DYNAMICS OF REINFORCED CONCRETE FRAMES SUBJECT TO HORIZONTAL AND VERTICAL GROUND MOTION

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SUMMARY

Effect of the vertical groung motion on the dynamic behaviours of reinforced concrete frames are examined. The first part relates the experiment to obtain the restorong force characteristic which considers an interaction between the horizontal and the vertical displacement. Earthquake response analysis in the second part shows that there is the case where the axial force in the column becomes more than 1.5 times of that by dead load, which may increase or decrease the maximum horizontal displacement, maximum shearing force in the column and other horizontal responses about 20% of that with no consideration on the vertical ground motion.

INTRODUCTION

Earthquake motion is essentially a three dimensional motion. A considerable of recorded, especially near the epicenter, accompanied intense vertical motion. In an aseismic design for such earthquakes, it seems necessary to take a consideration on the effect of the vertical ground motion,which have usually been neglected. Nonlinear response of structures against simultaneous action of the horizontal and the vertical ground motion will only be calculated exactly by considering an interaction between the displacements in both directions.

In the case of reinforced concrete frames, this interaction might be considers in two ways. First, the restoring force characteristic in the horizontal direction strongly depends on the axial force in the column. Secondly, the restoring force characteristic in the vertical direction depends on the horizontal displacement. In this investigation, these interactios are exprimentally examind and, applying the result, dynamic response of reinforced concrete (RC) frames subject to ground motion in both the horizontal and vertical directions are calculated and compared with that subject to horizontal motion only.

OUTLINE OF THE EXPERIMENT

Restoring force characteristic of the RC frames, whether in the horizontal direction or in the vertical direction, can be represented by the product of a skeleton curve and a hysteresis curve[1]. The purpose of the experiment is to get a rule of the effect of the fluctuation of the axial force in the column on both curves of horizontal restoring force characteristic and to get one of the effect of the horizontal displacement on both curves of restoring force characteristic in the vertical direction.

Fig.1 shows the test structure, of which reinforcement is so designed as to bending collapse of the column precede the shearing collapse of the column or the collapse of the beam. Fig.2 shows the loading system and Fig.3 shows the measuring system. All displacements are measured by dial gauge. Steel bar, which is simply supported at the center of joint, is to avoid the effect of the rotation of the column.

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Experiment is classified into six series as:

1. Monotonous loading in the horizontal direction with constant vertical load. Magnitude of the vertical load applied to a column is N=0, N=3t and N=6t, which correspond to the ratio of the applied load to the maximum compressive load being $\bar{N}=0$, $\bar{N}=1/6$ and $\bar{N}=1/3$ respectively.
2. Horizontal loading test with application of fluctuating vertical load, magnitude of which is N=0, N=3t and N=6t.
3. Repeated horizontal loading test with constant vertical load of N=0, N=3t and N=6t. Amplitude of the horizontal displacement is 5mm, 15mm and 30mm. Number of repetition in each amplitude is three.
4. Repeated horizontal loading test with application of fluctuating vertical load, magnitude of which is N=0, N=3t and N=6t.
5. Repeated vertical loading test at four positions of horizontal displacement, magnitude of which is 0, 5mm, 15mm and 30mm.
6. Mixed repeated loading test in horizontal and vertical directions. During one loading in horizontal direction, in each amplitude, vertical load is repeated four times. Horizontal displacement where the vertical load is repeated is 0.

In Fig.4, fine line and thick line respectively represents the experiment 1 and 2. Such symbols as $0\rightarrow3$, $3\rightarrow6$ on the thick line mean that the vertical load is changed from 0 to 3t and from 3t to 6t. It is seen from the Fig.1 that, when the magnitude of the vertical load is changed, skeleton curve approaches gradually, with increase in horizontal displacement, one which is obtained from the experiment with constant vertical load of that changed magnitude.

In each amplitude of the experiment 4, during the second cycle of horizontal loading, magnitude of the vertical load is arbitrary changed. In Fig.5 and Fig.6, the second hysteresis curves of experiment 3 and 4 are shown. Amplitude of the horizontal displacement is 5mm and 15mm. From these figures, it is seen that, when the magnitude of the vertical load is changed, hysteresis curve approaches gradually, with a change in horizontal load, one which is obtained from the experiment with constant vertical load of that changed magnitude.

It is seen from Fig.7 to Fig.10, which show the results of the experiment 5, that the stiffness of both skeleton curve and hysteresis curve in the vertical direction decreases with an increase of the horizontal displacement and that the hysteresis curve is almost like a straight line with slight hysteretic damping, regardless of the horizontal displacement and the magnitude of the vertical load.

Hysteresis curves in the vertical direction which are obtained from the experiment 6 are shown from Fig.11 to Fig.14. It is seen that, if the current horizontal displacement is zero, as the maximum horizontal displacement becomes large, hiyteresis curve in the vertical direction exhibits a hard spring characteristic. It is also seen that the shape of the hysteresis curve does not depend on the number of repetition of the horizontal loading, the amplitude of the vertical load and the number of the vertical loading.

Fig.15 shows the scheme of the stiffness degradation of the hysteresis curve in the horizontal direction, which is defined as:

$$\gamma_i = K_i / K_0 \qquad (1)$$

where, K_0 and K_i is the stiffness of the first and the i-th hysteresis curve. From Fig.16 to Fig.18, stiffness degradation in the experiment 3 (N =3t) and 6 are shown. Solid line shows the experiment 3 and dotted line shows the experiment 6. Ordinate is the stiffness degradation and abscissa is the number of repetition of the horizontal load. From these figures, it seems that the stiffness degradation decreases due to the repetition of the vertical load.

ANALYTICAL MODEL OF RESTORING FORCE CHARACTERISTIC IN THE HORIZONTAL DIRECTION

Restoring force characteristic in the horizontal direction is shown in Fig.19. Restoring force Q will be represented as the product of the skeleton curve F and the normalized hysteresis curve f or g. Q is expressed in six ways depending on the state of the response as follows:

$$Q = F_1(x) \quad \text{——} ①$$
$$Q = F_2(x) \quad \text{——} ②$$
$$Q = F_1(x_p)\cdot f_1(x/x_p) \quad \text{——} ③$$
$$Q = F_1(x_p)\cdot f_2(x/x_p) \quad \text{——} ④$$
$$Q = F_1(x_p)\cdot g_1(x/x_p) \quad \text{——} ⑤$$
$$Q = F_1(x_p)\cdot g_2(x/x_p) \quad \text{——} ⑥$$
$$\text{——————} \quad (2)$$

where, x is the horizontal displacement and x_p is an amplitude of the hysteresis curve. In Eq.2, ①~⑥ correspond to the state of the response as shown in Fig.19. Functions F, f and g ought to be expressed as a function of normalized vertical load $\bar{N}$. In Fig.19, $Q_{\bar{N}}$ is the maximum force when the applied vertical load is $\bar{N}$ and $X_{\bar{N}}$ is correspondig maximum force displacement.

Skeleton curve is given as:

$$\left.\begin{array}{l} F_1(x) \\ F_2(x) \end{array}\right\} = \pm\gamma\left(e^{\mp\alpha x} - e^{\mp\beta x} \right) \quad \text{———} (3)$$

where, α, β and γ are functions of $Q_{\bar{N}}$, $X_{\bar{N}}$ and $k_{\bar{N}}$ which determines the descending slope of the curve.

$$\alpha = \log k_{\bar{N}} / X_{\bar{N}}(k_{\bar{N}} - 1) \quad \text{———} (4)$$
$$\beta = k_{\bar{N}}\alpha \quad \text{———} (5)$$
$$\gamma = Q_{\bar{N}} / \left(e^{-\alpha X_{\bar{N}}} - e^{-\beta X_{\bar{N}}} \right) \quad \text{———} (6)$$

$X_{\bar{N}}$, $Q_{\bar{N}}$ and $k_{\bar{N}}$ are function of $\bar{N}$ and are given by next equations.

$$X_{\bar{N}} = X_0\ (-1.26\ \bar{N}^2 - 0.33\ \bar{N} + 1.0) \quad \text{———} (7)$$
$$Q_{\bar{N}} = Q_0\ \left(-(\bar{m}_{max} - 1.0)\ \left(\frac{\bar{N}}{0.4} - 1.0\right)^2 + \bar{m}_{max} \right) \quad \text{———} (8)$$
$$k_{\bar{N}} = 60.0\ (8.63\bar{N}^2 - 5.75\ \bar{N} + 1.0) \quad \text{———} (9)$$

where, X_0 and Q_0 are values of X and Q when $\bar{N}$=0.

Eq.7 and Eq.9 is based on the present experiment but Eq.8 is based on the A.I.J. 's formula[2] for the ultimate bending moment of the RC columns. Parameter $\bar{m}_{max}$ in Eq.8 is the ratio of ultimate bending moment with $\bar{N}$=1/3 to that with $\bar{N}$=0. $\bar{m}$ takes various values depending on the strength of the reinforcement, strength of the reinforcement and amount of reinforcement as is shown in Fig.20.

Hysteresys curve is given by the product of the skeleton curve and the normalized hysteresis curve. Function f, which represents a symmetric hysteresis curve, is expressed as:

$$\left.\begin{array}{l} f_1(x) \\ f_2(x) \end{array}\right\} = \mp A\,(x/x_p)^4 + B\,(x/x_p)^3 + (1-B)(x/x_p) \pm A \qquad (10)$$

Coefficient A and B in Eq.10 are related to the eqivalent viscous damping factor h_{eq} and slipping stiffness K_s as:

$$h_{eq} = 8A / 5\pi \qquad (11)$$

$$K_s = 1 - B \qquad (12)$$

So coefficient A and B are experimentaly determined.[4] A and B are functions of normalized vertical load $\bar{N}$ and normalized amplitude of the hysteresis curve $\bar{X}=x_p/X_{\bar{N}}$. A and B is given by

$$A(\bar{N},\bar{X}) = (5\pi/8)\{(0.36\,\bar{N} + 0.04)(\bar{X} - 0.5) + 0.1\} \qquad (13)$$

$$B(\bar{N},\bar{X}) = 1 + \{40.5(\bar{N} - 1/3)^4 + 1\}\{(\bar{X}/(3.0\,\bar{N} + 1.5)\}^2 - 1 \quad (\bar{X} < 2.5) \qquad (14)$$

$$B(\bar{N},\bar{X}) = 1.0 \qquad (X > 2.5). \qquad (15)$$

Function g, which represents the non-symmetric hysteresis curve is

$$\left.\begin{array}{l} g_1(x) \\ g_2(x) \end{array}\right\} = \left\{ \mp A\left(\frac{2(\bar{x} \mp 1)}{1 \pm \bar{x}_0} \pm 1\right)^4 + B\left(\frac{2(\bar{x} \mp 1)}{1 \pm \bar{x}_0} \pm 1\right)^3 + (1-B)\left(\frac{2(\bar{x} \mp 1)}{1 \pm \bar{x}_0} \pm 1\right) \pm (A - 1) \right\} \frac{1 \pm \bar{y}_0}{2} \pm 1 \qquad (16)$$

where,

$$\bar{x}=x/x_p, \quad \bar{y}=y/F(x_p), \quad \bar{x}_0=x_0/x_p, \quad \bar{y}_0=y_0/F(x_p) \qquad (17)$$

and x_0 and y_0 are coordinates where the velocity is reversed.

Stiffness degradation, which is defined in Eq.1, is conveniently expressed as[3]

$$\gamma_i = 1 / i^n \qquad (18)$$

where i is the number of repetition of the horizontal loading and n will be determined experimentally. In the present analysis, effect of the fluctuation of the vertical load on the stiffness degradation of restoring force characteristic in the horizontal direction is assumed to be able to neglect and is given by next simple relation, which contains normalised amplitude only.

$$n = 0.135\,(\bar{X} + 1) \qquad (19)$$

Fig.21 shows the scheme of the stiffness degradation in the analysis. It is assumed that the stiffness degradation occurs when the velocity is reversed on the hysteresis curve. Considering that Eq.18 is defined for the repeated loading with constant displacement amplitude or loading amplitude, number of repetition i is defined as the ratio of the total displacement on the hysteresis curve to four times of amplitude of the hysteresis curve.

ANALYTICAL MODEL OF RESTORING FORCE CHARACTERISTIC IN THE VERTICAL DIRECTION

Restoring force characteristic in the vertical direction is also represented by skeleton curve and hysteresis curve. One of the conspicuous point about these curves is that the origin is always moving according as the horizontal displacement changes. However, as it is difficult, at present, to take this effect in the analysis, origin of the skeleton curve is assumed to be fixed.

Skeleton curves, which are obtained in experiment 5, are normalized and shown together in Fig.22. An ordinate is normalized vertical load $\bar{N}$ and abscissa is strain in the vertical direction ε. Relation between $\bar{N}$ and ε is given by

$$\bar{N} = \sqrt{\frac{1}{a}\left(\varepsilon + \frac{b^2}{4a}\right)} - \frac{b}{2a} \qquad (20)$$

where, a and b are functions of horizontal displacement and are given by

$$a = (6.5\,\bar{X}_M + 0.45)\times 10^{-3}, \qquad b = 1.4 \times 10^{-3} \qquad (21)$$

In Eq.21, $\bar{X}_M$ is normalized maximum horizontal displacement i.e. the ratio of the maximum horizontal displacement to the maximum force displacement with application of the vertical load of $\bar{N}$=1/6.

In the experiment, hysteresis curve is obtained for the case where horizontal displacement is zero or is equalt to the maximum horizontal displacement, therefore, hysteresis curve for the intermediate horizontal displacement is linearly interpolated. Shape of the hysteresis curve changes from a straight line to a slipping curved line as the current horizontal displacement comes near to zero. It is not difficult to represent a hysteresis curve as a curved line, however, considering that it is time comsuming task for the computer analysis to get an intersection of the hysteresis curve and the skeleton curve, and considering that the axial force response shows little deference whether the hysteresis curve is represented as a straight line or the slipping curved line, hysteresis curve is transformed to a straight line and the hysteretic damping is considered as an equivalent viscous damping. It is seen from Fig.7 to Fig.14 that the hysteretic damping in the vertical direction is very small and is always about 5%, not depending on the horizontal displacement.

Stiffness of the hysteresis curve is given by

$$K_v = \frac{N_{max}}{3\,H\,\varepsilon_0} \qquad (22)$$

where, ε_0 is the strain in the vertical direction on the hysteresis curve with the application of vertical load of $\bar{N}$=1/3. H is structural height and N_{max} is the maximum compressive force. Then, relation between axial force N and vertical displacement z is obtained as:

$$N = \frac{N_{max}}{3\,H\,\varepsilon_0}\left(z - H(a\bar{N}_p^2 + b\bar{N}_p - 3\,\varepsilon_0) \right) \qquad (23)$$

where, $\bar{N}_p$ is the normalized axial force correspond to the intersection of the skeleton curve and hysteresis curve. ε_0 is determined from the experiment as:

$$\varepsilon_0(\bar{X}_M, x) = \left(\left(\frac{-17}{0.533\bar{X}_M + 1} + 22\right) + \left(\frac{17}{0.533\bar{X}_M + 1} - \frac{5}{1.8\bar{X}_M + 1} - 12\right)\frac{x}{X_M} \right) \times 10^{-4} \qquad (24)$$

where, $\bar{X}_M$ is normalized maximum horizontal displacement and X_M is the maximum horizontal displacement. Diagram of the analytical restoring force characteristic in the vertical direction is shown in Fig.23.

EXAMPLE

RC frame is transformed to a single degree of freedom model, of which structural parameters are shown in Table 1. Normalized axial force in the column with application of the dead load is $\bar{N}=1/6$. Earthquake data applied are El. Centro 1940 (EW, UD) and Taft 1952 (EW, UD). Maximum acceleration of the horizontal and the vertical component is amplified to 500 gal and 300 gal respectively. Numerical integration is carried out by Runge-Kutta Method and integration mesh is set equal to 1/50 of the shoter natural period.

Response is calculated for the three cases. In the first case, ground motion is applied only in the horizontal direction. Second and the third cases consider an vertical ground motion as well as horizontal ground motion. In the second case, restoring force characteristic in the vertical direction is assumed to be linear, the stiffness of which is set to the stiffness of the skeleton curve when dead load is applied. In the third case, restoring force characteristic in both the horizontal and the vertical directions is nonlinear.

Maximum responses for the three cases are shown in Table 2. Deference of the vertical response between the case 2 and case 3 is quite large, especially for the application of El. Centro earthquake. Increase or decrease of horizontal response due to the application of vertical ground motion comes near to -20% ~ +20 of those with no consideration on the fluctuating vertical load. It seems that the effect of the vertical ground motion appears more distinctly in case 2 than in case 3.

CONCLUSION

Effect of the mutual interaction between the horizontal displacement and the vertical displacement on the dynamics of RC frame is examined. There is the case where this effect can not be neglected, especially for the frames which are composed of rather strong concrete with rather poor longitudinal reinforcement. In such frames, it is desirable to increase an amount of transverse reinforcement in the columns.

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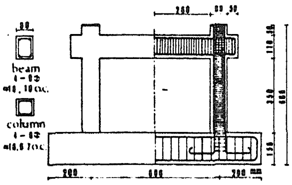

Fig.1
Test Structure

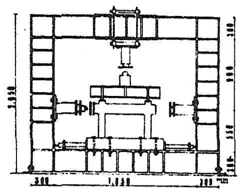

Fig.2
Loading System

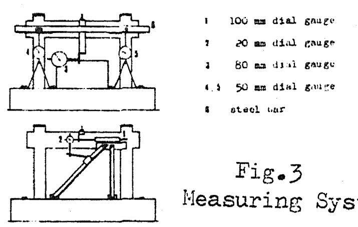

Fig.3
Measuring System

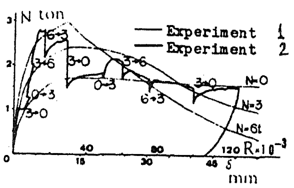

Fig.4

Skeleton Curve

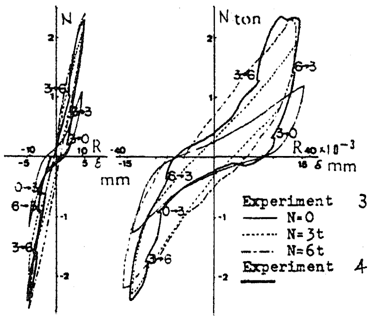

Fig.5 Fig.6
Hysteresis Loop

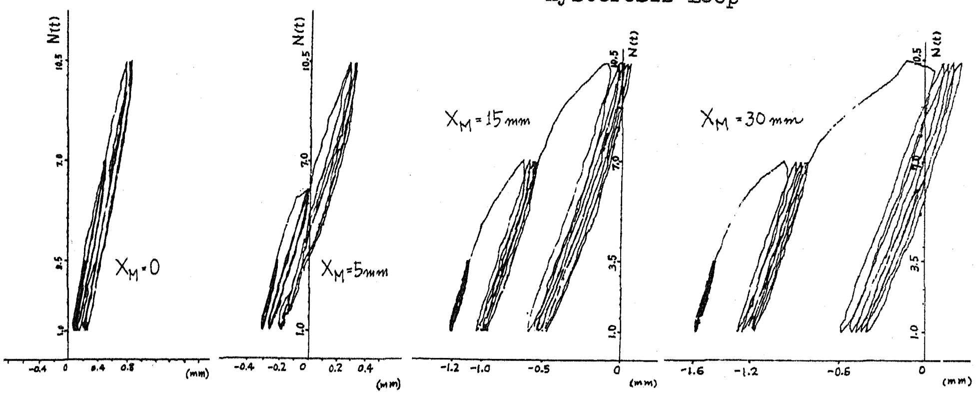

Fig.7 Fig.8 Fig.9 Fig.10
Restoring Force Characteristic in the Vertical Direction

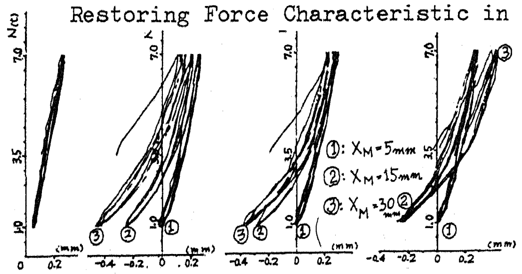

Fig.11 Fig.12 Fig.13 Fig.14
Restoring Force Characteristics
in the Vertical Direction

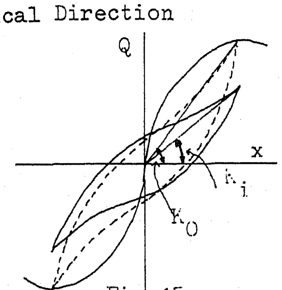

Fig.15
Stiffness degradation

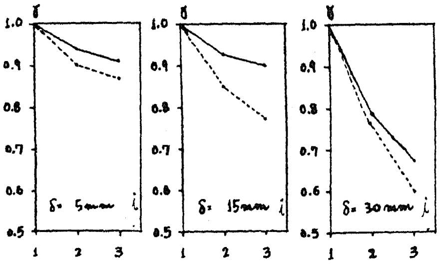

Fig.16 Fig.17 Fig.18
Stiffness Degradation

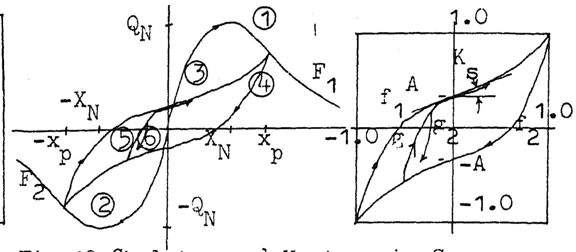

Fig.19 Skeleton and Hysteresis Curve

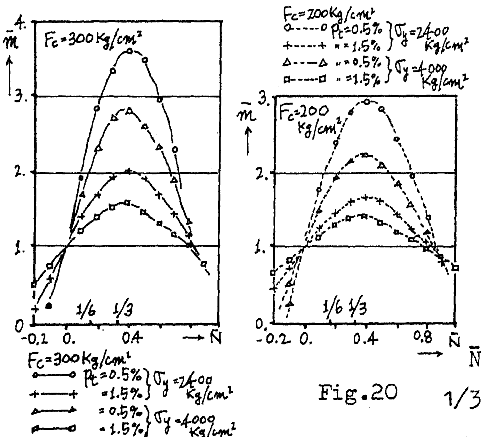

Fig.20

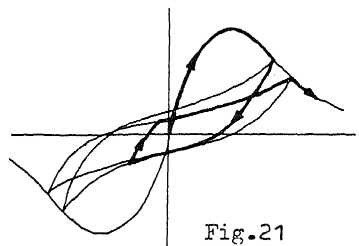

Fig.21
Stffness Degradation

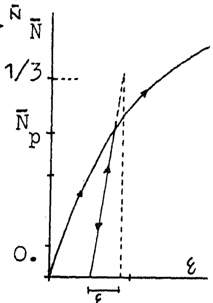

Fig.23

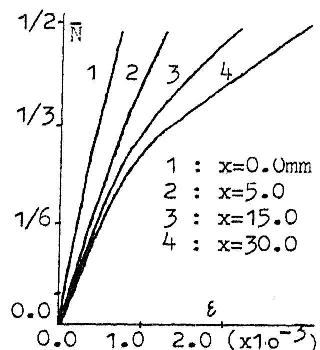

Fig.22 Skeleton Curve

Table 1. Structural Parameters

Weight	1000 t
$Q_{\bar{N}}$	300 t ($\bar{N}$=1/6)
$X_{\bar{N}}$	8.0 cm ($\bar{N}$=1/6)
N_{max}	6000 t
Period	T_H =0.54, T_V=0.10
Damping	h_H = 0.01, h_V= 0.05
$\bar{m}_{max}$	3.5

Table 2. Maximum Response

Input	Case	Horizontzl Response				Vertical Response			
		Disp. (cm)	Vel. (kine)	Acc. (gal)	Q (ton)	Disp. (cm)	Vel. (kine)	Acc. (gal)	N (ton)
El. Centro	1	13.13	70.54	762.67	299.1	-	-	-	-
	2	15.56	69.89	813.47	335.1	0.57	18.74	1120.4	2207.0
	3	12.11	69.96	800.36	339.7	1.20	16.67	915.0	1807.0
Taft	1	9.53	56.14	484.39	300.0	-	-	-	-
	2	10.49	62.94	520.85	326.5	0.30	4.10	248.04	1462.4
	3	8.12	60.47	520.09	334.74	0.72	6.88	327.0	1510.0