COMPUTED BEHAVIOR OF A TEN-STORY REINFORCED CONCRETE FRAME-WALL STRUCTURE FOR STATIC LATERAL LOADS

by

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SYNOPSIS

A procedure using cantilever beam models is presented for the inelastic analysis of a reinforced concrete frame-wall structure for static lateral loads. The wall members and frame members are replaced by three types of mechanical models which take into account inelastic behavior of a reinforced concrete cantilever beam. The procedure can be applied to reinforced concrete structures for inelastic analysis.

INTRODUCTION

This paper outlines a procedure and presents some results for the computation of the inelastic behavior of a frame-wall structure subjected to lateral loads.

The structure selected for analysis is one tested on the Illinois Earthquake simulator. The example structure to be analyzed (Fig. 1 consists of two ten-story three bay frames surrounding a slender shear wall. The shear wall is treated as a vertical cantilever beam which is subjected only to horizontal loading. The reinforcing schedule for the structure and assumed material properties of constituent members are listed in Tables 1 and 2.

A simplified approximate procedure is adopted for the analysis of this structure. The entire system is idealized as a plane structure composed of two systems as shown in Fig. 2. One of these systems is the isolated wall. The second system is a substitute frame structure which models the two parallel rigid frames as a single frame substructure. Geometric nonlinearities are assumed insignificant and are thus neglected in the analysis.

MECHANICAL MODELS

The Concentrated Spring Model - The concentrated spring model consists of a flexible elastic line element and a nonlinear rotational spring at the restrained end of the cantilever beam as shown in Fig. 3 [4]. Instead of analyzing this model, however, the equivalent model in the form of a simple beam is analyzed. The flexibility matrix [F] relating the incremental external moments $\Delta M'_A, \Delta M'_B$ to the incremental rotations $\Delta\theta'_A$, $\Delta\theta'_B$ is expressed as:

$$\begin{Bmatrix} \Delta\theta'_A \\ \Delta\theta_B \end{Bmatrix} = [F] \begin{Bmatrix} M'_A \\ M'_B \end{Bmatrix} \qquad (1)$$

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$$[F] = \begin{bmatrix} f_1 + f(\Delta M'_A) & f_2 \\ f_2 & f_1 + f(\Delta M'_B) \end{bmatrix} \quad (2)$$

where f_1 and f_2 are the rotational flexibilities due to flexural and shearing action in the flexible element. $f(\Delta M'_i)$ is the rotational flexibilities resulting from bond slip, inelastic action over the beam length at the end i. A stiffness matrix [k] can be obtained by inverting [F]. The incremental forces and displacements at the ends of the rigid portions ($\lambda \ell$) are related to those at the ends of the flexible element through a transformation matrix [T]. These are further related to corresponding quantities in global coordinates by another transformation matrix [C]. The resulting member stiffness matrix $[K_m]$ transformed to global coordinates is:

$$[K_m] = [C]^T [T]^T [k][T] [C] \quad (3)$$

The Multiple Spring Model - The multiple spring model is a line element model which is composed of a number of segments, each of which handles independently both the linear and the nonlinear action through the springs as shown in Fig. 4. This model is applicable to members which are exposed to a more general moment distribution than can be handled by the concentrated spring model [5].

The method of analysis with this model uses the flexibility matrix of each subelement in conjunction with transfer matrices. The flexibility matrix $[F_{ab}]$ of the cantilever beam AB, can be derived as:

$$[F_{ab}] = \sum_j E_{jb} F_{ij} E_{jb} \quad (4)$$

where $[F_{ij}]$ is the flexibility matrix of the element ij. $[E_{jb}]$ is the transformation matrix for elements between nodes j and b. The stiffness matrix $[K_{bb}]$ of the cantilever beam is obtained by inverting the flexibility matrix $[F_{ab}]$. The total stiffness matrix of an individual member $[K_{ab}]$ can then be obtained as:

$$[K_{ab}] = \begin{bmatrix} E_{ab} K_{bb} E_{ab}^T & -E_{ab} \\ -K_{bb} E_{ab}^T & K_{bb} \end{bmatrix} \quad (5)$$

In the global coordinates, using transformation matrix [C] , the member stiffness matrix $[K_m]$ for the multiple spring model is expressed as;

$$[K_m] = [C]^T[K_{ab}] [C] \quad (6)$$

The Layered Model - The layered model shown in Fig. 5 is a modification or alteration of the concentrated spring model. A layered cross section of length, L_p is assigned at the end of the cantilever beam and connected to an elastic line element. L_p is the anticipated length of inelastic action. The inelastic flexural action of the cantilever beam is calculated explicitly by the layered method which is derived from an overall moment-curvature relation reflecting the various stages of material behavior of concrete and steel in the layered section [1]. This model has the advantage that the layered concept can automatically take care of the change of flexural rigidity due to both a change in the moment and a change in the axis force.

Displacement D(M) at the free end of a cantilever beam is calculated from the idealized curvature distribution along its length, L. The end rotation is then computed as:

$$\theta = \frac{D(M)}{L} + R(M) \tag{7}$$

where R(M) is the rotation due to steel bar slip. The incremental rotation $\Delta\theta$ of the layered model can be expressed by the instantaneous rotational flexibility f_L:

$$\Delta\theta = f_L \Delta M \tag{8}$$

where ΔM is the incremental moment at the fixed end of a cantilever beam. The analytical procedure is similar to the case of the concentrated spring model. The flexibility matrix corresponding to Eq. (2) is:

$$[F] = \begin{bmatrix} f_L + f_s & f_2 \\ f_2 & f_L + f_s \end{bmatrix} \tag{9}$$

where f_s is the rotational flexibility due to shearing action. The member stiffness matrix $[K_m]$ can be obtained from Eq. (3) in a same manner.

FORCE - DEFORMATION RELATIONSHIPS OF ELEMENTS

The stress-strain relationship for concrete is constructed from a parabola combined with a straight line (Fig. 6a). A bilinear stress-strain relationship is adopted for the reinforcing steel (Fig. 6b).

The hysteresis rule used for the rotational springs of the concentrated spring model and the multiple spring model are an adaptation of those presented by Takeda et al [3] (Fig. 7).

For the layered model, the concrete inside the stirrup in the layered cross section is considered as confined while that outside is taken as unconfined. With a monotonically increasing load capacity, the unconfined concrete provides no contribution at strains greater than $\varepsilon_{cu} = 0.004$. To predict unloading from an inelastic state and subsequent reloading back into that inelastic range, the values of ε_e and ε_n are defined by the following equation [2].

$$\frac{\varepsilon_n}{\varepsilon_0} = 0.145 \left(\frac{\varepsilon_e}{\varepsilon_0}\right)^2 + 0.13 \left(\frac{\varepsilon_e}{\varepsilon_0}\right) \tag{10}$$

where ε_e is the strain on the envelope curve at which unloading starts. ε_n is the plastic strain remaining after all load has been released. ε_0 is the strain at which f'_c is attained.

With ε_e established, a linear equation is used for unloading and reloading between the ε_e point on the envelope curve and the ε_n point.

$$f_c = \frac{f_{en}}{\varepsilon_e - \varepsilon_n} (\varepsilon_c - \varepsilon_n) \tag{11}$$

where f_{en} is the concrete stress at which the concrete strain is ε_e.

ANALYTICAL PROCEDURE

The full-size structural stiffness matrix is accomplished by summing the member stiffness matrices $[K_m]$ in proper order.

$$\begin{Bmatrix} F_F \\ F_W \\ \hline F_H \end{Bmatrix} = \left[\begin{array}{cc|c} A_1 & 0 & R_1 \\ 0 & A_2 & R_2 \\ \hline R_1^T & R_2^T & E \end{array}\right] \begin{Bmatrix} D_F \\ D_W \\ \hline D_H \end{Bmatrix} \tag{12}$$

where $\{F_F\}$, $\{D_F\}$ ($\{F_W\}$,$\{D_W\}$) are the force and displacement vectors of the frame (wall) in terms of rotation and vertical motion. $\{F_H\}$, $\{D_H\}$ are the horizontal force and displacement vectors. Static condensation of vertical displacements and rotations yields;

$$\{F_H\} = [K_H]\{D_H\} \quad \text{then} \quad \{D_H\} = [K_H]^{-1} \{F_H\} \tag{13}$$

where

$$[K_H] = [E - (R_1^T A_1^{-1} R_1 + R_2^T A_2^{-1} R_2)] \tag{14}$$

COMPUTED RESULTS

The concentrated spring model is applied to model the frame members and the multiple spring model is applied to the wall members. The static load is applied to the structure in small increments with the same distribution pattern as that of a triangular shaped load.

The base shear-top story displacement relationship of the structure is shown in Fig. 8. Solid line is the case in which the layered model is applied only to the exterior first story columns. The curve using the layered model shows that the left column yields at the base at an early stage while the center and right columns do not yield at all during the loading process.

The moment-curvature hysteresis loops of the layered section in the layered model are shown in Fig. 9. The hysteresis loops of the layered section shift from one moment-curvature curve to another moment-curvature curve in order to reflect change in axial force. On the hysteresis loop with increasing (decreasing) axial force in the column, a stiffer (softer) slope than that used in the concentrated spring model results. The concentrated spring model's primary curve is based on a constant axial force of 4.45 KN. The concentrated spring model's primary curve then positions itself approximately as the mean curve between the stiffer and softer curves of the layered model.

A redistribution of base shear occurs between the wall and the various columns as the load increases on the structure (Fig. 10). Distribution of base shear in the wall changes from 78 per cent of the total in the elastic stage to 68 per cent when the wall yields at the base. There is then an accelerated decline down to 32 per cent at which time the columns yield at the base.

The base shear top story displacement relationship of the structure under one cycle loading is shown in Fig. 11. The moment-curvature hysteresis loops of the layered sections are shown in Fig. 12. For the first one quarter cycle of loading, the curves of No. 1 and No. 2 are the same as the ones just described for the case of monotonically increasing load. For the next half cycle of loading and unloading, the column layered section of No. 1 experiences a snap-through phenomenon. For computational reasons the natural stiffness for this portion is replaced by a small positive one. After this snap-through phenomenon has occurred, the column section again demonstrates stiffer flexural rigidity. How much depends upon the level of axial force. The column of No. 2 on the other side of the structure then experiences similar relationships of a form which appears antisymmetrical about the origin. The behavior of the layered cross section illustrated in the Fig. 13 shows how the steel and concrete strains in its cross section shift during one cycle loading.

SUMMARY

The mechanical models used in the study: the concentrated spring model, the multiple spring model and the layered model, show the inelastic behavior of the reinforced concrete frame-wall structure.

ACKNOWLEDGMENT

This study was supported by the National Science Foundation under Grant No. ENV 7422962.

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Table 1. Reinforcing Schedules for the Structure of FW-2

Number of Wires per Face, Walls No. 2g wire, Beams and Columns No.13g wire.

Story	Walls	Beams	Columns (Ext.)	Columns (Int.)
10-8	2	2	2	2
7-4	2	3	2	2
3	2	3	3	2
2-1	2	2	3	2

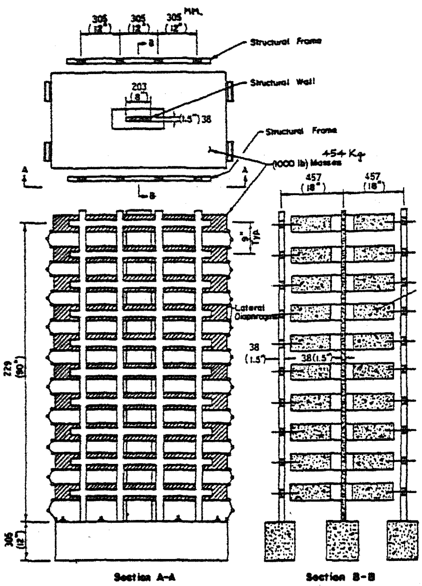

Fig.1 Reinforced Concrete Frame-Wall Structure

Table 2: Assumed Material Properties

Concrete	Compressive Strength	f'_c	(Mpa)	42.1
	Tensile Strength	f_t	(Mpa)	3.24
	Young Modulus	E_c	(Mpa)	30700
	Shear Modulus	G	(Mpa)	13100
	Strain at f_c	ε_o		0.003
	at Ultimate	ε_{cu}		0.004
Steel Reinforcement	Yield Stress	f_y	(Mpa)	338
	Ultimate Stress	f_u	(Mpa)	400
	Young Modulus	E_s	(Mpa)	200000
	Strain at f_y			0.0017
	at f_u			0.07

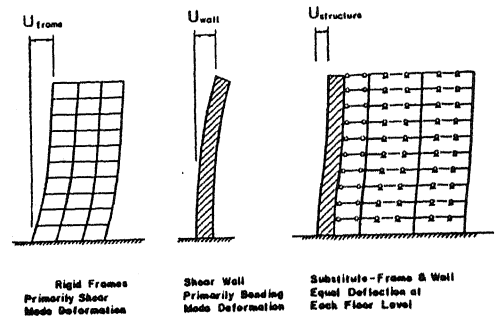

Fig.2 Deformation Modes of Frame-Wall Structure

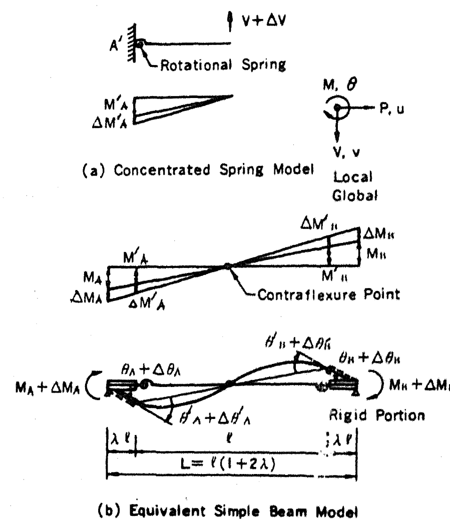

Fig.3 Concentrated Spring Model

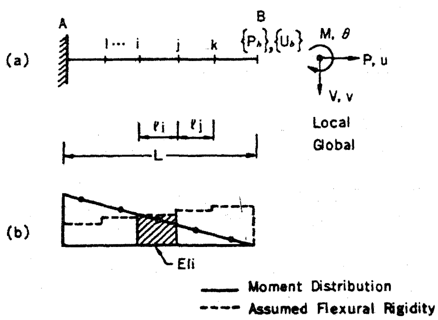

Fig.4 Multiple Spring Model

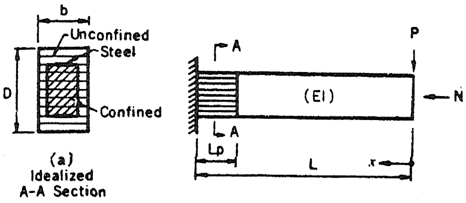

Fig.5 Layered Model

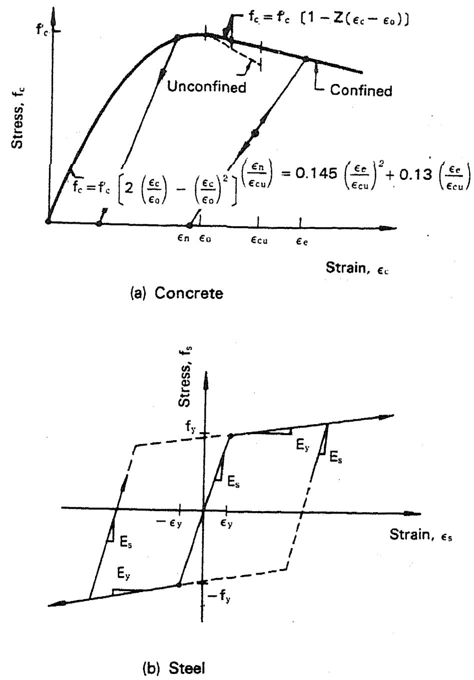

Fig.6 Idealized Stress-Strain Relationships

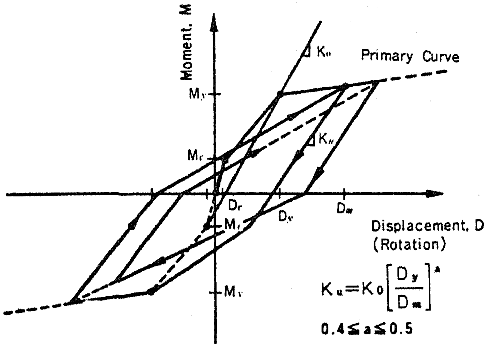

Fig.7 Trilinear Hysteresis Rule

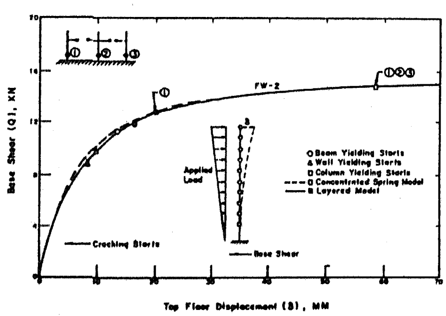

Fig.8 Base Shear-Top Displacement Relationships

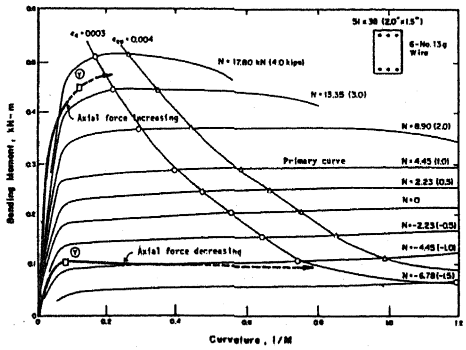

Fig.9 Moment-Curvature Relationships of the Layered Section

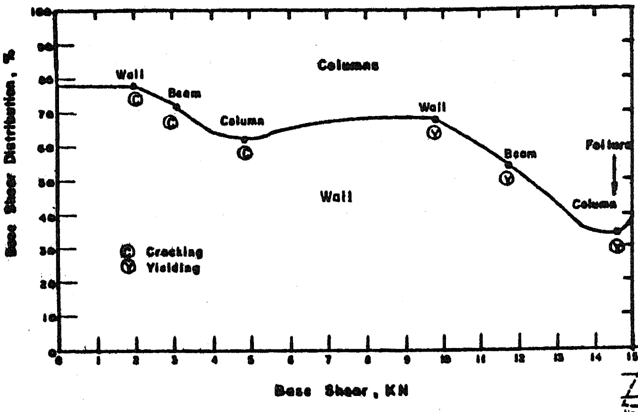

Fig.10 Redistribution of Base Shear

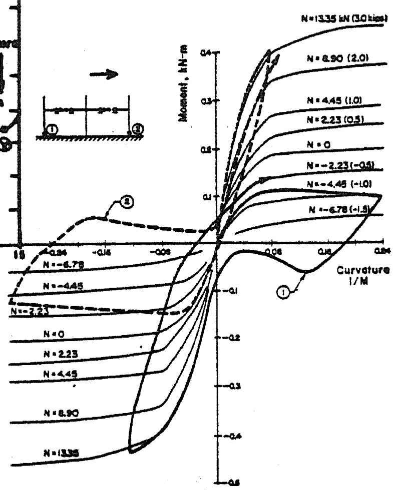

Fig.12 Moment-Curvature Relationships of the Layered Section for Single Cycle of Loading

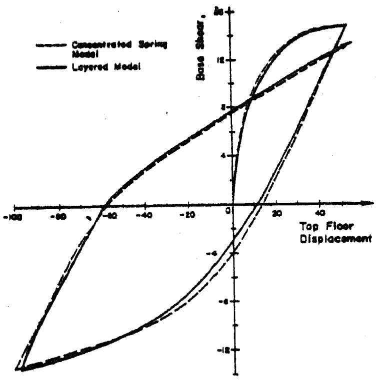

Fig.11

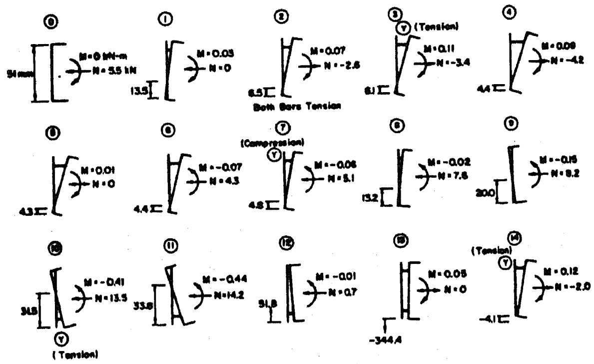

Fig.13 Behavior of the Layered Section