SEISMIC RESPONSE OF MULTI-SUPPORT STRUCTURES

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SUMMARY

Probabilistic models are proposed for determination of design responses in structures sensitive to phase differences among ground motion histories at their supports. Selection of design values is based on approximate proportionalities between responses corresponding to given probabilities of exceedance and standard deviations of those responses at the end of a segment of random noise taken as equivalent to earthquake excitation. Detailed practical rules are proposed for mode and component superposition as well as for definition of internal-load combinations to be used when checking safety conditions at given critical sections.

INTRODUCTION

Design values of internal forces produced by earthquakes are usually taken as those provided by an envelope to the values of those variables caused by the individual ground motion components, one at a time. When dealing with structures having small dimensions in plan, reasonable approximations to maximum response for the superposition of the various ground motion components can be obtained from a linear combination of maximum responses to each individual component at a given point of the ground-structure interface, provided modal responses can be taken as stochastically independent (1).

For structures extended in plan, such as long bridges, or founded on heterogeneous formations or irregular topography, such as dams, differences in ground motion among different supports or zones of the ground-structure interface may give place to quantitative and qualitative differences in internal actions as compared with those produced by in-phase motion of all supports. In addition, strong correlation may exist between pairs of parallel ground motion components at different points, and the correlation among the responses of a given mode to several (correlated) ground motion components can no longer be neglected.

The present paper explores several criteria for determining design responses, on the basis of the variance of the response to the various ground motion components, for a given assumption concerning the covariance structure of the generalized excitation[III]. Simplified rules for practical application are also discussed.

DYNAMICS OF LINEAR SYSTEMS SUBJECTED TO OUT-OF-PHASE SUPPORT DISPLACEMENTS

Let the excitation acting on a multi-support system be described by vector $X(t) = \{x_q\}$ of support displacements. Suppose the system is discretized and represented by the mass, stiffness and damping matrices M, K and C respectively, where support displacements are not taken as degrees of freedom. The vector Y(t) of structural response components (displacements, stresses) can be obtained as follows (2,3).

$$Y(t) = \sum_q x_q(t) \tilde{Y}_q + U(t) \quad (1)$$

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III More general assumptions are covered in Ref 2.

In this equation, $\sum_q$ is taken to all the supports, $\tilde{Y}_q$ is the vector of static response components produced by a unit displacement x_q, and U is the vector of dynamic response components, i.e. those measured at anty given instant with respect to the superposition of the static configurations associated with support displacements x_q at the same instant. U can be expressed in terms of modal vectors $Z_j = \{ z_{ij} \}$

$$U(t) = \sum_q \int_0^t \ddot{x}_q (\tau) \sum_j a_{jq} Z_j h_j (t - \tau) d\tau \tag{2}$$

Here a_{jq} is the participation factor of mode j that corresponds to displacements of support q, and $h_j(\tau)$ is the unit impulse response function for the mentioned mode. This factor can be determined from conditions Y = 0, $\dot{Y} = 0$, at $t = 0^+$, inmediately after application of a concentrated unit-area acceleration pulse $\ddot{x}_q = \delta(t)$. Thus (2,3),

$$a_{kq} = \frac{r_{qk}}{\omega_k^2 Z_k^T M Z_k} \tag{3}$$

where r_{qk} is the external reaction at support q when the system vibrates freely in its k'th mode having shape Z_k and natural frequency ω_k. From Eqs. 1 and 2, a given element Y_i of the response vector of the system can be expressed as follows

$$Y_i(t) = \sum_q x_q (t) \tilde{Y}_{iq} + \sum_q \sum_j a_{jq} Z_{ij} \int_0^t \ddot{x}_q(\tau) h_j(t-\tau) d\tau \tag{4}$$

In this equation, $\tilde{Y}_{iq}$ and Z_{ij} denote the (time independent) values of Y_i associated with vectors $\tilde{Y}_q$ and Z_j, respectively.

DYNAMIC RESPONSE ANALYSIS FOR STOCHASTIC GROUND MOTION

It will be assumed that x can be represented as the product of a Gaussian stationary process $W_q(t)$ with spectral density $G_q(\omega)$ by a deterministic envelope $A_q(t)$:

$$\ddot{x}_q(t) = A_q(t) \; W_q(t) \tag{5}$$

In order to determine the parameters that define $Y_i(t)$ in probabilistic terms, it is advantageous to express that function independently of $x_q(t)$, by means of an integral expressed exclusively in terms of $\ddot{x}_q(t)$ and of a response function $g_{iq}(t)$ wich takes into account both static and dynamic components. For an elementary accelerogram defined by a concentrated impulse $\ddot{x}_q(t) = \delta(t)$, one obtains $\dot{x}_q(t) = H(t)$ and $x_q(t) = tH(t)$, where $\delta(.)$ and H(.) are Dirac delta function and Heaviside step function, respectively. Y_i is then the unit impulse response function for acceleration of support q, and is given by the following equation, obtained from Eq. 4:

$$g_{iq}(t) = tH(t) \tilde{Y}_{iq} + \sum_j a_{jq} Z_{ij} h_j(t) \tag{6}$$

If the complete accelerogram at a given support is taken into account by integration of eq. 6 with respect to time, and if the contributions from all supports are added together, the following alternative to Eq. 4 is obtained:

$$Y_i(t) = \sum_q \int_0^t \ddot{x}_q(\tau) \; g_{iq}(t-\tau) d\tau \tag{7}$$

Taking into account Eq. 5, the last equation becomes

$$Y_i(t) = \sum_q \int_0^t A_q(\tau) W_q(\tau) g_{iq}(t-\tau)\, d\tau \tag{8}$$

Hence, the variance of $Y_i(t)$ is

$$\operatorname{var} Y_i(t) = \sum_r \sum_q \int_0^t \int_0^t A_q(\tau_1) A_r(\tau_2) R_{qr}(\tau_1,\tau_2) g_{iq}(t-\tau_1) g_{ir}(t-\tau_2)\, d\tau_1\, d\tau_2 \tag{9}$$

where $R_{qr}(\tau_1,\tau_2) = E[W_q(\tau_1) W_r(\tau_2)]$ is the cross-correlation function of W_q and W_r.

From Eqs. 5 and 8 and the approximate theory of Ref. 4 it is possible to determine the probability distribution of the maximum absolute value of $Y_i(t)$ during an earthquake, but the amount of computations involved makes application of that theory impractical; adoption of simpler criteria is advantageous. Herein it is assumed that the values of responses which correspond to a given probability of exceedance during an earthquake of given intensity are proportional to the maximum values reached by the respective standard deviations while ground motion lasts. Thus, if σ_0^2 is the maximum variance of ground acceleration during the earthquake and $\ddot{x}_0(p)$ is the value of that acceleration which corresponds to probability p of being exceeded, and if σ_R^2 and $x_R(p)$ are the corresponding values associated with a response variable R, the assumption proposed implies that if the design criterion adopted is based on equal exceedance probabilities for all design responses, then the ratio of the design value of R to the specified peak ground acceleration should equal σ_R/σ_0.

Ref. 2 contains expressions for the variance of a response variable for several alternative assumptions concerning the covariance structure of the excitation. Only one particular case is discussed here: that where ground accelerations at the various supports are represented by segments of stationary white noise travelling undistorted along the ground surface:

$$\ddot{x}_q(t) = W_0(t-\tau_q), \quad q = 1, \dots, N \tag{10}$$

In this equation, W_0 is white noise with spectral density G_0. The white-noise assumption is not restrictive, and the criterion derived from it can be applied to more general spectral shapes of excitation.

If the natural periods of the systems analyzed are short as compared with the duration of strong ground motion, maximum variances will be reached at the end of the excitation, i.e. for t=s:

$$\operatorname{var} Y_i(s) = \sum_q \sum_r \tilde{Y}_{iq} \tilde{Y}_{ir} I_{qr}(s) + 2 \sum_q \sum_r \sum_k a_{kr} \tilde{Y}_{iq} Z_{ik} I'_{kqr}(s) + \sum_q \sum_r \sum_j \sum_k a_{jk} a_{kr} Z_{ij} Z_{ik} I''_{jkqr}(s) \tag{11}$$

where, for the case defined by Eq. 10, making g(t) = tH(t),

$$I_{qr}(s) = \pi G_0 \int_0^s g(s-\tau_q-\tau) g(s-\tau_r-\tau)\, d\tau \tag{12a}$$

$$I'_{kqr}(s) = \pi G_0 \int_0^s g(s-\tau_q-\tau) h_k(s-\tau_r-\tau)\, d\tau \tag{12b}$$

$$I''_{jkqr}(s) = \pi G_0 \int_0^s h_j(s-\tau_q-\tau) h_k(s-\tau_r-\tau)\, d\tau \tag{12c}$$

Analytic expressions for these integrals are given in the Appendix.

SUPERPOSITION CRITERIA

The ground motion models given by Eq. 5 are reasonable representations of earthquake accelerograms, in spite of the fact that they ignore frequency-content variations during a given event. These models lead to sufficiently accurate estimates of the variances of ground accelerations and of responses which are determined mainly by those accelerations; however, they do not lead to accurate estimates of variances of quantities sensitive to ground velocities or displacements (such as the dynamic response of structures possessing moderate or long natural periods, or the stresses produced by phase differences among support displacements), unless the spectral density of ground displacements (and not only of ground accelerations) is closely represented or base-line corrections are applied to accelerograms given by Eq. 5 so as to produce zero final ground velocities. As a consequence, the proportionality of design responses with square roots of variances described above cannot always be directly applied, particularly in those instances in which the proportionality refers simultaneously to quantities sensitive to ground accelerations, velocities and displacements. This difficulty can be overcome if the proportionality assumption is applied individually to each term in Eq. 11: although the displacement variances predicted on the basis of Eq. 5 may be excessive, the ratios of the variances of responses proportional to displacements predicted on the same basis will be good estimates of the ratios of the actual variances. Thus, the following equation results:

$$Q^2 = \sum_q \sum_r \left(Q_q Q_r \alpha_{qr} D_{oq} D_{or} + 2 \sum_k Q_k a_{kr} Z_k \alpha'_{kqr} D_{oq} D_{kr} + \sum_j \sum_k a_{jq} a_{kr} Z_j Z_k \alpha''_{jkqr} D_{jq} D_{kr} \right) \tag{13}$$

In this equation, Q is the design response, D_{oq} and D_{or} are the peak ground displacements at supports q and r, respectively; D_{jq} and D_{kr} are the displacement spectral ordinates at the same supports for modes j and k; Q_q is the static response to a unit displacement of support q, and α_{qr}, α'_{kqr}, α''_{jkqr} are proportionality factors obtained as follows:

$$\alpha_{qr} = I_{qr}(s) / (J_{qq} J_{rr})^{1/2} \tag{14a}$$

$$\alpha'_{kqr} = I'_{kqr}(s) / (J_{qq} J''_{kr})^{1/2} \tag{14b}$$

$$\alpha''_{jkqr} = I''_{jkqr}(s) / (J''_{jq} J''_{kr})^{1/2} \tag{14c}$$

J_{qq} *and* J_{rr} *are the values of* I_{qq} (s) *and* I_{rr}(s) *when time is measured from the* instant when ground motion starts at supports q and r, respectively, and J''_{jk}, J''_{kr} are the corresponding values of I''_{jjqq} and I''_{kkrr} for the same time origins. In other words, the contributions to design responses are expressed in terms of peak ground displacements at all supports as well as of the modal responses to each ground motion component taken as acting simultaneously at all supports.

Figs. 1-4 illustrate the variation of α, α' and α'' (determined with Eqs. A1-A3) for several combinations of natural frequencies and time-lags. The latter are expressed as fractions of the duration s of the white noise segment used to represent the excitation. On firm ground, s can be taken as 20 sec.

DESIGN CRITERIA

It is assumed that design consists in determining probable combinations of internal forces at critical sections and verifying if those combinations lie within the specified safe regions. The criterion advocated herein for the determination of the mentioned combinations is based on the same concepts as that recommended in Ref. 1, but unlike the latter it takes into account the statistical correlation among support displacements. Both criteria assume that the joint probability distribution of the internal forces which determine the most unfavorable condition at a critical section is Gaussian multi-dimensional, and that a set of multidimensional ellipsoids can be built with centers at the expected values of the internal forces and principal axes in directions which are functions of the correlation coefficients, so that to each ellipsoid corresponds a number of standard deviations of each internal force from its expected value and hence a given probability of containing the load combination of interest. A given design is adequate if the ellipsoid which corresponds to a specified probability lies within the safe region and is tangent to its boundary (1). Because contructing the mentioned ellipsoid would be excessively difficult for practical densign, it is proposed to substitute the ellipsoid with a set of $2^N N$ points (combinations) where N is the number of cartesian components in each combination (IV). The design value of component Y_k in the j-th combination would equal $\gamma_{kj} y_k$, where y_k is the design value which would be assumed for Y_k if design were based exclusively on it, γ_{kj} equals ± 1 for $k = j$ and $\gamma_{jj}(\rho_{kj} \pm 0.3(1-\rho_{kj}^2))$ for $k \neq j$, and ρ_{kj} is the correlation coefficient between $Y_k(s)$ and $Y_j(s)$. An expression for $\text{cov}(Y_k(s), Y_j(s))$ of the type of Eq. 13 can readily be derived starting from Eq. 4.

Take for instance the case illustrated in Fig. 5 for the cross section of a reinforced concrete column subjected to axial load P and bending moment M with respect to one of its principal axes of inertia. The cases when the correlation coefficient ρ_{MP} equals 0 and 0.5 will be considered. Assume also that the dotted line represents the section's interaction diagram, and that p and m are the design values of M and P should each of them be considered separately. For each value of ρ_{MP}, N = 2 and the number of internal-forces combinations to be considered equals $2^2\,2 = 8$. For $\rho_{MP} = 0$, $\gamma_{MP} = \pm 0.3$, and for $\rho_{MP} = 0.5$, $\gamma_{MP} = \pm(0.5 \pm 0.225)$; hence Figs. 5a, b.

APPLICATIONS

Fig 6 shows some results of applying Eq. 13 to define the design responses of a fixed-end arch when both supports move out of phase. Responses to vertical and horizontal ground displacements are analyzed separately. In each case, results are expressed in terms of both the ratio τ_q/s of the time-lag between support displacements to the duration of the white-noise segment and the apparent wave propagation velocity along the ground surface. Both qualitative and quantitative deviations with respect to the case of in-phase support motion are evident.

Another case of interest is shown in Fig. 7, which represents a bridge built of simply supported spans resting on columns with ends fixed on the ground surface. Variables studied include bending moments at column bases and tensions on tie-bars connecting beam spans. Again, the sensitivity of design responses to phase differences is obvious.

CONCLUSIONS

The seismic response of extended-in-plan structures can be very sensitive to phase differences among the motions of different points in the foundation. Under the assumption of linear behavior it is possible to formulate approximate criteria to obtain design values and response superposition models which account for all ground motion components. The resulting expressions are determined by the covariance structure of the ground motion histories at the different supports.

REFERENCES

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(VI) The criterion proposed in Ref. 1 works with load vectors corresponding to each ground motion component.

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APPENDIX. VARIANCE INTEGRALS FOR SHIFTED IDENTICAL WHITE NOISE COMPONENTS

Analytical expressions for I_{qr}, I'_{kqr}, I''_{jkqr}, according to Eqs. 12a – c:

$$I_{qr}(t)=\frac{1}{3}(t^3-t_1^3)-\frac{1}{2}(\tau_q+\tau_r)(t^2-t_1^2)+\tau_q\tau_r(t-t_1) \qquad (A1)$$

$$\begin{aligned} I'_{kqr}(t)=\frac{1}{D_1^2\omega'_k}\Big\{ & e^{-\zeta_k\omega_k(t-\tau_r)}\Big[\big(B_1(1+\zeta_k\omega_k t)-A_1\omega'_k t-\omega'_k\tau_q D_1\big)\cos\omega'_k(t-\tau_r)+ \\ & +\big(A_1(1+\zeta_k\omega_k t)+B_1\omega'_k t-\zeta_k\omega_k\tau_q D_1\big)\sin\omega'_k(t-\tau_r)\Big] - \\ & -e^{-\zeta_k\omega_k(t_1-\tau_r)}\Big[\big(B_1(1+\zeta_k\omega_k t_1)-A_1\omega'_k t_1-\omega'_k\tau_q D_1\big)\cos\omega'_k(t_1-\tau_r)+ \\ & +\big(A_1(1+\zeta_k\omega_k t_1)+B_1\omega'_k t_1-\zeta_k\omega_k\tau_q D_1\big)\sin\omega'_k(t_1-\tau_r)\Big]\Big\} \end{aligned} \qquad (A2)$$

$$\begin{aligned} I''_{jkqr}(t)=\frac{e^{\gamma}}{2\omega'_j\omega'_k}\Big\{ & e^{-At}\Big[\frac{A\cos(Z_1-W_1t)+W_1\sin(Z_1-W_1t)}{A^2+W_1^2} - \\ & -\frac{A\cos(Z_2-W_2t)+W_2\sin(Z_2-W_2t)}{A^2+W_2^2}\Big] - \\ & -e^{-At_1}\Big[\frac{A\cos(Z_1-W_1t_1)+W_1\sin(Z_1-W_1t_1)}{A^2+W_1^2} - \\ & -\frac{A\cos(Z_2-W_2t_1)+W_2\sin(Z_2-W_2t_1)}{A^2+W_2^2}\Big]\Big\} \end{aligned} \qquad (A3)$$

In these equations,

- $t_1=\max(\tau_q,\tau_r)$
- $A_1=\zeta_k^2\omega_k^2-\omega_k'^2$
- $B_1=2\zeta_k\omega_k\omega'_k$
- $D_1=\zeta_k^2\omega_k^2+\omega_k'^2$
- $\gamma=\zeta_j\omega_j\tau_q+\zeta_k\omega_k\tau_r$
- $A=\zeta_j\omega_j+\zeta_k\omega_k$
- $Z_1=\omega'_j\tau_q+\omega'_k\tau_r$
- $Z_2=\omega'_j\tau_q-\omega'_k\tau_r$
- $W_1=\omega'_j+\omega'_k$
- $W_2=\omega'_j-\omega'_k$

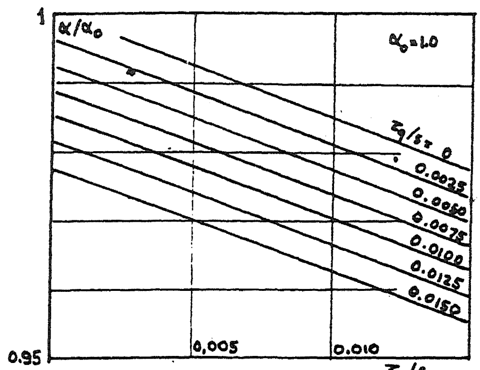


Fig. 1 Values of α

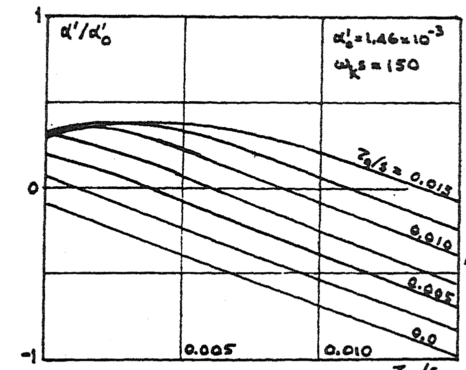


Fig. 2 Values of α'

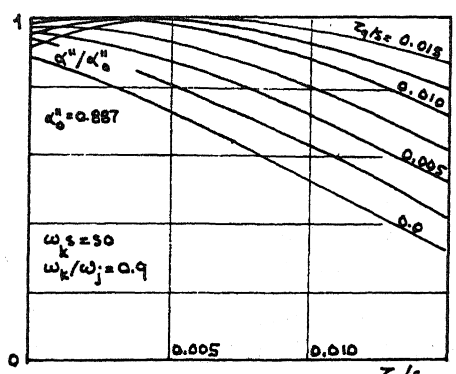


Fig.3 Values of α"

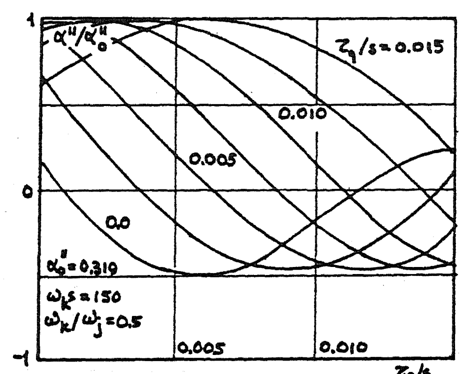


Fig.4 Values of α"

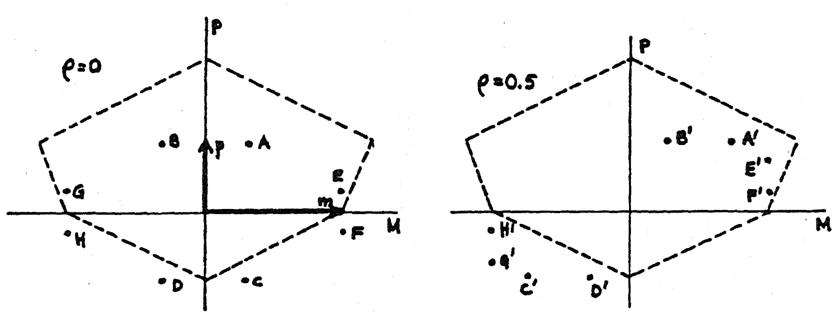
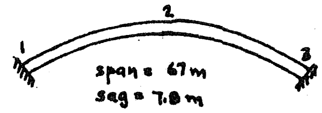
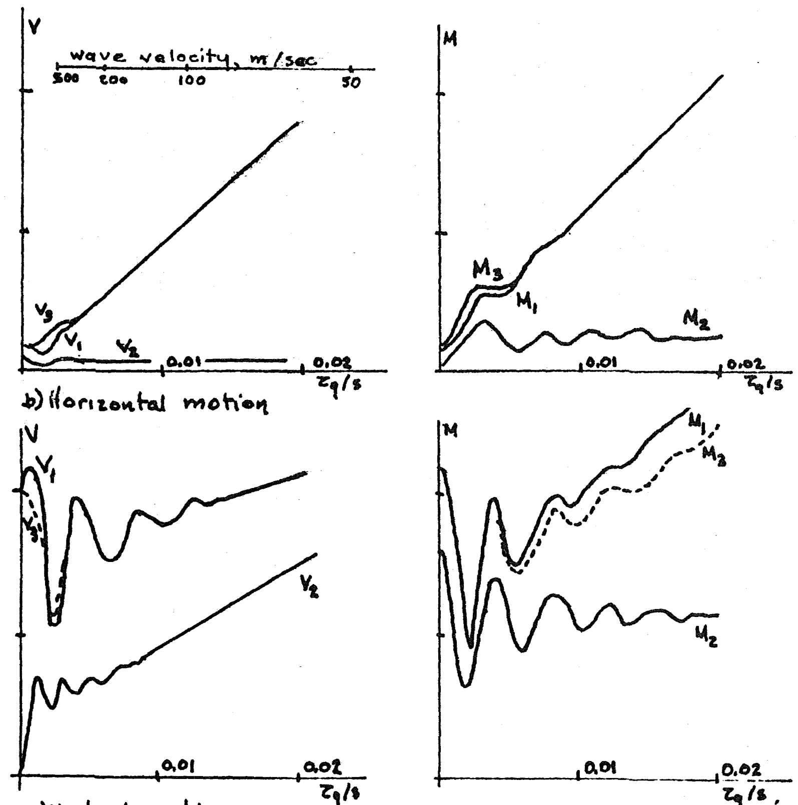


Fig.5 Superposition of correlated responses

a) Structure studied:

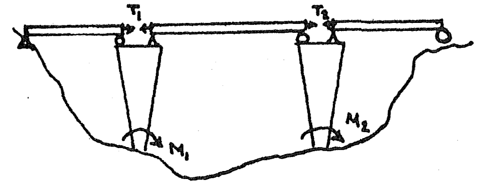




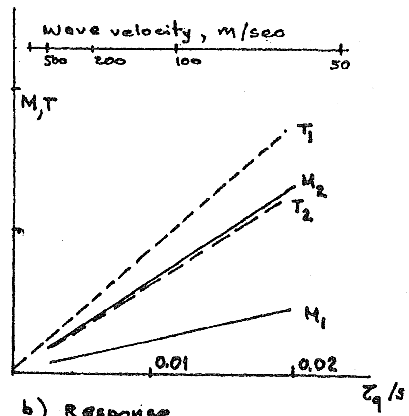
b) Horizontal motion

c) Vertical motion

Fig.6 Influence of phase differences in the seismic response of an arch



a) Bridge with spans connected by ties



b) Response

Fig.7 Bridge response