NONLINEAR EARTHQUAKE RESPONSE OF REINFORCED CONCRETE BUILDING FRAMES BY THE EQUIVALENT LINEAR METHOD

by

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SYNOPSIS

Nonlinear earthquake response of reinforced concrete building frames was calculated by the equivalent linear method and the result was compared with the computer-actuator on-line test data. The equivalent linear model used in the calculation consists of the origin-oriented hysteretic rule having tetra-linear skelton curve and the equivalent viscous damping ratio depending upon the experienced maximum response displacement in terms of ductility factor. The maximum response displacement obtained by the equivalent linear method were in good agreement with the on-line test results.

INTRODUCTION

Analytical model for earthquake response analysis on reinforced concrete members or frames with dominantly flexial behaviour have been proposed by many investigators [1,2,3,4]. Among them, an equivalent linear model is one of the prospective methods for the practical purpose of seismic design. A principle of earthquake response analysis using the equivalent linear model is to evaluate the hysteretic energy dissipation in inelastic range by equivalent viscous damping model [3,4]. However, the difficulty to use this method is laid upon the evaluation of such simple equivalent viscous damping model and the verification of the model. In this paper, a simple equivalent linear model is proposed obtained by the cyclic loading test, and the nonlinear earthquake response calculated by the model is compared with the computer-actuator on-line test results carried out by the identical reinforced concrete building frames with the cyclic loading test [5,6,7,8].

MODELING FOR THE EQUIVALENT LINEAR METHOD

For the nonlinear earthquake response analysis by the equivalent linear method, it is necessary to evaluate a) skelton curve to model the load-deformation characteristics under monotonic loading, b) stiffness reduction rule under unloading and reloading, and c) equivalent viscous damping ratio. In determining the skelton curve and the stiffness reduction rule, the on-line test data as well as the cyclic loading test data were used.

Cyclic Loading Test and On-Line Test: The cyclic loading test and the on-line test carried out were reported in the reference [6,7,8]. Analyzed frames and specimens are summarized in Table 1. Fig.2 shows the detail of the specimens for both test series (FD-series and FO-series). Six frames (FD-6 through FD-11 for FD-test series) and ten frames (FO-1 through FO-10 for FO-test series) of column yielding type were tested. Test variables were axial stress, initial natural periods and amplitudes of a ground motion.

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One frame (FD-6) of FD-series and two frames (FO-1,FO-6) of FO-series were tested in cyclic loading manner.

Skelton Curve: The skelton curve was determined as shown in Fig.3 for each test series. This curve is called a tetra-linear curve in this report and consists of four linear segments; the cracking point(c) defines the flexural cracking of concrete, the first yielding point (Y_1) the yielding of the upper layer reinforcement and the second yielding point (Y_2) the yielding of the middle layer reinforcement.

Stiffness Reduction Rule: Fig.4 shows the stiffness reduction ratio (K_i/K_1) due to the varing with time in the on-line test. Where, K_1 is the elastic stiffness. The peak-to-peak average stiffness (K_i) is expressed by the ratio of the incremental shear force ($Q_{i+1}-Q_i$) and the incremental response displacement ($X_{i+1}-X_i$) between the i-th peak and the (i+1)-th peak.

Fig.4 indicates the following tendency;

a) Stiffness reduction ratio decreased due to the varing with time until 3 sec, when the response displacement became maximum.
b) At this time, K_i/K_1 ratio were 0.05-0.3.
c) After the occurrence of the maximum response displacement, the stiffness reduction ratio (K_i/K_1) were almost constant, however, the shorter period frames (for example FD-10,FO-7,FO-8) showed slight increase of K_i/K_1.
d) At the end of tests, the K_i/K_1 ratio were 0.16-0.31.

Fig.5 show the relationships of the stiffness reduction ratio and the response displacement in terms of the average ductility factor after the occurrence of the maximum displacement. Average ductility factor(β) is defined as $|X_{i+1}-X_i|/2X_{y2}$, and X_{y2} is the second yielding displacement. A dashed line demonstrates the stiffness reduction ratio on the skelton curve (Fig.3).

The remarks obtained from Fig.5 are as follows;

a) Stiffness reduction ratio(K_i/K_1) at the occurrence of the maximum response displacement were around the dashed line calculated by the tetra-linear skelton curve. This ratio changed according to the amplitude of maximum response displacement.
b) After the occurrence of the maximum displacement, K_i/K_1 ratio increased to some extent due to the decreasing of average ductility factor (β), and this ratio were 0.16-0.31. This tendency was the same as that obtained in Fig.4.

From the above results, the unloading and reloading rule were assumed to that shown in Fig.7.

Equivalent Viscous Damping Ratio: The relationships between the equivalent viscous damping ratio and the peak displacement in terms of the ductility factor (β) is shown in Fig.6. The damping ratio shows the energy dissipation ratio in inelastic range(area of hysteretic loop) to the potential energy[9]. The ductility factor is defined by the second yielding displacement (X_{y2}). The results by the cyclic loading test(frame FD-6,FO-1,FO-6) and the analysis by the fibre model considered the nonlinear stress-strain relationships of concrete and steel [6,7,8] are shown in Fig.6.

The figure shows the equivalent viscous damping ratio increase according to the increase of ductility factor both in the test and the analysis.

Considering such tendency, the following relationships between the eqivalent viscous damping ratio and ductility factor were proposed;

$$h_{eq} = 0.0 \qquad (0.0 \leqq \mu < 0.4)$$

$$h_{eq} = 0.1 \cdot (0.7 \cdot \mu - 1/\sqrt{\mu}) + 0.13 \qquad (0.4 \leqq \mu < 4.0) \qquad \ldots\ldots.(1)$$

$$h_{eq} = 0.36 \qquad (4.0 \leqq \mu \quad)$$

where, ductility factor $\mu = X/X_{y2}$

EARTHQUAKE RESPONSE ANALYSIS

The differential equation for nonlinear earthquake response of one mass system by the equivalent linear method are as follows;

When the response displacement is on the skelton curve,

$$\ddot{X} + Q/M = - \ddot{X}_o \qquad \ldots\ldots.(2)$$

For unloading of reloading condition in inelastic range,

$$\ddot{X} + 2 \cdot h_{eq} \cdot \omega_e \cdot \dot{X} + \omega_e^2 \cdot X = - \ddot{X}_o \qquad \ldots\ldots.(3)$$

where, M : Mass of the system
X : Relative displacement of the system
$\dot{X}$: Relative velocity of the system
$\ddot{X}$: Relative acceleration of the system
$\ddot{X}_o$: Acceleration of ground motion
ω_e : Equivalent circular frequency of the system($=\sqrt{K_e/M}$)
K_e : Average stiffness of the system
h_{eq}: Equivalent viscous damping ratio of the system
Q : Restoring force(shear force) of the system

In order to solve the Eqs. 2 and 3, the following assumption in calculation were used;

a) The hysteretic model is assumed to be the origin-oriented model shown in Fig.7. The average stiffness(K_e) reduces gradually according to the increasing of the peak response displacement and so far as the displacement is less than the peak value, it is constant.
b) Eq. 1 controls the viscous damping ratio. The hysteretic energy dissipated in inelastic range is not considered in the origin-oriented model under unloading or reloading condition.
c) For numerical integration, the central difference method is used [11].

The acceleration record used for the ground motion was the 1968 Hachinohe (NS). The duration time was 12 seconds with zero data of 2 seconds. The amplitude of the acceleration was modified so that the ratio of the lateral strength of the frame in terms of the base shear coefficient to the peak ground acceleration normalized by the acceleration of gravity became 0.67-1.13 (Table 1).

RESULT OF ANALYSIS

Maximum Response Displacement: Fig.9 shows the maximum response displacement spectrum by the equivalent linear method and the on-line test. The results of response by the Degrading Tri-linear model, of which the skelton curve is shown in Fig.8, are also plotted. The Tri-linear curve was determined so that the complementary energy at the second yielding point became equal to that of the tetra-linear curve (Fig.3). The unloading and reloading rule were reported in the reference [1].

From the figure, it is recognized that the proposed analytical method; equivalent linear method, simulates well the maximum response displacement of the on-line test.

Response Displacement characteristics due to the Varing with Time: The characteristics of response displacement; amplitude of displacement and peak-to-peak period, can be seen from Fig.10. Fig.10(a) is an example which shows good correspondence between the proposed analysis and the on-line test(frame FD-9), and Fig.10(b) is one which shows the worst correspondence(FD-11) of all frames.

The increase of peak-to-peak period and the amplitude of response displacement of the on-line test can be approximated by the analysis before the occurrence of maximum response displacement. But after that time, the discrepancy between both simulations increases, especially in frame FD-11 which shows greater maximum displacement. The peak-to-peak period of the analysis decreases more than it particularly at the free vibration (10-12 sec). For other frames, this tendency is also recognized. This would be caused by adopting the fact that the stiffness reduction ratio (K_i/K_1) and the equivalent viscous damping ratio (h_{eq}) at the occurrence time of maximum displacement were used for those of unloading and reloading condition after at that time respectively.

Shear Force-Displacement Relationships: The restoring force of the analysis is evaluated as the term of $(2 \cdot h_{eq} \cdot M \cdot \omega_e \cdot \dot{X} + Q)$ in Eq. 3. This force is affected by the characteristics of equivalent viscous damping ratio(h_{eq}), equivalent circular frequency(ω_e) in terms of average stiffness(K_e) and relative velocity($\dot{X}$). Therfore, it was resulted that the shear force-response displacement relationships was not so similar to that of the on-line test. Fig.11 shows the example that the hysteretic loop shows the best fitting of all frames between the analysis [Fig.11(a)] and the on-line test [Fig.11(b)] for frame FD-9.

CONCLUDING REMARKS

In this paper, the equivalent linear method taking into account of the hysteretic energy and the stiffness reduction in inelastic range was presented for evaluating the behaviour of reinforced concrete building frames subjected to a ground motion. The results computed by the proposed model were compared to that by the computer-actuator on-line test developed by the authors, and the following concluding remarks were obtained;

1) As far as the maximum response displacement was concerned, the proposed equivalent linear model simulated well the results by the on-line test, while some discrepancy between the analysis and the test were observed in the time history of response displacement and the shear force-

displacement relationships.

2) Since the equivalent linear model easily took into account hysteretic energy dissipation by the characteristics of skelton curve, stiffness reduction ratio and equivalent viscous damping ratio obtained by cyclic loading test, the equivalent linear method would be useful in view of practical purpose.

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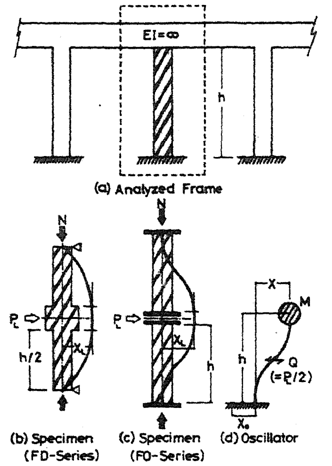
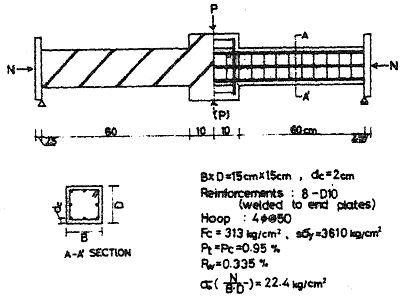
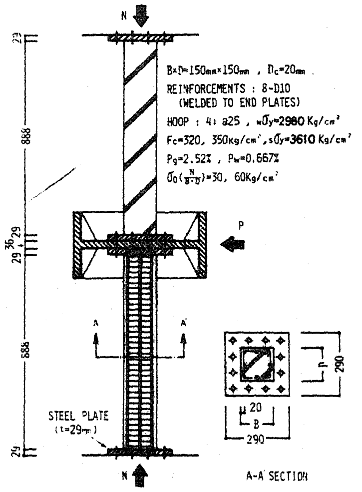


Fig.1 Analyzed Frame and Specimen



(a) FD-Test Series



(b) FO-Test Series

Fig.2 Detail of Specimen

Table 1 Characteristics of Frame and Ground Motion

(a) FD-Test Series

Frame ID	Initial Period T_1 (sec)	Mass M ($kg \cdot cm^{-1} sec^2$)	Peak Ground Acceleration $(\ddot{X}_o)_{max}$ (gal)	k_g^*	$\frac{k_y}{k_g}$ **
FD-6	Cyclic Loading Test				
FD-7	0.6	77.51	29.67	0.030	1.11
FD-8	0.4	34.45	66.76	0.068	1.15
FD-9	0.2	8.61	267.03	0.273	1.13
FD-10	0.15	4.84	474.74	0.484	1.13
FD-11	0.2	8.61	400.50	0.409	0.75

* $k_g=(\ddot{X}_o)_{max}/980$ ** $\frac{k_y}{k_g}=\frac{Q_y}{Mg} / \frac{(\ddot{X}_o)_{max}}{g}$

*** Axial stress σ_o=22.4 kg/cm^2 for all frames

(b) FO-Test Series

Frame ID	Initial Period T (sec)	Axial Stress (kg/cm^2)	Mass M ($kg cm^{-1} sec^2$)	Peak Ground Acceleration $(\ddot{X}_o)_{max}$ (gal)	$K_g = \frac{(\ddot{X}_o)_{max}}{980}$	$\frac{K_y}{K_g}$
FO-1	—	30	Cyclic Loading Test			
FO-2	0.2		18.54	202.2	0.206	1.03
FO-3	0.2		18.54	303.4	0.310	0.69
FO-4	0.4		74.17	50.6	0.052	1.01
FO-5	0.4		74.17	66.7	0.068	0.76
FO-6	—	60	Cyclic Loading Test			
FO-7	0.2		18.54	242.9	0.248	0.98
FO-8	0.2		22.29	266.5	0.272	0.76
FO-9	0.4		74.17	80.1	0.082	0.77
FO-10	0.4		74.17	91.0	0.093	0.67

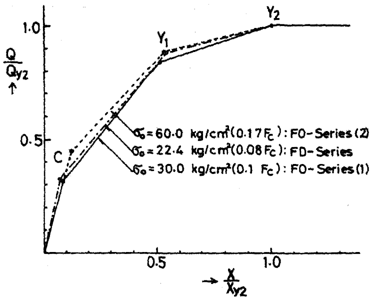
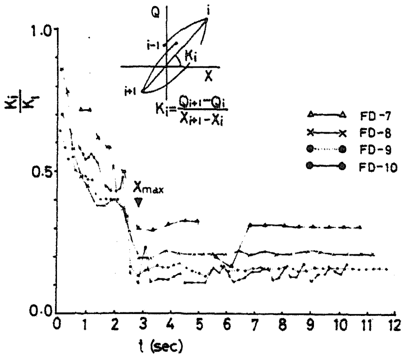
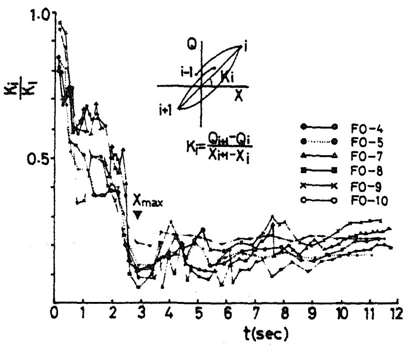


Fig.3 Tetra-Linear Skelton Curve



(a) FD-Test Series



(b) FO-Test Series

Fig.4 Stiffness Reduction Ratio due to Time Varing

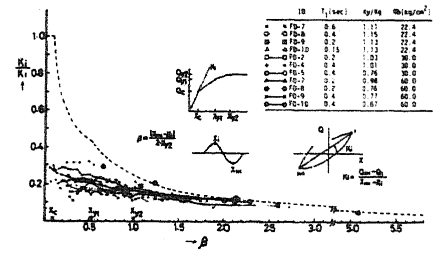


Fig.5 Stiffness Reduction Ratio vs. Displacement

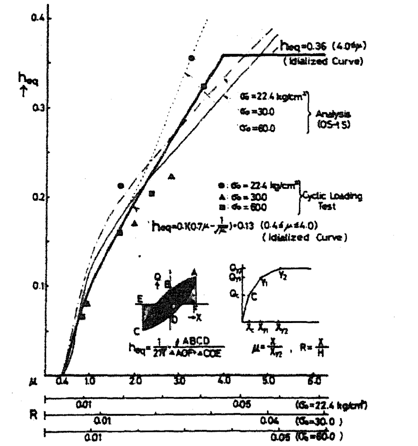


Fig.6 Equivalent Viscous Damping Ratio vs. Ductility Factor

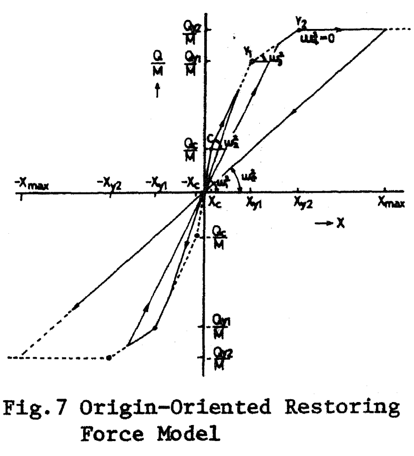


Fig.7 Origin-Oriented Restoring Force Model

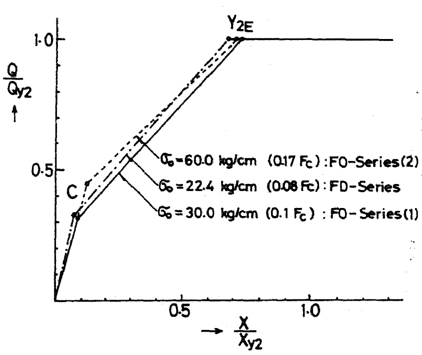
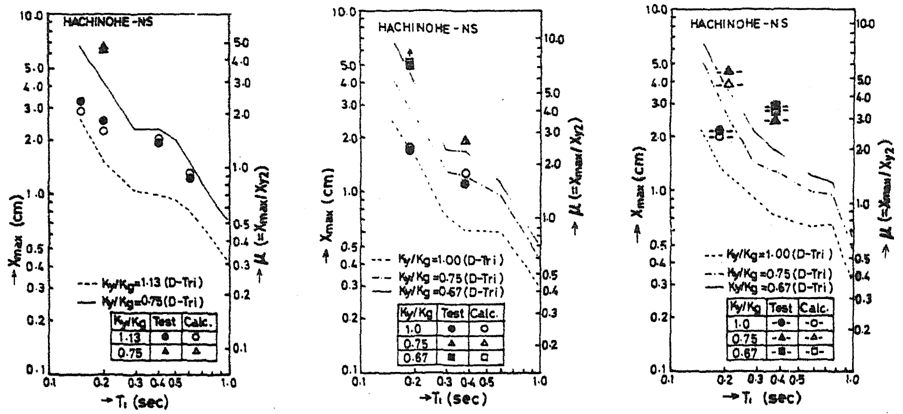


Fig.8 Equivalent Tri-Linear Skelton Curve

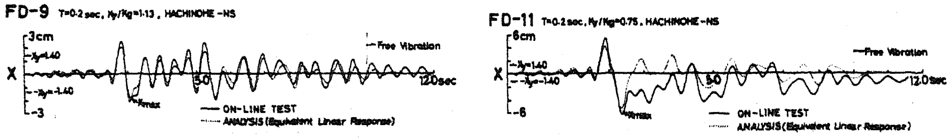


(a) FD-Test Series (FD-7 - FD-11)

(b) FO-Test Series(1) (FO-2 - FO-5)

(c) FO-Test Series(2) (FO-7 - FO-10)

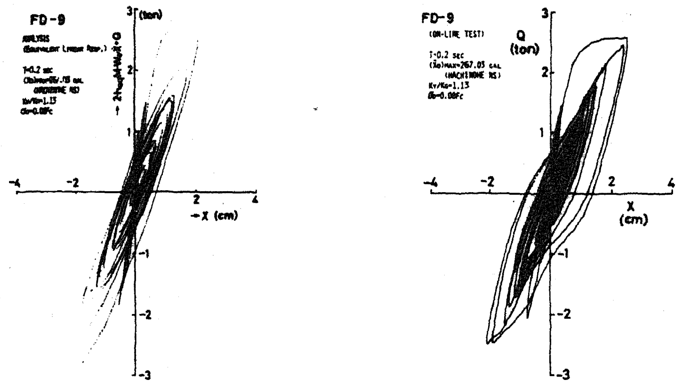
Fig.9 Maximum Response Displacement



(a) Frame FD-9

(b) Frame FD-11

Fig.10 Time History of Response Displacement



(a) Equivalent Linear Method

(b) On-Line Test

Fig.11 Shear Force - Displacement Relationships