ELASTO-PLASTIC DYNAMIC RESPONSE ANALYSIS OF FLEXURAL-SHEAR MODEL (AN APPRAISEMENT OF AN EXAMPLE OF THE APPLICATION OF PLANE-FRAME)

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SYNOPSIS

At present, the so-called equivalent-shear model (tri-diagonal maxtrix type) is often used as the structural dynamic model in elasto-plastic dynamic response analysis. However, the response values obtained are sometimes not resonable. In this article, the response for a flexural-shear model (full-matrix type) obtained by tracking the by application to an actual building design.

INTRODUCTION

The equivalent-shear model employed as a structural dynamic model faithfully represents the structural dynamic characteristics of buildings and the handling of elasto-plasticity is extremely simplified by assuming restoring force characteristics to be independent for each story. A more reasonable aseismic design for all types of response has been realized, which confirms the importance of the function of this model. However, the following problemmatical points have been indicated as regards this model.

1. Due to the continually increasing height of the buildings, gloval flexural deflection has become of primary importance and the restoring force characteristics cannot be regarded for each story independently but is now exceedingly complicated.

2. The response of the member level cannot be evaluated from the response of the story.

3. Regardless of the fact that the stiffness (rigidity) and the strength are continuous, at times plasticization is concentrated on a particular story.

In order to study above problems in more detail, it seems essential that the stiffness matrix of a building should be evaluated at all times by forming the elasto-plastic stiffness matrix of one member to that of another. Here, the matrix-hinge method is explained in the following section and a comparison of both models will be made.

PROBLEMS ON NUMERICAL CALCULATION

The vibration equation of a multi-mass system is expressed through the following formula:

$$[M]\{\ddot{x} + \ddot{x}_{0}\} + [C]\{\dot{x}\} + \{F\} = \{0\} \quad \cdots\cdots \quad (1)$$

"Eq. 1" is incremented and formed into "Eq. 2" if: $\{x\} = \{x'\} + \{\Delta x\}$, $\{F\} = \{F'\} + [k]\{\Delta x\}$ as follows:

$$[M]\{\Delta\ddot{x} + \ddot{x}_{0}\} + [C]\{\Delta\dot{x}\} + \{F'\} + [k]\{\Delta x\} = \{0\} \quad \cdots\cdots \quad (2)$$

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The solution can be obtained through step-by-step numerical calculation from linear acceleration or other method. A problem arising here is the treatment of the unbalance force due to plasticity originated at a certain incremental stage. That is, when the member end yields at a certain increment, generally, the yield point passed through the yield surface. This can be treated by such methods as returning to the previous step to make the unbalance force smaller by reducing the increment still further, by obtaining the increment up till the yield point due to linear projection at the member ends, etc. However, this is actually inapplicable for high storied framework where multi-degree of freedom and a large number of members are involved. Here, "Eq. 2" is solved only through the principal variable (dregree of freedom with mass) and by introducing the interior diaplacement due to unbalance force into the dependent variable (degree of freedom without mass) the accumulation was prevented. That is, in $\{F\} = [k]\{\Delta x\} + \{F'\}$ by resolving the main variable ($_0$) and dependent variable ($_1$), it can be expressed as "Eq. 3" and this $\{F'\}$ is the unbalance force for the preceding step.

$$\begin{Bmatrix} F_0 \\ F_1 \end{Bmatrix} = \begin{bmatrix} k_{00} & k_{01} \\ k_{10} & k_{11} \end{bmatrix} \begin{Bmatrix} \Delta x_0 \\ \Delta x_1 \end{Bmatrix} + \begin{Bmatrix} F'_0 \\ F'_1 \end{Bmatrix} = \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix} \quad \cdots\cdots \quad (3)$$

$$\{\Delta x_1\} = -[k_{11}]^{-1}[k_{10}]\{\Delta x_0\} - [k_{11}]^{-1}\{F'_1\} = [k^*_{10}]\{\Delta x_0\} + \underbrace{\{\Delta x^*_1\}}_{\text{interior displacement}}$$

$$\{F_0\} = \left[[k_{00}] - [k_{01}][k_{11}]^{-1}[k_{10}]\right]\{\Delta x_0\} + \{F'_0\} - [k_{01}][k_{11}]^{-1}\{F'_1\}$$

$$= [k^*_{00}]\{\Delta x_0\} + \{F'^*_0\} \quad \cdots\cdots \quad (4)$$

That is through this $\{F'^*_0\}$, $\{F'\}$ for "Eq. 2" indicates the restoring force vector. From this, the unbalance force generated in the preceding step is eliminated for the succeeding step. In addition ' indicates the preceding step, while an explanation of the symbols will be omitted.

COMPUTER ANALYTICAL METHOD AND ASSUMPTION UNDER ELASTO-PLASTICITY

For dynamic analysis of free shaped buildings, the following 4 systems come to mind. The conversion to each of these systems is made possible by preparing transformation matrix through correspondence of degree of freedom.

1. Member System ..... Indicates a system formed from member with columns and beams. For non-linear form of the member, the axial force-bending moment should conform to the Flow-Roll of the Mises. The yield surface is replaced with parabola.

2. Local System ..... Is defined by a sub-matrix through member collection. Consequently, a single system is practical even if folating in mid-air such as floor slabs.

3. Gloval System ..... Indicates the matrix for an entire building with a closed system which is not unstable, and consists of a collection of frames in a free plane.

4. Vibration System ..... Indicates the particle model of the discrete system composed of $[M]$, $[C]$, $[K]$ and is defined by the degree of freedom of the principal variable as treated in the preceding section.

ANALYTICAL MODEL

A rigid frame of 15 storied composite construction composed of 2 frames as shown in "Fig.1" is considered. In this case, after considering symmetry, 1/2 model was selected with fixed G floor, while analysis was conducted on the following 2 models:

1. Equivalent-shear model (E.S model) ..... The bi-linear restoring force model was considered, in which, the shearing rigidity was obtained from design load, (story restoring force/story deflection) and the yield strength was taken as the story restoring force when forming the mechanism. The distributions of shearing rigidity and yield strength are shown in "Fig.2".

2. Flexural-shear model (F.S model) ..... The elasto-plasticity of the member was pursued manually piece by piece through the hinge-method, as given in the analytical method.

The natural period of both models is shown in "Tab.1". In addition, as regards earthquake waves, the setting was made with the maximum acceleration of 0.5g which was the wave observed during the earthquake off Miyagi in 1978.

ANALYTICAL RESULTS AND APPRAISEMENT

The restoring force characteristics for the 3rd story is shown in "Fig.3a" and "3b". A comparison of both models shows the restoring force for F.S model as bi-linear and the assumption as based on the E.S model is found to be correct in all respects. However, for the upper stories complicated forms were observed. The restoring force of an rigid frame was found to be poli-linear in form as is generally believed. "Fig.4" shows the M-N profile at the base of column. Due to transition of the inflection point of the moment, the results are complicated. It can be seen that flow occurs in certain parts due to yield. "Fig.5" shows the Moment-Rotation relations of the beam ends. The relations are of the elasto-plastic type for the hinge-method. In addition, the ductility factor at the beam end is about 1.4 times that for the story. "Fig.6" shows the distribution of maximum story deflection. According to this figure, for the E.S model response, in spite of the fact that rigidity and strength are changing continuously as shown in "Fig.2", it is clear that story slippage phenomena is originated and that response is unnatural. However, for the F.S model response is seen to expand to the upper and lower stories and is smooth in the same manner as for linear (elastic) response. The reason for this is thougth to lie in the fact that in the case of beam yield, the column within the elastic range exerts a pulling-in force on the upper and lower stories as a long column, but in the case of column yield, the same form will be shown as for E.S model response.

As seen from the above, although it only applies to one model, in flexural-shear model response, a reasonable response form is obtained and it is believed that it will significantly affect future dynamic designs.

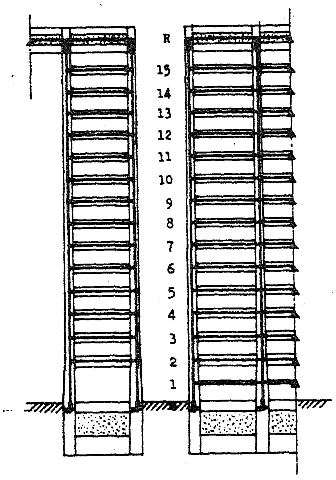

Fig.1 Frame model

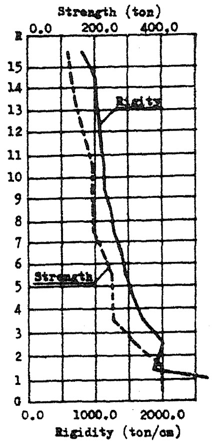

Fig.2 Rigidity and strength (E.S model)

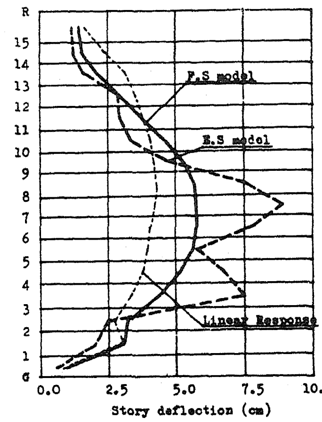

Fig.6 Comparison of the maximum story deflection

F.S model	
T1	1.48 sec
T2	0.53 "
T3	0.32 "
E.S model	
T1	1.48 sec
T2	0.50 "
T3	0.28 "

Tab.1 Natural period of the model

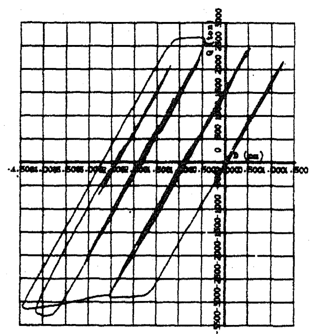

Fig.3a Q-D relation of the 3rd story (F.S model)

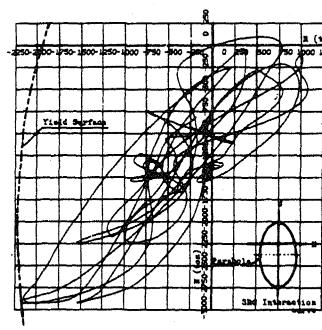

Fig.4 M-N relation at the column base of G floor

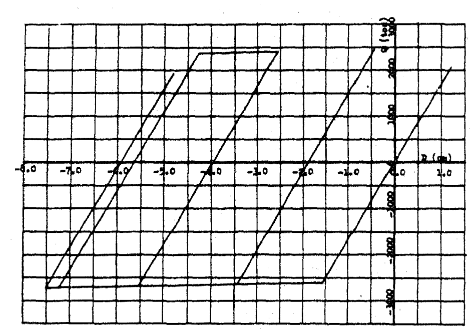

Fig.3b Q-D relation of the 3rd story (E.S model)

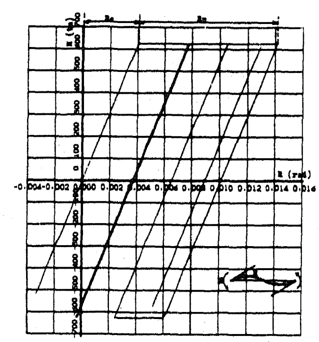

Fig.5 M-R relation of the beam end of 4th floor