SEISMIC ANALYSIS OF A DOMED CYLINDRICAL LIQUID STORAGE TANK

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SUMMARY

The problem under consideration is that of an elastic cylindrical liquid storage tank with an elastic, rotationally symmetric dome of arbitrary contour. The tank is attached to a rigid base slab and is filled to an arbitrary depth with a perfect, incompressible liquid. A finite element analysis is presented for the free vibrations of the coupled liquid-elastic system permitting determination of natural frequencies and mode shapes. The response of a partially or completely filled tank to an earthquake is also determined through use of finite elements.

BACKGROUND

Although experimental and simplified analytical treatments of liquid storage tanks subject to seismic excitation were offered as early as 1934, it was not until 1957 when G.W. Housner [1] offered expressions for the hydrodynamic pressures developed on the walls of a rigid tank subject to horizontal acceleration. Very recently the effect of tank flexibility on hydrodynamic forces induced in the tank was treated by A.S. Veletsos and J.Y. Yang [2]. Comprehensive experiments on both tall as well as relatively short (broad) tanks having either an open top or a flat roof have been reported by D.P. Clough [3] and A. Niwa [4]. Additional experiments as well as analysis have also been offered by K. Fujita [5].

ANALYSIS

Let us consider the small deformations of a shell of revolution whose axis is vertical and let the middle surface displacements be designated by u,v, and w in the meridional, tangential, and inward radial directions respectively. Also, ϕ is the slope between the vertical axis and a tangent to the shell in the meridional direction, s is the meridional distance along the element, and θ is a latitude coordinate as indicated in Fig. 1. The middle surface strain-displacement relations are:

$$\varepsilon_s = \frac{\partial u}{\partial s} - \phi' w \qquad (1)$$

$$\varepsilon_\theta = \frac{1}{r}\left[\frac{\partial v}{\partial \theta} + u\sin\phi + w\cos\phi\right] \qquad (2)$$

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$$\varepsilon_{s\theta} = \frac{1}{r}\left(\frac{\partial u}{\partial \theta}\right) - \frac{v}{r}\sin\phi + \frac{\partial v}{\partial s} \quad (3)$$

where

$$\phi' = \partial\phi/\partial s.$$

The shell theory due to Novozhilov is taken to be suitable for use here and the shell of revolution is idealized by a sequence of curved ring elements as indicated in Figure 1 with the slope of each element being given by

$$\phi = a_1 + a_2 s + a_3 s^2 \quad (4)$$

where the a_i are coefficients that are evaluated by requiring the slopes of the idealization and the real shell to be identical at nodal points. The middle surface displacements are represented by polynomials in the meridional direction(s) and a Fourier series in the circumferential (θ) direction. Then, the stress resultants may be computed from these displacement relations and the element stiffness matrices are found from the strain energy; the element mass matrices (including the effects of rotary inertia) are found from the kinetic energy expressed in terms of middle surface displacements.

The liquid in the cylindrical tank is discretized into annular elements of rectangular cross-section. These elements are formed by the intersection of concentric annular cylindrical surfaces with a set of horizontal planes. This three dimensional problem can then be transformed into a two dimensional one by developing the pressure p in a Fourier series in the circumferential direction. Here, the forced motion of the slab supported tank when excited by horizontal ground accelerations can be reasonably well described by consideration of only the first circumferential harmonic. Thus, the two dimensional problem involves determination of pressures at the nodes of a rectangular element lying in a diametral plane of the shell.

The equations describing liquid motion are those due to Laplace and Bernoulli together with boundary conditions describing the liquid free surface as well as compatibility of velocities of the liquid and an adjacent element in the elastic tank wall. An appropriate variational functional for the liquid is

$$I = \int_{t_1}^{t_2} (T - \Pi - W)dt \quad (5)$$

where T, Π, and W represent the kinetic energy, the potential energy of the liquid, and the work done on the liquid respectively. These are given by:

$$T = (1/2)\rho_f \int_V \nabla\Phi \cdot \nabla\Phi dv \quad (6) \qquad \Pi = (1/2)\int_F \xi(\rho_f\, g\xi)\, ds \quad (7)$$

$$W = \int_{\Sigma} \rho_f (\frac{\partial w}{\partial t}) \Phi ds \qquad (8)$$

where ρ_f denotes liquid density and ξ is the deviation of the liquid elevation from the static configuration. In the present investigation, in (5), it is most convenient to investigate the dynamic problem in terms of the liquid dynamic pressure p. The linear shape function assumed for variation of liquid pressures between nodes of the rectangular liquid element makes it possible to determine the various terms in (5) which include the liquid element stiffness matrix and mass matrix as well as the shell-liquid coupling matrix.

This finite element formulation leads to the coupled natural frequencies of the liquid-tank system. The response of the system to base excitation is found by partitioning the shell (cylinder plus dome) generalized displacement vectors into components associated with the known displacements at the supporting slab with all other components being associated with the off-base nodes. Such a formulation is well-suited to being written in terms of partitioned matrices representing the equations of motion.

In the present investigation four separate computer programs have been developed to yield (a) the mass and stiffness matrices of the shell, (b) the added mass matrix due to the liquid, (c) the frequencies and mode shapes of the liquid-elastic tank system, and (d) the nodal displacements, force and moment resultants in the elements due to base excitation by an arbitrary ground motion. These programs, in FORTRAN-IV, are available upon request from the authors. The case of a cylindrical tank of steel, wall thickness 1 inch, radius 720 inches, altitude 480 inches, having a torispherical dome of central altitude of 347.5 inches, and also 1 inch thick was considered to be filled with water to the top of the cylindrical portion. The base was clamped and subject to the artificial earthquake PSEQGN. The finite element program indicated the radial displacement at the junction of the top of the cylindrical tank with the bottom of the dome as indicated in Figure 2.

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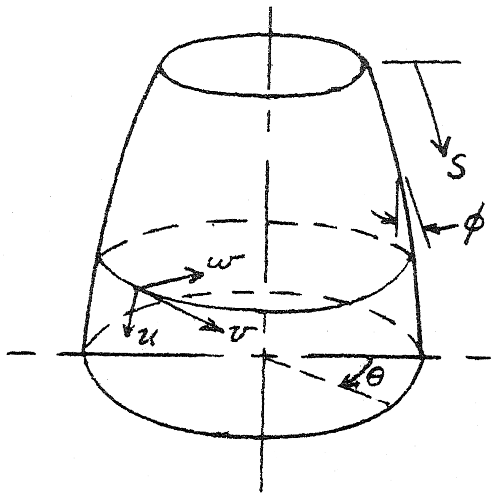

FIGURE 1. Coordinate System for Shell of Revolution

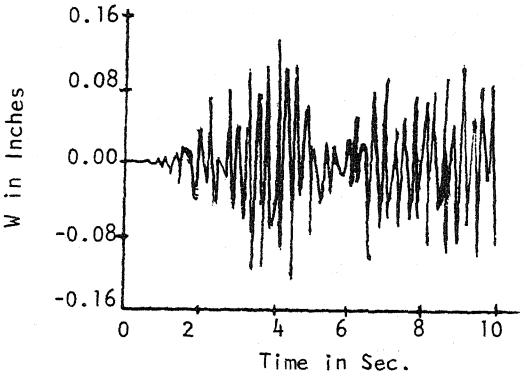

FIGURE 2
Radial Displacement at Node 21