SIMPLIFIED METHOD FOR COMPUTATION OF EARTHQUAKE INDUCED SHEARS AND OVERTURNING MOMENTS IN REGULAR MULTISTOREY STRUCTURES

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SUMMARY

Design charts for the approximative determination of earthquake induced shears and overturning moments are presented. The charts enable the elastic solution for any regular multistorey structure to be obtained by linearly combining the solutions of two basic models: the flexural and the shear beam. Lateral displacements of the structure due to uniform static lateral load are the information needed for the computation.

OBJECT AND SCOPE

Current seismic design codes of most countries use simple distribution of seismic force over the height of a building to determine shears and overturning moments, which does not take into account the type of the structure and the higher mode effects. The most important parameters determining the dynamic characteristics of a structure are (beside the mass distribution) the magnitude and the shape of deformation the structure undergoes when loaded laterally. Knowing the deformation and the mass distribution throughout the height of a building it is possible to determine quite accurately the natural periods and the associated mode shapes of the structure, which represent the basis of the elastic dynamic analysis.

According to the type of lateral deformation different mathematical models ranging from flexural beam to shear beam can be used for regular multistorey buildings. The flexural beam model represents closely a structure with uncoupled shear walls, while the shear beam model represents a frame structure with rigid girders. Other types of structures (coupled shear walls, frames with flexible girders, all types of shear wall frame assemblies) may be idealized as cantilevers deforming both in shear and bending.

Response spectrum method has been used and shears and overturning moments in two basic models have been determined using the exact analytical solutions. The solution for any other model can be obtained by linearly combining the solution of both basic models. The combination coefficients have been numerically determined as a function of a single parameter: the ratio of the lateral displacement at the top and the middle due to the uniform static lateral load. In Fig.1 design charts and formulae for computation of shears and overturning moments in regular (with constant

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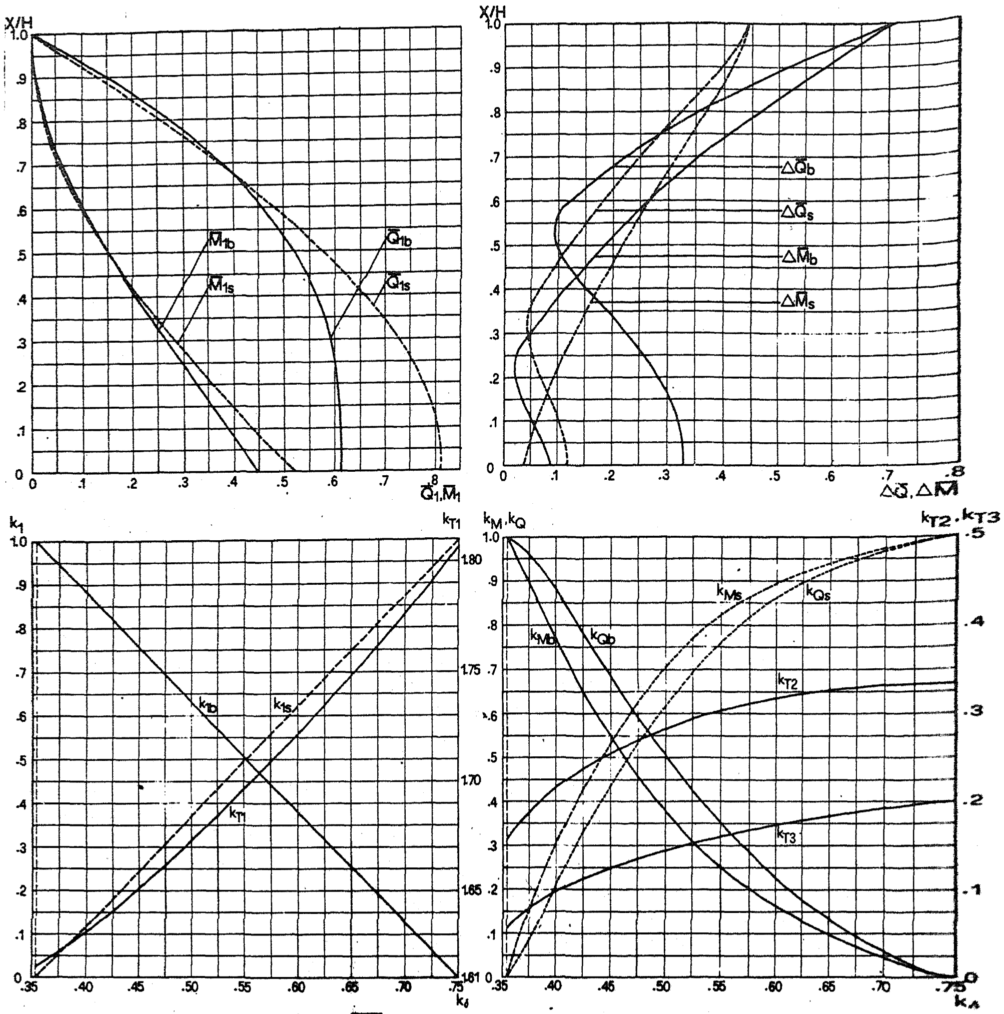

$k_\delta = \delta_M/\delta_T \quad , \quad T_1 = k_{T1}\sqrt{\delta_T}$

$T_2 = k_{T2}T_1 \; , \; T_3 = k_{T3}T_1 \; , \; c = S_{a2}/S_{a1}$

$\overline{Q}_1 = k_{1b}\,\overline{Q}_{1b} + k_{1s}\,\overline{Q}_{1s}$

$\Delta\overline{Q} = \sqrt{\Sigma\,\overline{Q}^2 - \overline{Q}_1^{\,2}}/\overline{Q}_1 = k_{Qb}\Delta\overline{Q}_b + k_{Qs}\,\Delta\overline{Q}_s$

$\overline{M}_1 = k_{1b}\,\overline{M}_{1b} + k_{1s}\,\overline{M}_{1s}$

$\Delta\overline{M} = \sqrt{\Sigma\,\overline{M}^2 - \overline{M}_1^{\,2}}/\overline{M}_1 = k_{Mb}\Delta\overline{M}_b + k_{Ms}\,\Delta\overline{M}_s$

$$\overline{Q} = \overline{Q}_1\sqrt{1 + (c\,\Delta\overline{Q})^2}\;,\quad \overline{M} = \overline{M}_1\sqrt{1 + (c\,\Delta\overline{M})^2}$$

$$Q = m\,H\,S_{a1}\,\overline{Q}\;,\qquad M = m\,H^2 S_{a1}\,\overline{M}$$

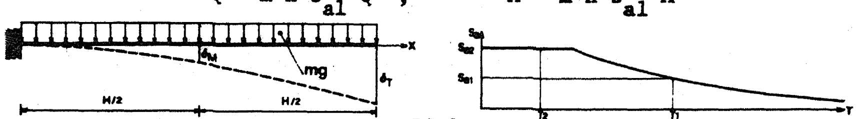

FIG.1

properties throughout the height of the building) multistorey structures are given. The higher mode effects and arbitrary design response spectrum can be taken into account.

NUMERICAL EXAMPLE

As a numerical example a ten-storey shear wall frame, as shown in Fig.2, is investigated. The design spectrum according to Fig.3 is used. Lateral displacements at the top and at the middle of the structure due to the uniform lateral static load mg are determined by using tables in [1]:

$$\delta_T = 0.735 \text{ m} \ , \ \delta_M = 0.409 \text{ m} \ , \ k_\delta = 0.409/0.735 = 0.556$$

From Fig.1 we find

$$k_{T1} = 1.70 \ , \ k_{T2} = 0.31 \ , \ k_{T3} = 0.165$$

$$k_{b1} = 0.49 \ , \ k_{s1} = 0.51 \ , \ k_{Qb} = 0.34 \ , \ k_{Mb} = 0.24 \ , \ k_{Qs} = 0.77 \ , \ k_{Ms} = 0.83$$

Hence

$$T_1 = 1.70 \sqrt{0.735} = 1.46\text{s}, \ T_2 = 0.31 \times 1.46 = 0.45\text{s}, \ T_3 = 0.165 \times 1.46 = 0.24\text{s}$$

$$S_{a1} = (0.075/1.46) \times 9.81 = 0.50 \text{ m/s}^2, S_{a2} = S_{a3} = 0.15 \times 9.81 = 1.47 \text{ m/s}^2$$

$$c = 1.47/0.50 = 2.94$$

The calculated shears and overturning moments are plotted in Fig.4. The comparison of these values and values, determined by elastic dynamic analysis using a standard computer program,has shown that the difference was about 3 %. The distribution of shears and moments to shear wall and frame may be performed according to usual static procedures, e. g. [1].

REFERENCE

[1] Rosman R., Statik und Dynamik der Scheibensysteme des Hochbaues, Springer-Verlag 1968.

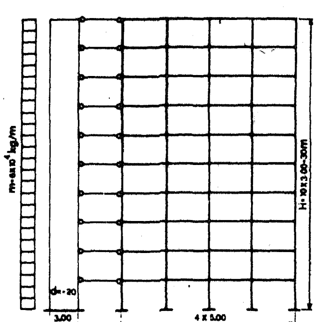

FIG.2

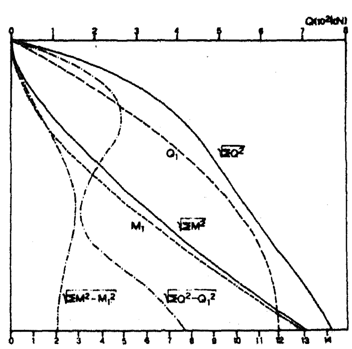

FIG. 4

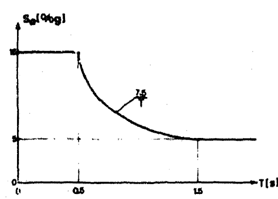

FIG. 3

APPENDIX : ANALITICAL EXPRESSIONS

$$Q_n(x) = m\ S_{an}\ \Gamma_n \int_x^H \Phi_n(x)dx$$

$$M_n(x) = m\ S_{an}\ \Gamma_n \int_x^H x\ \Phi_n(x)dx - x\ Q_n(x)$$

$$\Gamma_n = \frac{\int_0^H \Phi_n(x)dx}{\int_0^H \Phi_n^2(x)dx}$$

$$\bar{x} = x/H$$

Shear beam

$$\omega_n = b_n \sqrt{G\ A_s/(m\ H^2)}\ , \quad \Phi_n(\bar{x}) = \sin b_n\bar{x}$$

$$b_n = (n - 0.5)\ \Pi\ , \qquad \Gamma_n = 2/b_n$$

$$Q_n(\bar{x}) = m\ H\ S_{an}\ \Gamma_n/(b_n) \cdot \cos b_n\bar{x}$$

$$M_n(\bar{x}) = m\ H^2 S_{an}\ \Gamma_n/(b_n{}^2)\ [(-1)^{n+1} - \sin b_n\bar{x}\ ]$$

Flexural beam

$$\omega_n = b_n{}^2 \sqrt{E\ J/(m\ H^4)}\ , \quad \Phi_n(\bar{x}) = A_n\ SHM_n(\bar{x}) + CHM_n(\bar{x})$$

$$b_1 = 1.8751\ , \quad b_2 = 4.6941\ , \quad b_3 = 7.8548$$

$$A_1 = -0.7341\ , \quad A_2 = -1.0185\ , \quad A_3 = -0.9992$$

$$\Gamma_1 = 0.7830\ , \quad \Gamma_2 = 0.4339\ , \quad \Gamma_3 = 0.2544$$

$$CHM_n(\bar{x}) = ch\ b_n\bar{x} - \cos b_n\bar{x}\ , \quad CHP_n(\bar{x}) = ch\ b_n\bar{x} + \cos b_n\bar{x}$$

$$SHM_n(\bar{x}) = sh\ b_n\bar{x} - \sin b_n\bar{x}\ , \quad SHP_n(\bar{x}) = sh\ b_n\bar{x} + \sin b_n\bar{x}$$

$$Q_n(\bar{x}) = m\ H\ S_{an}\ \Gamma_n/(b_n)\ [-A_n\ CHP_n(\bar{x}) - SHM_n(\bar{x})]$$

$$M_n(\bar{x}) = m\ H^2 S_{an}\ \Gamma_n/(b_n{}^2)\ [A_n\ SHP_n(\bar{x}) + CHP_n(\bar{x})]$$