretic damping is used for both the soil and the structure, not only because it is physically realistic, but also because its presentation by complex stiffness gives the same results as those obtained by the summation of the energy dissipation in the components of the system.

MODEL OF A SOIL-STRUCTURE SYSTEM

The idealization of an actual soil-structure system is shown on Fig. 1 as a lumped mass multy-degree-of-freedom system, which can sway and rock in the ground. The energy dissipation or the damping of each story and that of the soil for swaying and rocking can be expressed by their respective complex stiffnesses,

$$K_j^* = K_j(1 + i\alpha_j) \qquad (1)$$

where α_j is a hysteretic damping coefficient of the j-th element of the system.

The equation of motion of the fixed base structure is:

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$$[M]\{\ddot{x}'^*\}+[K^*]\{x'^*\}=-\ddot{x}_0^*(t)[M]\{1\} \tag{2}$$

where: $[M]$ and $[K^*]$ are mass matrix and complex stiffness matrix of the fixed base structure, $\{\ddot{x}'^*\}$, $\{x'^*\}$ – complex acceleration and displacement vectors of the fixed base structure, $\ddot{x}_0^*(t)$ – ground acceleration in complex form. In case the same structure is supported on a flexible foundation (see Fig. 1), two degrees of freedom will be added: the horizontal translation of the base and the rotation of the base about a horizontal axis. The equations of motion of the soil-structure system are:

$$\begin{aligned} m_f(\ddot{x}_f^* + \ddot{x}_0^*) + K_f^* x_f^* &= -\{1\}^T[M](\{\ddot{x}^*\} + \ddot{x}_0^*\{1\}) \\ [M]\{\ddot{x}^*\} + [K^*]\{x'^*\} &= -\ddot{x}_0^*[M]\{1\} \\ I_\varphi \ddot{\varphi}^* + K_\varphi^* \varphi^* &= -\{h\}^T[M](\{\ddot{x}^*\} + \ddot{x}_0^*\{1\}) \end{aligned} \tag{3}$$

where: m_f and I_φ – mass and mass moment of inertia of the foundation. From Fig. 1 can be seen that

$$\{x\} = x_f\{1\} + \varphi\{h\} + \{x'\} \tag{4}$$

Substitution from Eq. 4 into Eq. 3 leads to

$$[M]_s\{\ddot{x}^*\}_s + [K^*]_s\{x^*\}_s = -\ddot{x}_0^*[M]_s\{B\} \tag{5}$$

where: $[M]_s$ – mass matrix of the soil-structure system, $[K^*]_s$ – complex stiffness matrix of the soil-structure system, $\{\ddot{x}^*\}_s$, $\{x^*\}_s$ – complex acceleration and displacement vectors of the system, $\{B\}$ – vector of external excitation,

$$[M]_s = \left[\begin{array}{c|c|c} m_f + \{1\}^T[M]\{1\} & \{1\}^T[M] & \{1\}^T[M]\{h\} \\ \hline [M]\{1\} & [M] & [M]\{h\} \\ \hline \{h\}^T[M]\{1\} & \{h\}^T[M] & I_\varphi + \{h\}^T[M]\{h\} \end{array}\right]$$

$$[K^*]_s = \left[\begin{array}{c|c|c} K_f^* & & \\ \hline & [K^*] & \\ \hline & & K_\varphi^* \end{array}\right], \quad \{\ddot{x}^*\}_s = \left\{\begin{array}{c} \ddot{x}_f^* \\ \hline \{\ddot{x}'^*\} \\ \hline \ddot{\varphi}^* \end{array}\right\}, \quad \{x^*\}_s = \left\{\begin{array}{c} x_f^* \\ \hline \{x'^*\} \\ \hline \varphi^* \end{array}\right\}, \quad \{B\} = \left\{\begin{array}{c} 1 \\ \hline \{0\} \\ \hline 0 \end{array}\right\}$$

It is clear that the system, presented by Eq. 5 does not possess classical normal modes. If the assumed mode shapes are orthogonal, Eq.5 can be decoupled by the following coordinate transformation:

$$\{x^*\}_s = [A^*]\{\eta^*\} \tag{6}$$

where $[A^*]$ is matrix of complex mode shapes. It is not difficult to prove that the complex mode shapes are actually orthogonal and the j-th decoupled differential equation of motion is

$$\ddot{\eta}_j^* + \omega_j^{*2}\eta_j = -\xi_j^*\ddot{x}_0^* \tag{7}$$

The complex natural frequency ω_j^* can be determined by the solution of the homogenous part of Eq.5, which leads to the well known classical eigen value problem, i.e.

$$\left|[K^*]_s - \omega^{*2}[M]_s\right| = 0 \qquad (8)$$

The natural frequency, ω_j, and the corresponding modal damping coefficient, α_j, of the j-th mode of vibration are included into the complex eigenvalue of the same mode,

$$\omega_j^* = \omega_j(1 + i\alpha_j). \qquad (9)$$

An advantage of the considered theoretical model is the possibility to find the modal damping together with the natural frequency of any mode of vibration of the system.

METHOD OF ANALYSIS

In order to study the participation of the components in the overall damping of each mode of vibration of the soil-structure system, a model of a three story structure is chosen (see Fig.2). To make the analysis more clear some simplifications are involved. It is assumed that all masses of the structure as well as all story heights are equal. The mass of the foundation is expressed by the masses of the structure ($m_f = rm$). Some discrete values of r are used in computation of modal damping. The mass moment of inertia of the system about the horizontal axis of rotation in the soil is:

$$I = I_\varphi + \{h\}^T[M]\{h\} \qquad (10)$$

The mass moment of inertia of the foundation, I_φ, is too small in comparison with the second term of Eq.10 and it is neglected. By this approximation the number of the modes of vibration decrease by one.

The structure is supposed to be of shear type and all stories have equal stiffnesses and damping coefficients. Thus, $K_1^* = K_2^* = K_3^* = K(1+i\alpha_s)$. Nondimentional stiffness ratio coefficients are introduced:

$$\lambda = \frac{Kh^2}{3K_\varphi} = \frac{\varphi h}{\delta}\,; \qquad \bar{\lambda} = \frac{K}{3K_f} = \frac{\delta_f}{\delta} \qquad (11)$$

where: K - real stiffness of the stories, K_φ, K_f - real stiffnesses of the ground for rotation and horizontal translation, δ - unit horizontal displacement at the top of the fixed base structure, φ - rotation of the base, due to unit force, applied at the top, δ_f - unit horizontal displacement of the foundation, h - height of the structure.

The ratio between $\bar{\lambda}$ and λ, $u = \bar{\lambda}/\lambda$, is a constant quantity for a given structure, founded on certain soil conditions. It is a function of the ratio between the equivalent radius of the foundation and the height of the structure. Three values of u are chosen - 0,05, 0,1 and 0,2, corresponding to the possible variety of dimentions of real structures and values of Poisson's ratio.

The expansion of Eq.8 leads to a n-th order equation in ω^{*2}. This equation can be divided into two equations. The first one is for the real part and the second is for the imaginary. The roots of the first equation are the squares of the real natural frequencies of the system modes. Each modal frequency, ω_i , is associated with the modal damping coefficient, α_i , which can be calculated by the second equation. The simplifying assumptions mentioned above lead to simple expressions for the modal natural frequencies and damping coefficients:

$$\omega_i = p_i(\lambda)\frac{K}{m} \qquad (12)$$

and

$$\alpha_i = a_i(\lambda)\alpha_s + b_i(\lambda)\alpha_\varphi + c_i(\lambda)\alpha_f \qquad (13)$$

where: K, m - stiffness and lumped mass of the stories, $p_i(\lambda)$ - frequency factor, function of the stiffness ratio λ , $a_i(\lambda)$, $b_i(\lambda)$, $c_i(\lambda)$ - participation factors of the damping values of the components, function of the stiffness ratio - λ , α_s - damping coefficients of the structure, α_φ, α_f - damping coefficients for rocking and swaying of the foundation into the soil.

ANALYSIS OF THE RESULTS

The solution of Eq. 8 is performed for the consequent values of λ from $\lambda = 0$ to $\lambda = 5$. The case of $\lambda = 0$ corresponds to the fixed-base structure. When $\lambda = 5$, the displacement at the top of the structure, due to base rotation is about 83,3% of the total displacement. This case corresponds to a comparatively rigid structure, founded on a soft soil. The results of the computation are presented graphically. Fig. 3 illustrates the change of the frequency factor, p_i , as a function of the stiffness ratio, λ , for the values of u = 0,1. The solid line is for r = 1 and the dash line is for r = 3. It is evident that the mass ratio r plays an important role for the change of the modal frequencies of the higher modes of vibration, due to the soil-structure interaction. As it was mentioned above the number of modes is reduced from five to four. The fifth mode of vibration will appear, if the mass moment of inertia of the foundation is not neglected. The natural frequency of this mode is comparatively high and is not of practical interest.

The participation factors a_i, b_i and c_i as a function of the stiffness ratio, λ , are shown on Fig.4 and Fig. 5 for the mass ratio r = 1 and r = 3 respectively. The sum of a_i, b_i and c_i is equal to one for every mode of vibration. Therefore, b_i and c_i are presented as addends to a_i . It can be seen that the participation of the soil damping into the overall damping of the soil-structure system increases with the increase of λ for the first and second modes of vibration. For the first mode of vibration the influence of the rocking in the soil is predominant, while the damping of the second mode is more affected by the damping of the swaying motion. The participation factor, b_i, which accounts for the damping of the rocking motion is comparatively small for the higher modes and practically it can be neglected. The degree of the curves $a_i(\lambda)$ or $c_i(\lambda)$ increases with the increase of the number of the mode. The $c_3(\lambda)$ for the third mode of vibration has an extremum for the same value of λ, for which $p_s(\lambda)$ has an inflection point. This corresponds to the maximum horizontal translation of the foundation. The nature of the participation factors for the fourth mode of vibration is different from that of the other modes. The participation of the soil damping starts from maximum and decreases with the increase of λ. For small values of λ this is related to the predominant deformation in the soil rather than in the structure. It can be seen that the participation factors are much affected by the increase of the mass ratio r, i.e. by the increase of the foundation mass for all modes of vibration with the exception of the first mode. Further investigations show that the participation factors for the first mode of vibration are not influenced by the number of the stories of the superstructure, in case all stories are assumed to have equal stiffness and damping coefficients.

For practical application of the presented results it can be assumed that damping of the first mode of vibration is due to energy dissipation in the structure and in the ground, only for rocking motion. The swaying motion can be neglected. Thus, the order of the mass matrix and the stiffness matrix of the soil-structure system will decrease by one. Assuming that the fixed-base structure has classical normal modes and applying a coordinate transformation, the mass matrix and the stiffness matrix can be replaced by the generalized mass M_1 and the generalized stiffness K_1 of the first mode of vibration. The solution of Eq.8 leads to a simple expression for evaluation of the modal damping of the first mode of vibration of the soil-structure system:

$$\alpha = \frac{\alpha_s T_s^2 + \alpha_\varphi T_\varphi^2}{T_s^2 + T_\varphi^2} \qquad (14)$$

where: α - damping of the system, α_s - damping of the fixed-base structure, α_φ - damping of the soil for rocking motions, T_s - first mode natural period of the fixed-base structure, T_φ - natural period of rocking motion of the structure as a rigid body.

For the higher modes of vibration the rocking motion can be neglected, but this simplification does not lead to a simple type of equation, such as Eq.14.

CONCLUSION

The analysis of the results for the participation of the damping of the components in the overall damping of the soil-structure system, based on a hysteretic damping model, can explain the variety of the modal damping values.

The proposed Eq.14 for evaluation of the first mode damping value can be used in design practice.

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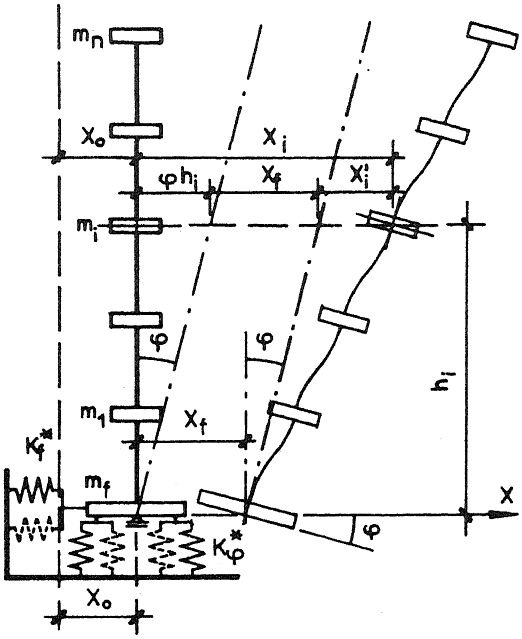

Fig. 1

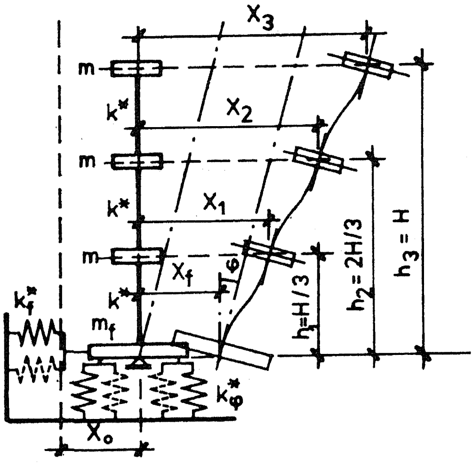

Fig. 2

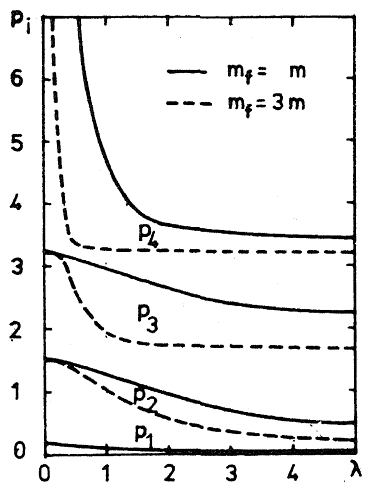

Fig. 3

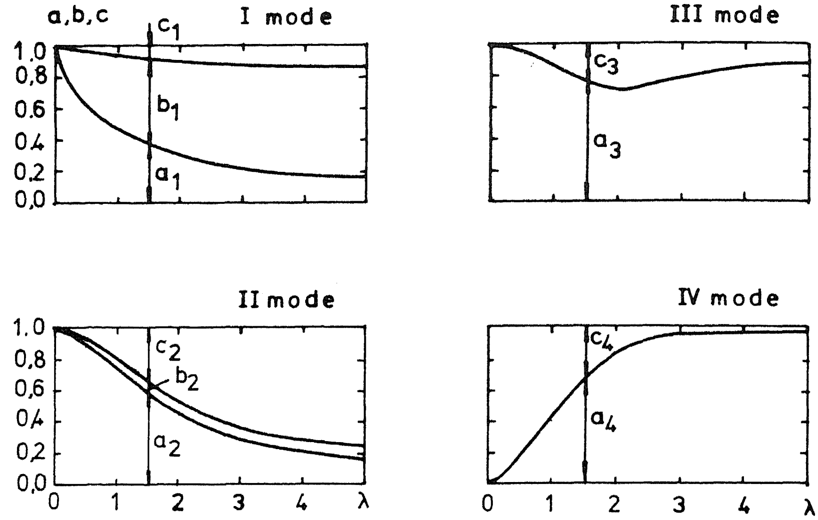

Fig.4 Participation factors for $m_f = m$

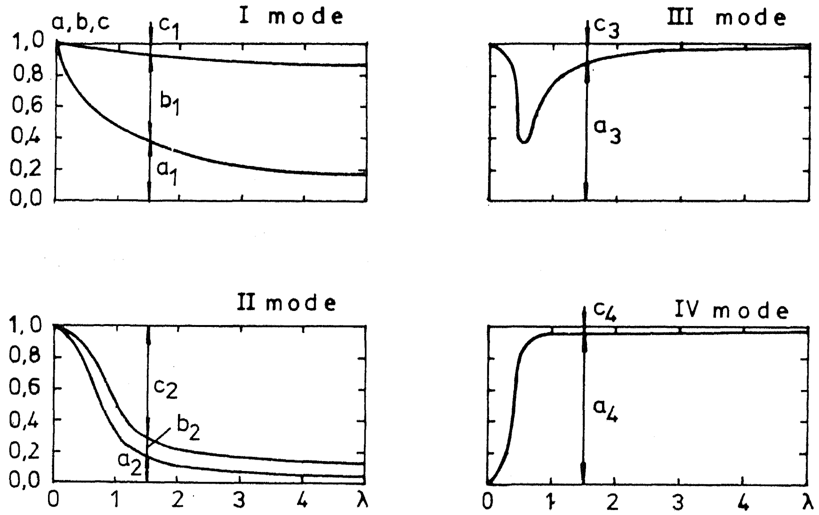

Fig.5 Participation factors for $m_f = 3m$