FREE VIBRATION OF ARCHES
WITH AXIAL LOADING

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INTRODUCTION

Although the free vibration of arches has been the subject of numerous investigations, only a limited amount of numerical data has yet been reported, particularly with respect to the higher natural frequencies and modes of the system. The studies reported in Refs. 1, 3 and 4 are an exception to this statement which provide a comprehensive analysis of the free vibrational characteristics of uniform, circular arches vibrating in their own planes. The studies in Refs. 3 and 4 are based on a theory which neglects the effects of either rotatory inertia or shearing deformation, or both. The effects of these factors are considered in Ref. 1 on the assumption that the effect of the axial forces on the deformation of the arch is negligible.

The present investigation, however, supplies an analysis of the in-plane, free vibration of uniform, circular arches considering the effects of both the axial load and the rotatory inertia but neglecting the effect of the shearing deformation in the arch. Both hinged- and fixed- ended arches having an angle of opening of 90° are considered. The arch is considered to be subjected to a dead load, q, of various values uniformly distributed along the arch-axis.

The method of analysis used in the study is that of finite elements. The method essentially involves considering the arch as a series of small segments; then developing the mass, stiffness and nodal force matrices for the whole system to solve for the natural frequencies and the associated modes. The method considers large deflections but small strains. It is worth noting that the displacement functions used in deriving the element matrices throughout the solution are those corresponding to linear theory for the unloaded segment.

ANALYSIS OF RESULTS

Numerical solutions are obtained for the eight lowest natural frequencies of both fixed- and hinged- ended arches having an angle of opening of 90°. The related numerical data is presented in Ref. 2. The solutions are obtained giving due regard to the effects of th- axial forces induced by the dead weight of the arch. The circular natural frequencies are expressed in the form

$$p = \frac{C}{S^2} \sqrt{\frac{EI}{m}}$$

where S = the length of the arch axis; EI = the flexural rigidity of the cross-section; m = mass per unit length of the arch; and C is a dimensionless frequency coefficient which depends on the boundary conditions, the slenderness ratio of the arch, S/r, and the intensity of the lateral load, q. The values of the dimensionless frequency parameter, C_n, for the first eight natural modes of vibration of hinged and fixed arches are obtained as a function of S/r for several values of the load ratio, q/q_{cr}. Here q represents the load per unit of length of the arch axis and q_{cr}. is the associated buckling load assuming the arch axis to be inextensible.

A close study of the results reveals that the frequency coefficients tend to decrease with q along the parts for which the deformations are due primarily to flexure. It is further observed that the decrease in C_n with q is moreor less independent of the slenderness ratio, S/r. Based on the results, it is further concluded that the reduction in the frequency values corresponding to the flexural modes may be estimated from the equation

$$p = p_o \sqrt{1-q/q_{cr}}$$

where p_o is the frequency of the arch with q = 0. The approximate values as determined by using this equation compares quite well with those obtained from the solution of the problem.

However, along the parts where the vibration is predominantly extensional, the frequency curves have an unexpected behaviour. The frequency coefficient tends to increase with increasing intensity of the grav ity load, q. This trend is caused by the reduction in the average or effective radius of the arch due to the uniform gravity load, q. Under the influence of the load, the arch does not remain circular, but its average or mean radius decreases, and this causes a corresponding increase in the frequency. It should be recalled that the frequencies of the extensional modes are inversely proportional to the arch radius.

In the course of the investigation, a study has also been made of the buckling load of the arch considering the extensibility of the arch axis. The buckling loads of fixed- and hinged- ended arches with 90° angle of opening have been found as a function of the slenderness ratio, S/r. It is worth noting that with increasing S/r, the values of the buckling load in each case approaches to the buckling load as obtained from the inextensible theory.

Moreover, information has been gathered on the shapes of the natural modes of arches for several different combinations of parameters involved. The data revealed that the differences between a loaded and unloaded arch are not significant, indicating that the effect of axial forces on the natural modes are relatively unimportant.

CONCLUSION

A study of the data based on a theory considering both the effect of the axial load and the rotatory inertia shows that the axial load decreases the frequencies of the predominantly flexural modes and increases the frequencies of most of the extensional modes in an arch. Further it has been shown that the buckling load of the loaded arch is a function of the slenderness ratio; but tends to approach asymptotically to the buckling load as obtained from the inextensible theory, as the slenderness ratio increases.

REFERENCES

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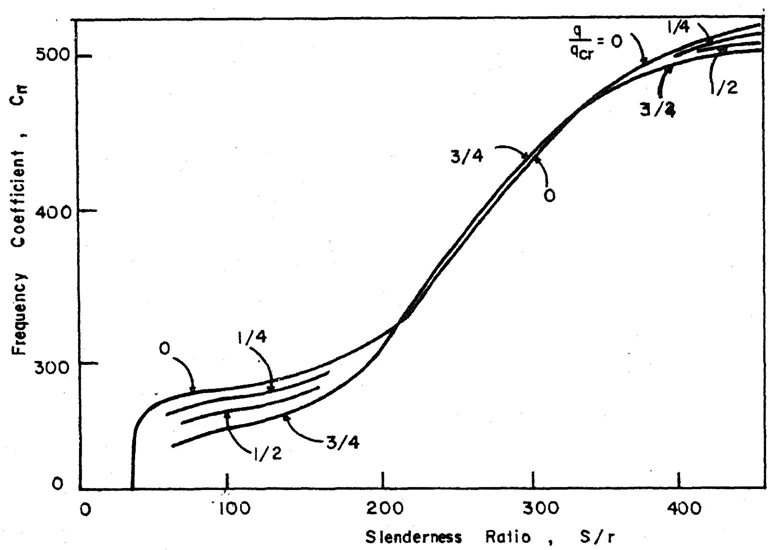

FIG. 1 _ FREQUENCY CURVES FOR THE 3rd. SYMMETRIC MODE OF FIXED ARCHES

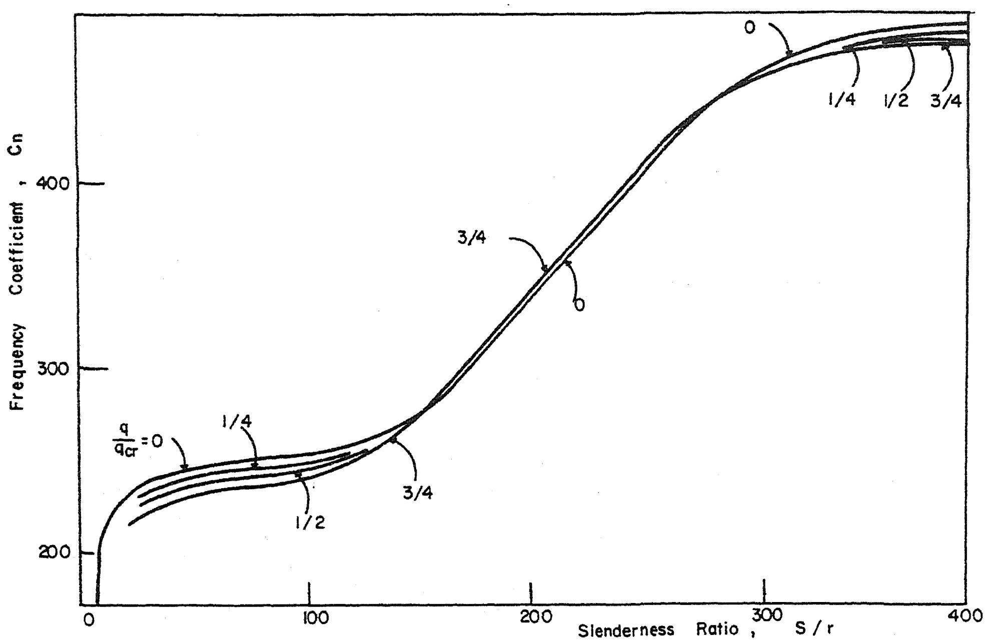

FIG. 2 _ FREQENCY CURVES FOR THE 3rd. SYMMETRIC MODE OF HINGED ARCHES

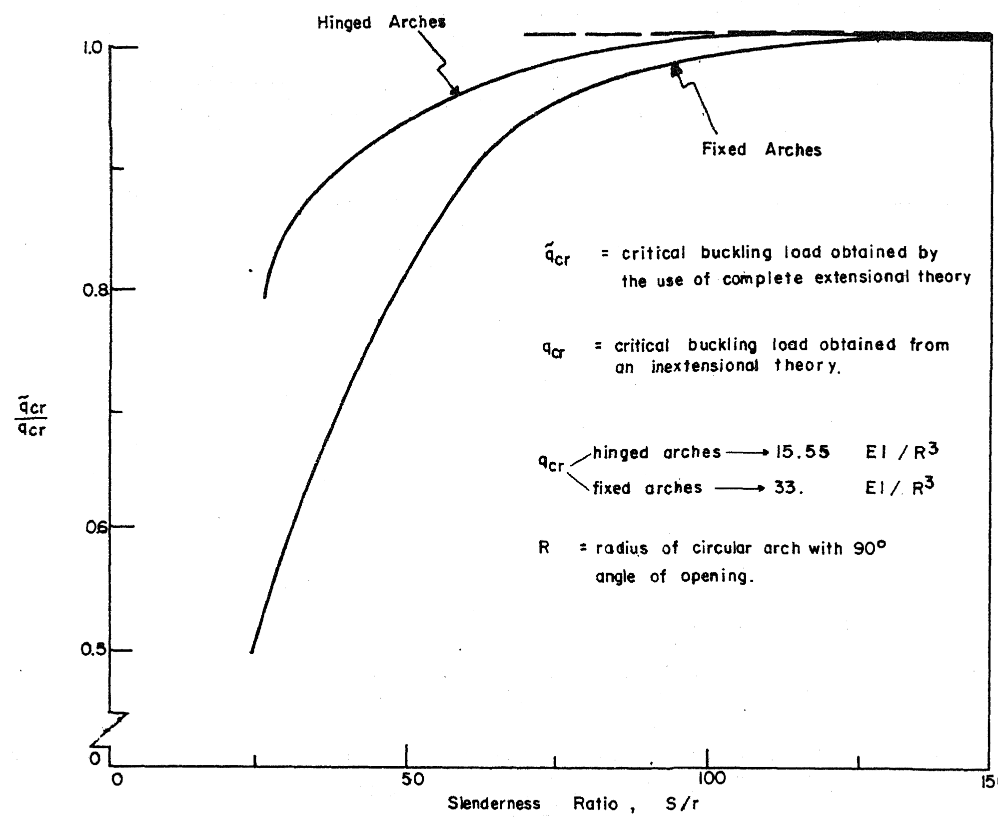

FIG. 3 _ NORMALIZED BUCKLING LOADS FOR ARCHES