PROBLEMS IN THE USE OF ROOT-SUM-SQUARE SOLUTIONS FOR THREE-DIMENSIONAL DYNAMIC ANALYSIS OF BUILDINGS

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SUMMARY

Numerous computer programs have been developed for linear structural analysis of three-dimensional (3-D) frame and shear wall buildings subjected to both static and earthquake loadings. These programs are very helpful and have been used worldwide by engineers for structure design of buildings.

In the earthquake analysis of buildings, most engineers prefer the square-root-of-the-sum-of-the-squares, or root-sum-square (RSS), method in the design of structural members. However, if the engineer is not aware of the errors associated with using this method for 3-D problems, some members will be substantially underdesigned. A new RSS 3-D method is shown to improve the results of modal combination.

INTRODUCTION

In earthquake analysis of buildings, it is assumed for the majority of structures that the floors are rigid in their own plane and that horizontal lateral loads act at floor levels, transferring the loads to columns and shear wall elements through rigid floor diaphragms. This results in three displacement degrees of freedom at each floor level: translation in the X and Y directions and rotation about the vertical axis. (Note, however, that tall, slender buildings have rotation about the two horizontal axes also.) The building's structural stiffness matrix is thus expressed in terms of the X and Y translations as well as the diaphragm rotation Θ.

In dynamic analysis, the response of the building is defined as the motion of each floor level described by the X, Y, Θ coordinate system, and the mass of the building is assumed to be lumped at each floor level. The vibration mode shapes are therefore calculated in the three directions of the component X, Y, and Θ coordinates. The actual vibration mode shapes, however, occur in the principal Φ_u, Φ_v, and Φ_θ axes, which when determined from the structural properties are not always identical to the coordinate axes of the structural geometric input.

In Fig. 1, diagrams a and b show the condition when the coordinate axes X, Y, Θ and Φ_u, Φ_v, Φ_θ are congruent. In this situation, if earthquake motion is not input in the X or Y direction, the calculated response of members a and c will not be a correct RSS solution. Diagrams c and d of Fig. 1 show the condition when X, Y and Φ_u, Φ_v are not congruent, a much more complicated case; even if the earthquake motion is input in the X or Y direction, the calculated response of members a and c will not be correct. In both cases, the error is caused by inappropriate application of the RSS method.

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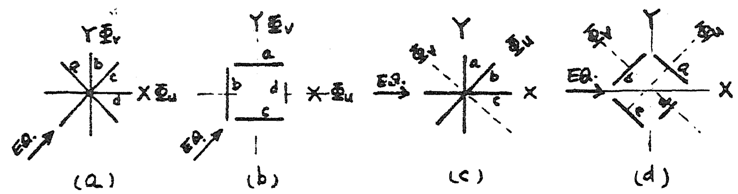

Figure 1--Structure plan and earthquake input direction

PROBLEMS OF MODAL RESPONSE COMBINATION

The general problems associated with modal response combination can be listed as follows:

1. The RSS method was developed for response spectrum analysis of two-dimensional (2-D) structures, with computer programs combining the response parameters for all calculated modes by the RSS method.
2. The principal axes (Φ_u, Φ_v, Φ_t) are defined from the vibrational frequencies and mode shapes of the structure, and the geometric coordinate axes of the structure (X, Y, Z) are selected by the engineer. In 2-D structures, X, Z and Φ_u, Φ_t are congruent; in 3-D structures, however, X, Y, Z and Φ_u, Φ_v, Φ_t do not always match.
3. If the earthquake input motion is applied in the direction of a principal axis, the response of the structure is likely to be a 2-D vibration; therefore, the 2-D RSS method is appropriate.
4. The results of member forces obtained by the 2-D RSS method are suspect when the earthquake input motion is applied in a direction other than that of a principal axis, because the input force is divided into two or three components acting along the principal axes. The members located along principal axes exhibit 2-D vibration only; however, the inclined members exhibit 3-D vibration and require proper combination of the 3-D force components.
5. If the structure is unsymmetric, a rotational component will be present and should be included in the combination also.

THE RSS 3-D METHOD

To determine the resultant response in 3-D structures, the principal components, Φ_u, Φ_v, and Φ_t, should be combined, either before the RSS method is applied (note that for purposes of member design no negative sign is allowed):

$$\sqrt{\Sigma(u_i + v_i + t_i)^2} \qquad \text{3-D RSS}$$

or after the RSS of each component has been calculated:

$$\sqrt{\Sigma u_i^2} + \sqrt{\Sigma v_i^2} + \sqrt{\Sigma t_i^2} \qquad \text{RSS 3-D}$$

When the structural members are not located along the principal axes of the structure, or when the structure is unsymmetric and its vibration mode shape includes rotation about the vertical axis, using the conventional 2-D RSS method produces errors and can result in substantially underdesigned structural members:

$$\text{2-D RSS} < \text{RSS 3-D} < \text{absolute sum}$$

NUMERICAL EXAMPLES

The following examples are for a two-story building with the story mass and earthquake response spectrum shown in Fig. 2. The mass moment of inertia was assumed to be small so that the rotational modes would be the highest modes, thus making the 3-D components more distinct. Both examples show the member shear forces at the first story.

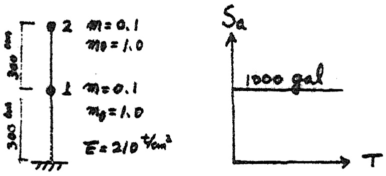

Figure 2--Story mass and earthquake response spectrum

Example 1 is a typical axisymmetric building. Eight identical shear wall panels, 12 cm x 250 cm, are oriented as shown in Fig. 3. The modal properties shown in Table 1a indicate that the mode shapes (Φ_u, Φ_v, Φ_θ) and the structural geometric axes (X, Y, θ) are congruent. When earthquake motion is applied along the principal axis Φ_u (X), the modal and RSS panel shears shown in Table 1b result. Because panels a, b, c, and d all exhibit 2-D vibration in the Φ_u (X) direction only and the structure has no response in the Φ_v (Y) direction, the 2-D RSS method can be used with confidence. The greatest force is distributed to panel d; there is no force on panel b; and the forces on panels a and c are identical.

If the earthquake motion is not input along a principal axis, the input force is separated into two components, and modal and RSS panel shears such as those shown in the example given in Table 1c for input motion inclined 45° to the Φ_u axis will result. In the example shown here, 0.707 S_a is applied in the Φ_u direction, and 0.707 S_a is applied in the Φ_v direction. Panels b and d, located along principal axes, exhibit 2-D vibration in the Φ_v or Φ_u direction only; but panels a and c, inclined to principal axes, respond in 3-D vibration to the component forces from the Φ_u and Φ_v directions at each time step. Therefore, combination of the two components is required before the RSS method can be applied. Table 1d shows improved results for panels a and c with this proposed use of the RSS 3-D method. Note that the results for panels a and c shown in Table 1d are congruent with those for panels b and d shown in Table 1b.

In Example 2, three 12-cm-x-500-cm panels and one 12-cm-x-250-cm panel are oriented as shown in Fig. 4. This configuration is common in nonsymmetrical building systems. The mode shapes for the principal axes are shown in Table 2a. The input (X, Y) axes and the principal (Φ_u, Φ_v) axes are not congruent. Moreover, the mode shapes are coupled with rotational modes about the vertical axis. Modal and RSS panel shears are shown in Table 2b. Even if earthquake motion is applied in the X direction, a problem similar to that just described, in which motion is applied at 45° to the principal axes, will result. Furthermore, a force component arises in panels a and c as a result of rotation about the vertical axis that is induced by vibration along the other principal axis at each time step, and this force should be included in the combination of force components. The RSS 3-D method improves the results, as shown in Table 2c. Panels a and c receive the same force from the Φ_v direction; however, force from the Φ_u direction causes rotation, which reduces the force on panel a but increases it on panel c. (Note that for member design purposes, deduction of tortional effects is not allowed.)

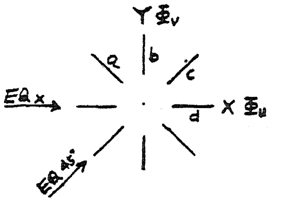

Figure 3--Plan, Example 1

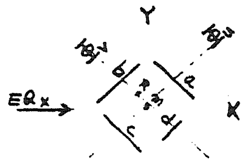

Figure 4--Plan, Example 2

TABLE 1--RESULTS FOR EXAMPLE 1

1a

Axis	Mode Shape 1 ($1\phi_v$)	 2 ($1\phi_u$)	 3 ($2\phi_u$)	 4 ($2\phi_v$)	 5 ($1\phi_\theta$)	 6 ($2\phi_\theta$)
X_2	0.	3.0114	0.9651	0.	0.	0.
X_1	0.	0.9651	-3.0114	0.	0.	0.
Y_2	3.0114	0.	0.	0.9651	0.	0.
Y_1	0.9651	0.	0.	-3.0114	0.	0.
θ_2	0.	0.	0.	0.	0.9523	0.3052
θ_1	0.	0.	0.	0.	0.3052	-0.9523

1b

Mode	Period	Mode Shape Direction, ϕ	Panel Member a	 b	 c	 d
1	0.1496	v	0.	0.	0.	0.
2	0.1496	u	26.271	0.	26.271	37.153
3	0.0225	u	6.958	0.	6.958	9.839
4	0.0225	v	0.	0.	0.	0.
5	0.0014	θ	0.	0.	0.	0.
6	0.0002	θ	0.	0.	0.	0.
		RSS	27.177	0.0	27.177	38.434

1c

Mode	Period	Mode Shape Direction, ϕ	Panel Member a	 b	 c	 d
1	0.1496	v	-18.577	26.271	18.577	0.
2	0.1496	u	18.577	0.	18.577	26.271
3	0.0225	u	4.920	0.	4.920	6.958
4	0.0225	v	-4.920	6.958	4.920	0.
5	0.0014	θ	0.	0.	0.	0.
6	0.0002	θ	0.	0.	0.	0.
		RSS	27.177	27.177	27.177	27.177

1d

Recommended 3-D RSS Shear Force

Panel a

3-D RSS $\sqrt{(-18.577 + 18.577)^2 + (4.920 - 4.920)^2} = 0.$

RSS 3-D $-\sqrt{18.577^2 + 4.920^2} + \sqrt{18.577^2 + 4.920^2} = 0.$

Panel c

3-D RSS $\sqrt{(18.577 + 18.577)^2 + (4.920 + 4.920)^2} = 38.434$

RSS 3-D $\sqrt{18.577^2 + 4.920^2} + \sqrt{18.577^2 + 4.920^2} = 38.434$

Remarks--

1. The equations for 3-D RSS and RSS 3-D are not mathematically identical; however, in all of the examples given when the modal responses are combined, the results are congruent. For computer use, the RSS 3-D equation is recommended; for hand-check purposes, the 3-D RSS equation is more convenient.

TABLE 2--RESULTS FOR EXAMPLE 2

2a

Axis	Mode Shape 1 ($1\phi_u$)	 2 ($1\phi_v$)	 3 ($2\phi_u$)	 4 ($2\phi_v$)	 5 ($1\phi_\theta$)	 6 ($2\phi_\theta$)
X_2	-2.1294	-2.1294	-0.6824	-0.6824	0.0019	0.0006
X_1	-0.6824	-0.6824	2.1294	2.1294	0.0006	-0.0019
Y_2	-2.1294	2.1294	-0.6824	0.6824	0.0019	0.0006
Y_1	-0.6824	0.6824	2.1294	-2.1294	0.0006	-0.0019
θ_2	0.0008	0.	0.0003	0.	0.9523	0.3052
θ_1	0.0003	0.	0.0008	0.	0.3052	-0.9523

2b

Mode	Period	Mode Shape Direction, ϕ	Panel Member a	 b	 c	 d
1	0.1164	u	-35.576	91.481	35.576	20.329
2	0.0772	v	55.905	0.000	55.905	0.
3	0.0175	u	-9.422	24.227	9.422	5.384
4	0.0116	v	14.806	0.000	14.806	0.
5	0.0002	θ	0.0001	0.0001	0.0001	0.
6	0.00003	θ	0.0000	0.0000	0.0000	0.
		RSS	68.549	94.635	68.549	21.030

2c

Recommended 3-D RSS Shear Force

Panel a

3-D RSS $\sqrt{(-35.576 + 55.905)^2 + (-9.422 + 14.806)^2} = 21.030$

RSS 3-D $-\sqrt{35.576^2 + 9.422^2} + \sqrt{55.905^2 + 14.806^2} = 21.030$

Panel c

3-D RSS $\sqrt{(35.576 + 55.905)^2 + (9.422 + 14.806)^2} = 94.635$

RSS 3-$ $\sqrt{35.576^2 + 9.422^2} + \sqrt{55.905^2 + 14.806^2} = 94.635$

2. Because each modal maximum occurs at a different time and has a unique sign, using the RSS 3-D method to calculate the maximum design value is as realistic as the 2-D RSS is for a 2-D structure. Note, however, that neither method approaches the answer obtained by using the absolute-sum method.
3. In a multistory building, the principal axes of the structure are not congruent at each story level for each mode. The incongruence is a function of the complex distribution of the story mass and stiffness. Further study is needed to develop a system to properly analyze complex 3-D structures by the response spectrum method.

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