NONLINEAR DYNAMIC ANALYSIS OF STRUCTURES USING VISCOPLASTIC MATERIAL MODEL

S.K. Thakkar[I]

SUMMARY

This paper presents an application of visco-plastic material model to the solution of materially nonlinear dynamic problems in finite element formulation. A general formulation of equations of motion which include the effects of both material and geometric nonlinearity is considered An implicit scheme is used to integrate equations of motion. Elastic - plastic solutions are obtained by using visco-plastic alogrithm as a solution technique. Numerical solutions of some dynamic problems were obtained to demonstrate the application of this approach. The visco-plastic approach with implicit time stepping scheme is seen to provide an economic approach to deal with the nonlinear dynamic problems.

INTRODUCTION

The development of efficient and accurate techniques for nonlinear dynamic analysis of structures has attracted much attention in recent years. The nonlinear dynamic response analysis of structures is of considerable practical importance for better assessment of safety, better understanding of inelastic behaviour and economic design. The complete study of nonlinear response of a structure subjected to an earthquake motion is still a matter of research rather than a routine design. A more general approach to the problems of nonlinear dynamic response analysis is offered by the use of numerical techniques such as finite element methods. Recently the application of visco-plastic material model is made for solving quasi-static (8) and dynamic problems (3,5,7). The visco-plastic model rather than purely plastic model is postulated to allow for the rate effects to be considered. Elastic-plastic behaviour can be obtained either from the degenerated visco-plastic model or by obtaining steady state solutions using pseudo-time stepping.

The purpose of this paper is to present a formulation for dynamic analysis of structures with material nonlinearity through visco-plastic model and geometric nonlinear behaviour within the Largangian framework. The equations of motion are solved by direct step-by-step Newmark implicit scheme (4). Some dynamic problems were solved to demonstrate the application of this approach and the results compared with other existing methods.

FORMULATION FOR NONLINEAR DYNAMIC RESPONSE

Equations of Motion

The incremental form of equations of motion for nonlinear system during the time increment Δt can be written as follows,

$$\underset{\sim}{M}\,\Delta\underset{\sim}{\ddot{u}}_t + \underset{\sim}{C}\,\Delta\underset{\sim}{\dot{u}}_t + \Delta\underset{\sim}{F}_t = \Delta\underset{\sim}{R}_t \qquad \ldots (1)$$

I Professor of Earthquake Engineering, University of Roorkee, Roorkee, India.

in which $\underset{\sim}{M}$ and $\underset{\sim}{C}$ are mass and damping matrices. The incremental vectors $\Delta \ddot{u}_t$ etc. represent the change of the quantity over the time step Δt. $\Delta \underset{\sim}{F}_t$ and $\Delta \underset{\sim}{R}_t$ are the change in the internal resisting force and external force respectively and can be written as,

$$\Delta \underset{\sim}{F}_t = \int_v \underset{\sim}{B}^T d\underset{\sim}{\sigma} \, dv + \underset{\sim}{K}_\sigma \Delta \underset{\sim}{u}_t$$

$$\Delta \underset{\sim}{R}_t = \underset{\sim}{R}_{t+\Delta t} - \underset{\sim}{M}\ddot{\underset{\sim}{u}}_t - \underset{\sim}{C}\dot{\underset{\sim}{u}}_t - \underset{\sim}{F}_t$$

$$\underset{\sim}{F}_t = \int_v \underset{\sim}{B}^T \underset{\sim}{\sigma} t \, dv \qquad \ldots (2)$$

in which $\underset{\sim}{B}$ and $\underset{\sim}{K}_\sigma$ are the kinematic and initial stress matrix respectively. The $\underset{\sim}{B}$ matrix for geometric nonlinear problems can be written as,

$$\underset{\sim}{B} = \underset{\sim}{B}_o + \underset{\sim}{B}_n \qquad \ldots (3)$$

where $\underset{\sim}{B}_o$ and $\underset{\sim}{B}_n$ respectively represent the strain matrices arising out of linear and nonlinear terms. All the equations are referred to include geometric nonlinear effects in Lagrangian framework. Using equations (2), equations of motion in incremental form becomes,

$$\underset{\sim}{M}\Delta\ddot{\underset{\sim}{u}}_t + \underset{\sim}{C}\Delta\underset{\sim}{u}_t + \int_v \underset{\sim}{B}^T d\underset{\sim}{\sigma} dv + \underset{\sim}{K}_\sigma \Delta \underset{\sim}{u}_t = \underset{\sim}{R}_{t+\Delta t} - \underset{\sim}{M}\ddot{\underset{\sim}{u}}_t - \underset{\sim}{C}\dot{\underset{\sim}{u}}_t - \underset{\sim}{F}_t \qquad \ldots (4)$$

Stress-strain relations

The stress-strain relations are established by assuming the total strain to be separable into elastic and visco-plastic components and then using Hooke's law. The incremental form of stress-strain relationship can be written as,

$$d\underset{\sim}{\sigma} = \underset{\sim}{D} \, (\underset{\sim}{B}\Delta\underset{\sim}{u}_t - d\underset{\sim}{\varepsilon}^{vp}) \qquad \ldots (5)$$

Using equation (5) in equation (4), incremental form of equation can be written as,

$$\underset{\sim}{M}\Delta\ddot{\underset{\sim}{u}}_t + \underset{\sim}{C}\Delta\underset{\sim}{u}_t + \underset{\sim}{K}_t\Delta\underset{\sim}{u}_t = \underset{\sim}{R}_{t+\Delta t} - \underset{\sim}{M}\ddot{\underset{\sim}{u}}_t - \underset{\sim}{C}\dot{\underset{\sim}{u}}_t - \underset{\sim}{F}_t + d\underset{\sim}{F}_{vp} \qquad \ldots (6)$$

$\underset{\sim}{K}_t = \underset{\sim}{K} + \underset{\sim}{K}_\sigma$ (tangent stiffness matrix), $\underset{\sim}{K} = \int_v \underset{\sim}{B}^T \underset{\sim}{D}\underset{\sim}{B} \, dv$ and

$d\underset{\sim}{F}_{vp} = \int_v \underset{\sim}{B}^T \underset{\sim}{D} d\underset{\sim}{\varepsilon}^{vp} \, dv$ (pseudo force)

Visco-plastic law

The visco-plastic law governing the occurrence of visco-plastic strains is related in the following form,

$$\frac{d\underset{\sim}{\varepsilon}^{vp}}{dt'} = \dot{\underset{\sim}{\varepsilon}}^{vp} = \gamma \langle \phi(F) \rangle \frac{\partial Q}{\partial \underset{\sim}{\sigma}} \qquad \ldots (7)$$

in which t' is a scalar parameter representing pseudo time, γ is a fluidity parameter, Q is the plastic potential, for associated flow rule, $Q \equiv F$. The function $\phi(F)$ is a conditional function defined as follows:

$$< \Phi(F) > = \phi(F) \qquad \text{for } F \geq 0$$

$$< \Phi(F) > = 0 \qquad \text{for } F < 0 \qquad \ldots (8)$$

Choice of function $\phi(F)$ must be based on the rate dependent properties of materials. The simplest form chosen in this study is,

$$\phi(F) = F \qquad \ldots (9)$$

Numerical integration of equations of motion

The expressions for incremental velocity and displacement for Newmark implicit scheme (4) in terms of two free parameters γ' and β can be written as follows,

$$\Delta \dot{\underset{\sim}{u}}_t = \gamma' \Delta t \; \ddot{\underset{\sim}{u}}_{t+\Delta t} + (1 - \gamma') \Delta t \; \ddot{\underset{\sim}{u}}_t$$

$$\Delta \underset{\sim}{u}_t = \Delta t^2 \ddot{\underset{\sim}{u}}_{t+\Delta t} + (0.5 - \beta) \Delta t^2 \ddot{\underset{\sim}{u}}_t + \Delta t \dot{\underset{\sim}{u}}_t \qquad \ldots (10)$$

Using equations (10), $\Delta \ddot{\underset{\sim}{u}}_t$ and $\Delta \dot{\underset{\sim}{u}}_t$ can be expressed in terms of $\Delta \underset{\sim}{u}_t$, $\dot{\underset{\sim}{u}}_t$ and $\ddot{\underset{\sim}{u}}_t$ in equation (6) which leads to equation of following form,

$$\underset{\sim}{K}^* \Delta \underset{\sim}{u}_t = \underset{\sim}{R}^* \qquad \ldots (11)$$

in which,

$$\underset{\sim}{K}^* = \underset{\sim}{K} + \frac{\underset{\sim}{M}}{\beta \Delta t^2} + \frac{\underset{\sim}{C}}{\beta \Delta t}$$

$$\underset{\sim}{R}^* = \underset{\sim}{R}_{t+\Delta t} - \underset{\sim}{M} \left(\ddot{\underset{\sim}{u}}_t - \frac{\dot{\underset{\sim}{u}}_t}{\beta \Delta t} - \frac{\ddot{\underset{\sim}{u}}_t}{2\beta} \right) + \underset{\sim}{C} \left(\frac{\gamma'}{\beta} \dot{\underset{\sim}{u}}_t + \frac{\Delta t}{2\beta} \ddot{\underset{\sim}{u}}_t - \Delta t \ddot{\underset{\sim}{u}}_t - \dot{\underset{\sim}{u}}_t \right)$$

$$- \underset{\sim}{F}_t + d\underset{\sim}{F}_{vp} \qquad \ldots (12)$$

Solving equation (11) for $\Delta \underset{\sim}{u}_t$, the quantities $\ddot{\underset{\sim}{u}}_{t+\Delta t}$, $\dot{\underset{\sim}{u}}_{t+\Delta t}$ and $\underset{\sim}{u}_{t+\Delta t}$ can be obtained by making use of equation (10).

METHOD OF SOLUTION

The method of computing $d\underset{\sim}{F}_{vp}$, pseudo force in equation (12) is an important step in obtaining elastic-plastic solution. The method employed herein uses viscoplasticity as a solution technique and consists in obtaining steady state solution. In this approach an arbitraty value of γ is chosen and pseudo time stepping is used to let an equlibrium state, $F = 0$ be attained. The following steps are involved,

(i) If at the begining of a dynamic time step Δt, state of stress falls outside the yield surface ($F>0$), compute visco-plastic strain rate $\dot{\underset{\sim}{\epsilon}}^{vp}$ from equation (7).

(ii) Compute visco-plastic strain increment $d\underset{\sim}{\varepsilon}^{vp} = \dot{\underset{\sim}{\varepsilon}}^{vp} dt'$, dt' = pseudo time step and must be within stability limits for Euler integration (2).

(iii) Compute incremental pseudo force vector, $d\underset{\sim}{F}^{n}_{vp} = \int_V \underset{\sim}{B}^T \underset{\sim}{D} d\underset{\sim}{\varepsilon}^{vp} \, dv$.

(iv) Solve the equation $\underset{\sim}{K}^* \, d\underset{\sim}{u}_i = d\underset{\sim}{F}^n_{vp}$, $d\underset{\sim}{u}_i$ is the correction to $\underset{\sim}{u}_t$, compute updated $\underset{\sim}{u}_{t+\Delta t}$, $\ddot{\underset{\sim}{u}}_{t+\Delta t}$ from equation (10).

(v) Compute the incremental changes in strain and stresses,

$$d\underset{\sim}{\varepsilon}_n = \underset{\sim}{B} \Delta \underset{\sim}{u}_t$$

$$d\underset{\sim}{\sigma}_n = \underset{\sim}{D} \, (\underset{\sim}{B} \Delta \underset{\sim}{u}_t - d\underset{\sim}{\varepsilon}^{vp}) \qquad \ldots (13)$$

(vi) Compute the stress, strain and visco-plastic strain at t'_{n+1},

$$\underset{\sim}{\sigma}_{n+1} = \underset{\sim}{\sigma}_n + d\underset{\sim}{\sigma}_n$$

$$\underset{\sim}{\varepsilon}^{vp}_{n+1} = \underset{\sim}{\varepsilon}^{vp}_n + d\underset{\sim}{\varepsilon}^{vp} \qquad \ldots (14)$$

$$\underset{\sim}{\varepsilon}_{n+1} = \underset{\sim}{\varepsilon}_n + d\underset{\sim}{\varepsilon}_n$$

(vii) Repeat the steps (i) to (vi) till the steady state is reached, that is, $\dot{\underset{\sim}{\varepsilon}}^{vp}$ approaches zero.

(viii) The stresses, strains, displacements, velocities and acceleration at the end of steady state are the starting condition for the next dynamic time step.

The pseudo stiffness matrix $\underset{\sim}{K}^*$ is updated at each step of dynamic and pseudo time stepping. The above procedure is computationally efficient as pseudo time stepping is required only in those steps in which the yielding occurs. For materially nonlinear problems $\underset{\sim}{K}^*$ remains constant throughout the dynamic and pseudo time stepping and it is necessary to triangularize $\underset{\sim}{K}^*$ only once. The visco-plasticity has the additional advantage of its capability to deal with the situation of non-associated flow rule without any additional computing effort.

NUMERICAL EXAMPLES

Elastic-plastic small displacement response of simply supported beam

Figure 1 shows the elastic-plastic response of simply supported beam by visco-plastic approach. The dynamic response obtained by tangent stiffness approach using implicit scheme (1) is also shown in this figure. The two solutions are in good agreement.

Elastic-plastic small displacement response of a spherical shell

Figure 2 shows the elastie-plastic response of a spherical shell subjected to suddenly applied radial load. Comparison of the response is shown between the present study using visco-plastic approach and tangent stiffness approach (1). Both the approaches use the same integration parameters of Newmark implicit scheme. The two solutions are seen to be in good agreement.

Elastic-plastic large displacement response of a simply supported plate

Figure 3 illustrates the dynamic large elastic-plastic response of a simply supported plate subjected to a uniformly distributed load. The results compare well with those obtained by using explicit scheme (6).

Elastic-plastic large displacement response of a spherical shell

Figure 4 shows the dynamic elastic large displacement response of a spherical shell cap obtained by visco-plastic approach and explicit scheme (6). The results compare well by the two methods.

CONCLUSIONS

An application of visco-plastic material model to include material nonlinearity with the Newmark implicit scheme has been demonstrated for finite element system. The formulation used here also includes the effect of geometric nonlinearity in the Lagrangian framework which requires the computation of tangent stiffness matrix at every time step. The dynamic response results compare very well with those obtained by other techniques. The visco-plastic alogrithm with implicit time stepping scheme provides economic approach to deal with material nonlinear problems.

ACKNOWLEDGEMENTS

The author gratefully acknowledges the financial support of Science Research Council, U.K. with its assistance the work reported herein was carried out at University College of Swansea, U.K.

REFERENCES

1. Bathe, K.J., Ozdemir, H. and Wilson, E.L. (1974), 'Static and Dynamic Geometric Nonlinear Analysis', Report No. UC5csm 74-4, Structural Engineering Laboratory, University of California, Berkeley, California.

2. Cormeau, I., (1975), 'Numerical Stability in Quasi-Static Elasto Visco-plasticity', International Journal for Numerical Methods in Engineering, Vol. 9, 109-127.

3. Nagrajan, S. and Popov, E.P., (1975), 'Nonlinear Dynamic Analysis of Axisymmetric Shells', International Journal for Numerical Methods in Engineering, 9, 535.

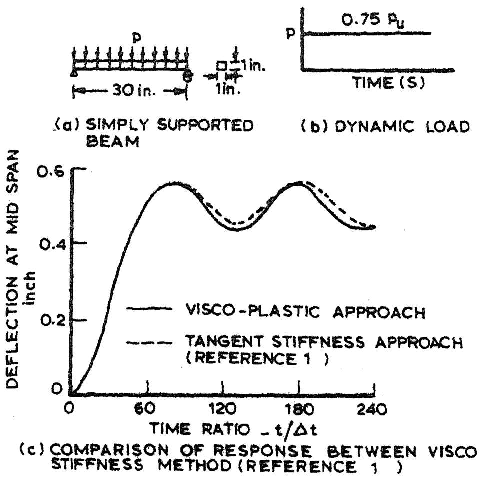
4. Newmark, N.M., (1959), 'A Method of Computation for Structural Dynamics', Proceedings, ASCE, EM3, 67-94.

5. Owen, D.R.J., Hinton, E. and Shantaram D. (1976), 'Non-linear Dynamic Transient Analysis of Plates Using Parabolic Isoparametric Elements', Proceedings of International Conference on Finite Elements in Engineering, University of Adelaide, Australia, Dec. 6-8.

6. Shantaram, D., (1976), 'Dynamic Transient Analysis of Structures Including Geometric and/or Material Nonlinearity Effects', Ph.D. Thesis, University of Wales, U.K.

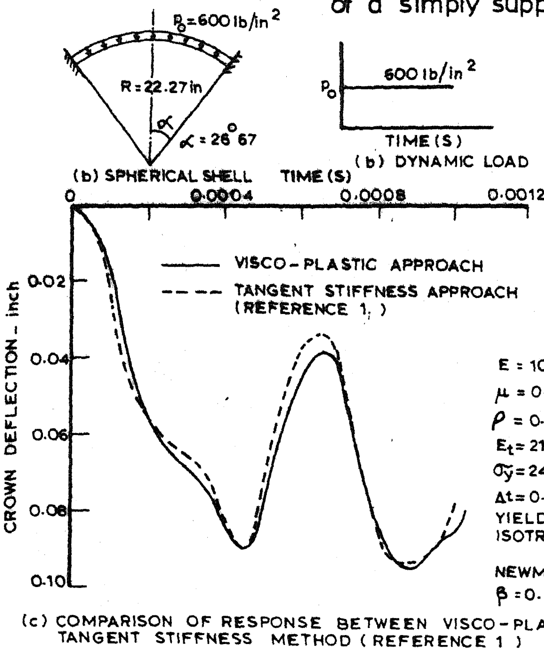
7. Thakkar, S.K. and Stagg, K.G. (1976), 'Non-linear Dynamic Analysis of Stress-Wave-Propagation Problems using Parabolic Isoparametric Finite Elements', Report of Science Research Council Project, University of Wales, Swansea.

8. Zienkiewicz, O.C. and Cormeau, I.C., (1974), 'Visco-plasticity and Creep in Elastic Solids - A Unified Numerical Solution Approach', International Journal for Numerical Methods in Engineering, Vol. 8, 821-845.



$E = 30000000\ lb/in^2$; $\mu = 0.3$; $\rho = 0.0007333\ lb\ s^2/in^4$; $\Delta t = 0.00005\ s$
$\sigma_y = 50000\ lb/in^2$; YIELD LAW – VON-MISES; P_u = ULTIMATE LOAD OF BEAM; FLOW RATE FUNCTION – $\phi(F) = F$; $\gamma = 1.0$
NEWMARK SCHEME: $\beta = 0.25$, $\gamma' = 0.50$; $\delta = 0.0$

Fig. 1 – Dynamic small displacement elastic-plastic response of a simply supported beam



$E = 10500000\ lb/in^2$
$\mu = 0.3$
$\rho = 0.000245\ lb\ s^2/in^4$
$E_t = 210000\ lb/in^2$
$\sigma_y = 24000\ lb/in^2$
$\Delta t = 0.000005\ s$
YIELD LAW – VON MISES WITH ISOTROPIC HARDENING
NEWMARK SCHEME: $\beta = 0.25$, $\gamma' = 0.5$; $\delta = 0.0$

Fig. 2 – Dynamic small displacement elasto-plastic response of a spherical shell

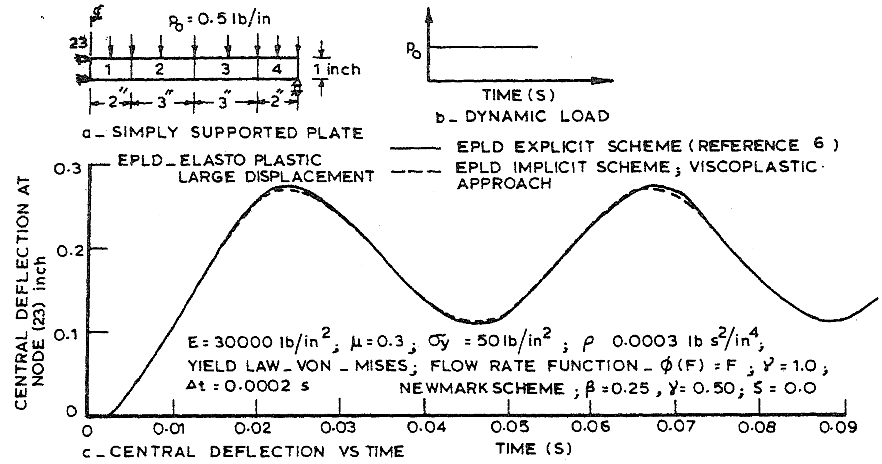


Fig. 3 - Dynamic elastic-plastic large displacement response of a simply supported circular plate

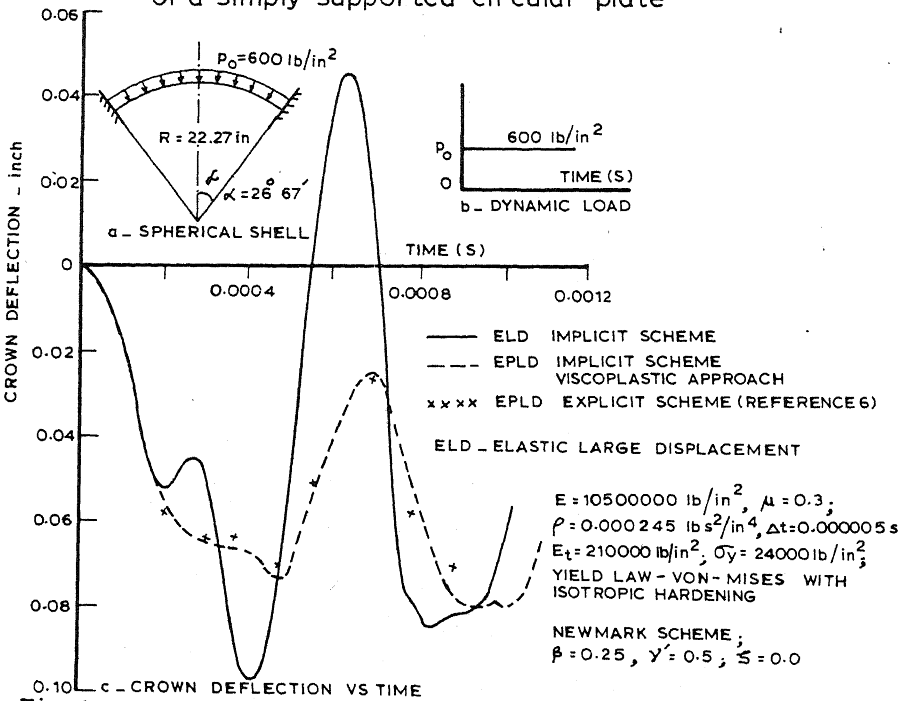


Fig. 4 - Dynamic elastic - plastic large displacement response of a spherical shell