ELASTO-PLASTIC RESPONSE ANALYSIS OF SEISMIC-RESISTANT FRAMES VIA MATHEMATICAL PROGRAMMING †

by

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SUMMARY

A parametric linear programming approach was recently developed |1-3| to follow the structural evolution of elasto-plastic frames under given quasi-static loading histories. Such a procedure is generalized in this paper to the analysis of non-linear response of dynamically loaded structures. The method is formulated with reference to elastic-perfectly plastic flexural characteristics of structural members. Two different integration methods are introduced and a comparative efficiency analysis is performed by means of some numerical examples that illustrate the computational aspects of the procedure. For this purpose the time-history analyses of two-storey frames subject to actual recorded ground motions are developed.
Finally the guidelines for extending the approach to more complex material constitutive laws are presented and discussed.

INTRODUCTION

The inelastic response of seismic resistant frames is generally investigated in order to design structures capable of responding in a ductile manner to strong earthquakes |4|. For this purpose different computer codes for non-linear dynamic analysis were developed (Refs 5,6,7 among others). Generally these programs integrate numerically in the time the differential equations of motion with iteration on the stiffness matrix whenever the resulting stresses fall out the yielding surface. Therefore they consist of step-by-step procedures because of both the variability in time of the ground acceleration and the non-linearity of the material constitutive law.
Other programs that do not take into account the actual plastic zone spread but concentrate the inelastic deformation in single sections, were also developed |8| to perform the dynamic analysis of plane frames in a way more efficient from a computational viewpoint. In this context the iteration on the stiffness matrix is avoided by the introduction of suitable procedures of stress redistribution |8|, that in any case leads to approximated results. This paper aims at the formulation of a dynamic analysis approach capable of determining directly the times at which inelastic deformations rise or local unloadings occur at any section. The equation of motion is then modified at each of these times.
When the inelastic deformations are assumed to be concentrated in single sections the solution of elastic-plastic frames may be regarded as the superposition of the elastic responses to the external actions and to the plastic strains taken as dislocations |9|. Thus, when one considers a discretized structure subject to static loads, the structural elasto-plastic behaviour may be investigated by mathematical programming methods |10-12|. The-

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se techniques permit the structural evolution to be followed under any given statical loading history and provide the current values of displacements, stresses and strains. Particularly the hinge-by-hinge analysis of elastic-plastic steel frames may be performed in a very efficient way by solving a suitable parametric linear programming (P.L.P.) problem |1-3| . The P.L.P. constraints express the yielding conditions at the critical sections and the algorithm provides the stress distributions and the inelastic deformations increments required to satisfy everywhere the material constitutive law.

In this paper the P.L.P. mathematical programming approach is generalized to the analysis of non-linear response of dynamically loaded frames. For this purpose a numerical three-point recurrence scheme for integration of second order equations |13|,|14| is considered and the equivalent statical loads are calculated for each time step. These loads are then introduced in the constraints of the P.L.P. approach. If the yielding condition boundary is achieved in a new section or a local unloading occurs during the time increment, such a procedure is able to evaluate the actual discontinuity point so that the governing matrices can be modified at the relevant time. The proposed method is derived for the elastic-perfectly plastic idealization of the actual material constitutive law. However the extension to bilinear or piecewise linear relationships does not présent theoretical difficulties. In particular the guidelines for formulating a procedure capable of approximating the dynamical analysis of reinforced concrete frames is presented.

GOVERNING RELATIONS

MDOF non-linear systems

The only generally applicable method for the analysis of arbitrary multi-degree-of-freedom (MDOF) non-linear systems is the numerical step-by-step integration of the coupled equations of motion |13|. The response history is divided into short time increments and the response is calculated during each increment for a linear system whose properties were determined at the beginning of the interval. Thus the non-linear analysis is approximated as a sequence of analyses of successively changing linear systems.
For this purpose the incremental equation of motion

$$\underline{m}\ \underline{\Delta\ddot{u}}(t) + \underline{c}(t)\ \underline{\Delta\dot{u}}(t) + \underline{k}(t)\ \underline{\Delta u}(t) = \underline{\Delta p}(t) \tag{1}$$

at any instant of time t, is generally written in the form:

$$\underline{k}_o(t)\ \underline{\Delta u}(t) = \underline{\Delta p}_o(t) \tag{2}$$

fully equivalent to a static incremental-equilibrium relationship.
In Eq. (1) $\underline{k}(t)$ is the stiffness matrix of the considered structure at time t and $\underline{\Delta u}(t)$ is the vector of the increments of the displacements representing the displaced shape of the structure; $\underline{c}(t)$ is the damping matrix at time t and $\underline{\Delta\dot{u}}(t)$ the incremental velocity vector; $\underline{m}$ is the mass matrix and $\underline{\Delta\ddot{u}}(t)$ is the incremental acceleration vector, both defined for the specified set of displacement coordinates. Note that it has been assumed that the mass matrix $\underline{m}$ does not change with time. Furthermore $\underline{\Delta p}(t)$ is the vector of the increments of the externally applied loads $\underline{p}(t)$ and all the variations are referred to the considered time step. If a ground motion is analysed, p(t) becomes $-\underline{m}\ \underline{I}\ a(t)$, where a(t) is the ground acceleration.
In Eq. (2) the dynamic behaviour is accounted for by the inclusion of inertial and damping effects in the effective-load ($\underline{\Delta p}_o$) and stiffness ($\underline{k}_o$)

terms. With reference to a three-point recurrence scheme for second order equation (Newmark algorithm) |13|, the last terms can be written, for $\underline{k}(t)$ and $\underline{c}(t)$ constant during the step, as:

$$\underline{k}_o(t) = \underline{k}(t) + \underline{m}/(\beta\ \Delta t^2) + \gamma\underline{c}(t)/(\beta\ \Delta t) \tag{3}$$

$$\Delta\underline{p}_o(t) = \Delta\underline{p}(t) + \underline{m}\{\dot{\underline{u}}(t)/(\beta\Delta t) + \tfrac{1}{2}\ddot{\underline{u}}(t)/\beta\} + \underline{c}(t)\{\gamma\dot{\underline{u}}(t)/\beta + (\tfrac{1}{2}\gamma/\beta - 1)\Delta t\ \ddot{\underline{u}}(t)\} \tag{4}$$

where $\underline{u}(t)$, $\dot{\underline{u}}(t)$ and $\ddot{\underline{u}}(t)$ denote the displacements, velocities and accelerations at time t and Δt is the time interval defining the integration step. The most common scheme with $\gamma = 1/2$ and $\beta = 1/6$ corresponds to a linear-acceleration assumption,which leads to a quadratic variation of the velocity vector and a cubic variation of the displacement vector during each Δt:

$$\underline{u}(\tau) = \underline{u}(t) + \dot{\underline{u}}(t)\ \tau + \ddot{\underline{u}}(t)\ \tau^2/2 + \Delta\ddot{\underline{u}}\ \tau^3/(6\ \Delta t) \qquad \text{for } 0 \leq \tau \leq \Delta t \tag{5}$$

Under this hypothesis, for any given time increment, the analysis procedure consists of the following operations:

1) $\underline{u}(t)$ and $\dot{\underline{u}}(t)$ are known either from values at the end of preceding step or as initial condition; $\underline{c}(t)$ and $\underline{k}(t)$ for the interval Δt are evaluated and the initial acceleration is given by:

$$\ddot{\underline{u}}(t) = \underline{m}^{-1}\{\underline{p}(t) - \underline{c}(t)\ \dot{\underline{u}}(t) - \underline{k}(t)\ \underline{u}(t)\} \tag{6}$$

that is a rearrangement of the equation of equilibrium for time t;

2) $\Delta\underline{p}_o(t)$ and $\underline{k}_o(t)$ are computed from Eqs. (3) and (4);

3) the displacement increment vector $\Delta\underline{u}(t)$ is given by Eq. (2) and with it:

$$\Delta\dot{\underline{u}}(t) = 3\ \Delta\underline{u}(t)/\Delta t - 3\ \dot{\underline{u}}(t) - \Delta t\ \ddot{\underline{u}}(t)/2 \tag{7}$$

$$\Delta\ddot{\underline{u}}(t) = 6\ \Delta\underline{u}(t)/\Delta t^2 - 6\ \dot{\underline{u}}(t)/\Delta t - 3\ \ddot{\underline{u}}(t) \tag{8}$$

4) the displacements and velocities at the end of the increment are given by:

$$\underline{u}(t + \Delta t) = \underline{u}(t) + \Delta\underline{u}(t)\ ;\ \dot{\underline{u}}(t + \Delta t) = \dot{\underline{u}}(t) + \Delta\dot{\underline{u}}(t) \tag{9}$$

The analysis for the considered step is finished and the process is repeated for the next time interval. Thus the Newmark procedure is founded on the solution of Eq. (2) that means on the performance of an incremental elasto-plastic static analysis when elasto-plastic MDOF frames are considered.
For following developments, it is worth noting that if the acceleration increment vector $\Delta\ddot{\underline{u}}_\theta(t)$ for the time step $\Delta t/\theta$ is expressed by:

$$\Delta\ddot{\underline{u}}_\theta(t) = \Delta\ddot{\underline{u}}(t)/\theta \tag{10}$$

then the corresponding incremental $\Delta\dot{\underline{u}}_\theta(t)$ and $\Delta\underline{u}_\theta(t)$ vectors are given by:

$$\Delta\dot{\underline{u}}_\theta(t) = \Delta t\ \ddot{\underline{u}}(t)/\theta + \Delta t\ \Delta\ddot{\underline{u}}(t)/(2\theta) \tag{11}$$

$$\Delta\underline{u}_\theta(t) = \Delta t\ \dot{\underline{u}}(t)/\theta + \Delta t^2\ \ddot{\underline{u}}(t)/(2\theta^2) + \Delta t^2\ \Delta\ddot{\underline{u}}(t)/(6\theta^2) \tag{12}$$

and hence

$$\underline{u}(t + \Delta t/\theta) = \underline{u}(t) + \Delta\underline{u}_\theta(t)\ ;\ \dot{\underline{u}}(t + \Delta t/\theta) = \dot{\underline{u}}(t) + \Delta\dot{\underline{u}}_\theta(t) \tag{13}$$

PLP approach to the static analysis of elasto-plastic frames

Consider an elasto-plastic plane frame and introduce the following assumptions: a) geometry changes do not influence the equilibrium equation (small displacements); b) flexural action predominates; c) shear, axial force, axial shortening and instability effects are neglected; d) inelastic rotations concentrate in single sections (critical sections) and give rise to rotation Θ(plastic hinge hypothesis); e) the flexural characteristics relevant to the critical sections are idealized to be elastic-perfectly plastic and non-holonomic. Furthermore let the frame be discretized in perfectly-elas tic elements connected by N critical sections and let $\underline{s}(t)$ and $\underline{\Theta}(t)$ denote the values of the R externally applied loads and the inelastic rotations at the N critical sections that define a known initial solution at any time t. Suppose that the structure is also acted upon by a set of permanent actions $\underline{w}$. At any time t, the actual bending moments $\underline{M}(t)$ at the N critical sections may be expressed as the sum of the elastic bending moments due to the permanent actions $\underline{w}$ ($\underline{M}_{Ew}$), to the applied loads $\underline{s}(t)$ ($\underline{M}_{E1}\underline{s}(t)$) and to the inelastic rotations $\underline{\Theta}(t)$ at the N critical sections taken as dislocations:

$$\underline{M}(t) = \underline{M}_{Ew} + \underline{M}_{E1}\,\underline{s}(t) + \underline{Z}\,\underline{\Theta}(t) \qquad (14)$$

In Eq. (14) $\underline{M}_{E1}$ is the NxR matrix whose j-th column means the elastic bending moments at dhe N critical sections due to the j-th load s_j made equal to the unit and $\underline{Z}$ is the NxN matrix of the influence coefficients.

The material constitutive law requires that a positive (negative) increment of the plastic rotation may only occur at a critical section if the bending moment assumes its positive (negative) yielding value. Let the vectors $\underline{M}_L^+$ and $-\underline{M}_L^-$ denote the positive and negative yielding moments.

Under the above assumptions, the incremental elastic-perfectly plastic analysis of the structure may be performed by solving the following parametric linear programming (P.L.P.) |1||2|:

$$\begin{aligned}
&\max -(\tilde{\underline{\xi}}(t)\,\underline{\Delta\Theta}^+(t) + \tilde{\underline{\zeta}}(t)\,\underline{\Delta\Theta}^-(t) + \tilde{\underline{\mu}}(t)\,\underline{\omega}^+(t+\Delta t) + \tilde{\underline{\nu}}(t)\,\underline{\omega}^-(t+\Delta t)) && a)\\
&\underline{Z}\,\underline{\Delta\Theta}^+(t) - \underline{Z}\,\underline{\Delta\Theta}^-(t) + \underline{\omega}^+(t+\Delta t) = \underline{M}_L^+ - \underline{M}_{Ew} - \underline{M}_{E1}\underline{s}(t) - \underline{Z}\,\underline{\Theta}(t) - \underline{M}_{E1}\underline{s}(\Delta t,t) && b)\\
&-\underline{Z}\,\underline{\Delta\Theta}^+(t) + \underline{Z}\,\underline{\Delta\Theta}^-(t) + \underline{\omega}^-(t+\Delta t) = \underline{M}_L^- + \underline{M}_{Ew} + \underline{M}_{E1}\underline{s}(t) + \underline{Z}\,\underline{\Theta}(t) + \underline{M}_{E1}\underline{s}(\Delta t,t) && c) \quad (15)\\
&\underline{\Delta\Theta}^+ \geq 0\ ;\ \underline{\Delta\Theta}^- \geq 0\ ;\ \underline{\omega}^+ \geq 0\ ;\ \underline{\omega}^- \geq 0 && d)
\end{aligned}$$

In Eq. (15) $\underline{\omega}^+$ and $\underline{\omega}^-$ are vectors of slack variables and the vectors $\underline{\Delta\Theta}^+(t)$ and $\underline{\Delta\Theta}^-(t)$ denote the positive and negative increments of the inelastic rotations at the N critical sections. The actual inelastic rotations $\underline{\Theta}(t+\Delta t)$ at any time $(t+\Delta t)$ are given by $\underline{\Theta}(t+\Delta t) = \underline{\Theta}(t) + (\underline{\Delta\Theta}^+(t) - \underline{\Delta\Theta}^-(t))$.

The orthogonality relations due to the constitutive law are satisfied by defining the components of the vectors $\underline{\xi}(t)$, $\underline{\zeta}(t)$, $\underline{\mu}(t)$ and $\underline{\nu}(t)$ ($\tilde{}$ = transpose) in the following way: $\xi_i=0$, $\zeta_i=1$, $\mu_i=1$, $\nu_i=0$ at the positive plastic hinges; $\xi_i=1$, $\zeta_i=0$, $\mu_i=0$, $\nu_i=1$ at the negative plastic hinges; $\xi_i=1$, $\zeta_i=1$, $\mu_i=0$ and $\nu_i=0$ at the remaining sections. The objective function (15a) attains a zero valued maximum for the actual solution of the problem |1||2|. This solution can be expressed in the form |15|:

$$\{\underline{\Delta\tilde{\Theta}}_B^+, \underline{\Delta\tilde{\Theta}}_B^-, \underline{\tilde{\omega}}_B^+, \underline{\tilde{\omega}}_B^-\}^T = \underline{d}(t) + \underline{B}(t)\,\underline{s}(\Delta t,t) + \underline{A}(t)\{\underline{\tilde{\omega}}^+_{NB}, \underline{\tilde{\omega}}^-_{NB}, \underline{\Delta\tilde{\Theta}}^+_{NB}, \underline{\Delta\tilde{\Theta}}^-_{NB}\}^T \qquad (16)$$

where the suffix B means the basic variables and the suffix NB marks the non-basic variables (T = transpose of the whole vector). Note that for the arbitrary j-th critical section $\Delta\Theta_j^+$ (or $\Delta\Theta_j^-$) is non-basic if ω_j^+ (or ω_j^-) is basic and vice versa, according to the material constitutive law.

Once Eq. (16) is known for any time t, it is possible to determine |1||2| a critical value Δt_c of the time increment as the lower value for which at time $(t+\Delta t_c)$ either the j-th plastic rotation increment ($\Delta\Theta_{Bj}^+$ or $\Delta\Theta_{Bj}^-$) or the j-th slack (ω_{Bj}^+ or ω_{Bj}^-) relevant to the arbitrary j-th critical section becomes zero. The last case means that a plastic hinge forms; the first that a local unloading occurs. Eq. (16) holds as long as the time is lower than or equal to $t+\Delta t_c$. If the structural analysis must proceed after Δt_c, the basic plastic rotation increment or the basic slack that assumes a zero value at $(t+\Delta t_c)$ must become non basic, and the orthonormal variable basic. Therefore Eq. (16) must be changed and that can be performed by a pivotal step on problem (15) |1||2|. In this way the structural stiffness matrix is automatical

ly modified. Furthermore, as at each step the inelastic rotation increments are stored in the R.H.S. of Eq.(15) (vector $\underline{d}$ in Eq.(16)), the non-holonomy of the material constitutive law is respected.
Therefore, once Eq.(16) is determined by solving problem (15) at time $\bar{t}$ and the relevant Δt_c is estimated, the analysis procedure consists, for each time increment with $(t+\Delta t)\leq(\bar{t}+\Delta t_c)$, of the following operations:

i) inelastic rotations and slack variables are determined by Eq.(16);
ii) the bending moment distribution at $(t+\Delta t)$ is obtained by Eq.(14);
iii) the displacements at given points are calculated by any elastic method.

Phase iii) can be performed, for instance, by means of a virtual work approach. The incremental analysis may also be performed allowing for the axial force effects. For this purpose a suitable piecewise linear yielding condition is assumed and hence a greater number of constraints must be considered. Moreover, if the axial shortening effects are significant, the corresponding matrix of the influence coefficients must be introduced in the L.H.S. of the constraints (15a) and (15b).

PLP DYNAMIC ANALYSIS

Consider now a R-degree-of-freedom elastic-perfectly plastic frame subjected to a set of dynamic loads $\underline{p}(t)$. For the steps during which no plastic hinge forms and no local unloading occurs, the motion is governed by Eq.(1), whose solution may be expressed, from Eq.(2), by:

$$\underline{\Delta u}(t) = \underline{k}_o^{-1}(t)\ \underline{\Delta p}_o(t) \tag{17}$$

These displacements induce in the structure a bending moment distribution fully equivalent to the distribution due to the statical loads $\underline{s}(\Delta t,t)$:

$$\underline{s}(\Delta t,t) = \underline{k}(t)\ \underline{\Delta u}(t) = \underline{k}(t)\ \underline{k}_o^{-1}(t)\ \underline{\Delta p}_o(t) \tag{18}$$

By substituting Eq.(18) in Eq.(15), the dynamic behaviour, governed by the incremental equation of motion (1), can be investigated by solving the PLP approach summarized in Eq.(15). Moreover the whole elasto-plastic dynamic analysis can be performed by means of a Newmark integration procedure. In particular Eq.(16) can be employed to solve the equivalent static analysis and to obtain the bending moment distribution. Then the searched displacements are evaluated by any elastic method and hence velocities and accelerations are computed by Eqs.(7) and (9) and by Eq.(6) respectively. Finally Eqs.(3) and (4) provide the effective load and stiffness terms for next step. However the main aspect of the PLP approach of Eq.(15) is the evaluation of the critical value Δt_c of the time increment. Then if $\Delta t\leq\Delta t_c$ Eq.(16) can be employed to solve the problem at $(t+\Delta t)$; otherwise a rearrangement of basic and non-basic variables is required at $(t+\Delta t_c)$. But for dynamic problems the dependence of $\underline{s}(\Delta t,t)$ on time increment is complex (see Eq.(18)) and so a cumbersome step-by-step procedure needs to determine Δt_c from this viewpoint. Therefore an alternate, more efficient, procedure is formulated.
Firstly assume that $\underline{s}(\tau,t)$ is a linear function of the time increment τ for which: $\underline{s}(0,t)=0$ and $\underline{s}(\Delta t,t)$ is determined by Eq.(18) (Δt is the time increment relevant to the Newmark integration procedure). Under this assumption a fictitious critical time increment Δt_k, at which the first variable leaves the basis, can be easily estimated by the PLP rules. Then the possible values of the ratio $\varepsilon=\Delta t_k/\Delta t$ at time t are:

i) $\varepsilon=0$, meaning that either a plastic hinge forms or a local unloading occurs at time t; in this case $\Delta t_c=\Delta t_k=0$;
ii) $\varepsilon=1$, meaning that either a plastic hinge forms or a local unloading occurs at time $t+\Delta t_k$; in this case $\Delta t_c=\Delta t_k=\Delta t$;

iii) $\varepsilon>1$, meaning that no plastic hinge forms and no local unloading occurs during $(t, t+\Delta t)$; hence the dynamic analysis may proceed;

iv) $\varepsilon<1$, meaning that either a plastic hinge forms or a local unloading occurs during $(t, t+\Delta t)$; hence a rearrangement of Eq.(16) variables is required at time $t+\Delta t_c$, where Δt_c is unknown.

For the first three cases the computational effort of determining Δt_c is avoided. In the last case the dynamic analysis may not proceed before the actual Δt_c is evaluated. For this purpose solve the step analysis under the assumption that $\underline{k}$ and $\underline{c}$ do not change during Δt, so that $\Delta\underline{\ddot{u}}(t)$ can be obtained by Eq.(8) and the displacements vary according to Eq.(5) during $(t, t+\Delta t)$. In this way the actual relationship between $\underline{s}(\tau,t)$ and τ can be made explicit by Eq.(18). Furthermore let h denote the element of the L.H.S. vector in Eq.(16) that leaves the basis (i.e. becomes zero) at $t+\Delta t_k$ when a linear relationship $\underline{s}(\tau,t)$ is introduced. The same variable is assumed to become zero at $t+\Delta t_c$ when the actual relationship is introduced. Then, by substituting Eq.(5) in Eq.(18) and this in the h-th row of Eq.(16), Δt_c is determined by solving the cubic equation:

$$0 = d_h(t) + B_h(t)\ \underline{k}(t)\{\underline{\dot{u}}(t)\ \Delta t_c + \underline{\ddot{u}}(t)\ \Delta t_c^2/2 + \Delta\underline{\ddot{u}}(t)\ \Delta t_c^3/6\ \Delta t\} \quad (19)$$

where $d_h(t)$ and $B_h(t)$ are the h-th element of $\underline{d}$ and row of $\underline{B}$ respectively. Once Δt_c is known, Eq.(2) is rewritten at time t for the shorter time increment Δt_c and the whole process is repeated. If the linear acceleration assumption fits well the actual structural behaviour (i.e. Δt is sufficiently small), at the end of this step $\Delta t_k/\Delta t_c=1$ is obtained, Eq.(16) is changed by pivotal rules at $t+\Delta t_c$ and the time increment Δt is used for the next integration step. If $\Delta t_k/\Delta t_c \neq 1$, the process is iterated until the actual Δt_c is obtained (NEWMARK INTEGRATION PROCEDURE; METHOD I).

Improvements of the above approach can be obtained by modifying the time integration procedure. For this purpose it is worth noting that, if Δt is sufficiently small, Eq.(19) provides directly a Δt_c estimation within the round-off tolerances. Therefore it is unnecessary to rewrite Eq.(2) for Δt_c and to repeat the analysis. The linear acceleration assumption, in fact, involves that, for $\theta=\Delta t/\Delta t_c$:

$$\underline{\ddot{u}}(t+\Delta t_c) = \underline{\ddot{u}}(t) + \Delta\underline{\ddot{u}}(t)/\theta \quad (20)$$

and the corresponding velocity and displacement vectors at $(t+\Delta t_c)$ are given by expressions like Eqs.(11),(12) and (13). Then Eq.(16) is modified and the initial conditions for the next step are the ones determined at $(t+\Delta t_c)$ (NEWMARK INTEGRATION PROCEDURE; METHOD II). In this way no iteration is required when a plastic hinge forms or a local unloading occurs.

This second approach is similar to the four-point recurence scheme called the Wilson-θ procedure. This consists in making use at each step of Eq.(20) with $\theta>1.37$. Therefore the possibility of formulating a third integration approach, that uses either $\theta=1.4$ at each step for which Eq.(16) must not be modified in $(t, t+\Delta t)$ or $\theta=\Delta t/\Delta t_c$ otherwise, was also analysed. But the "overshot" (i.e. overestimation of velocities and displacements during the first integration steps |16|) discouraged from implementing this approach.

The main advantage of the proposed Newmark procedures is that of avoiding any bending moment overshoting. Thus approximated techniques for redistributing the overshoting moments |8| are not required. Moreover that is obtained by only solving the cubic Eq.(19), without the introduction of expensive procedures breaking up the time step into smaller steps when yielding has occurred and integrating the equation of motion for these smaller intervals.

Further, as the ductility limit-state is generally predominant, it is worth noting that for the PLP approach the inelastic rotations are the main variables and hence the rotation capacity requirements can be easily verified.

NUMERICAL EXAMPLES

The steel frames of Figs. 1 and 4 are considered. They are acted by the dead loads w_1 and w_2 and are at rest at t = 0. Masses are lumped at the floor levels, each having one horizontal DOF. Axial force effects are taken into account, while axial shortening is neglected. Firstly calculations are developed with the ground motion of Fig. 2a) on the first structure, having a fictitious constant diagonal damping matrix $\underline{c}$ = (0.05,0.05).With t=0.005 the two proposed Newmark- PLP procedures give the same response histories(see Fig. 2).The required computer time is approximately the same. All the checks on the yielding time, performed by method I, are in fact satisfied. However that might be peculiar of the studied example. The frames were then analysed under the ground acceleration record of Fig. 3. Here viscous damping is assumed through a damping matrix (depending on the stiffness matrix) which produces equal modal damping (0.5%) in all modes (results in Figs. 4, 5).

CONCLUSIONS AND EXTENSIONS

The PLP approach has been only discussed under the assumption that the flexural characteristics in the critical sections may be idealized as elastic perfectly plastic relationships. But no problem arises if a bilinear (or, more generally, a piecewise linear) model having a post yield stiffness is employed. In this case only a given diagonal hardening matrix must be introduced to multiply $\underline{\Delta\Theta}$ in the L.H.S. of Eqs. (15b) and c)). A particular trilinear (elastic, cracking, yielding) moment-inelastic rotation law may also be accounted for in order to idealize the behaviour of reinforced concrete sections. For this purpose, in addition to the inelastic rotations, one must take account in Eq. (15) of the rotations relevant to the first cracking stage |10||11|. Reversibility can be actually attributed to these rotations, and hence their present value must not be stored in the R.H.S. of Eqs. (15a) and b)). However for both steel and concrete problems a model that takes into account the loss of stiffness following any yielding is required |13||17|. If this behaviour can be concentrate in single sections |17|, the incremental nature of the PLP approach makes it capable of improvement.

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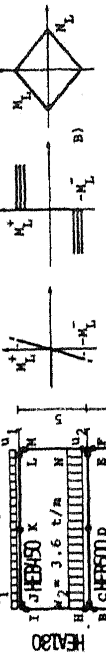


Fig.1-A)1st analysed frame and its discretization; B)flexural characteristics and interaction domain

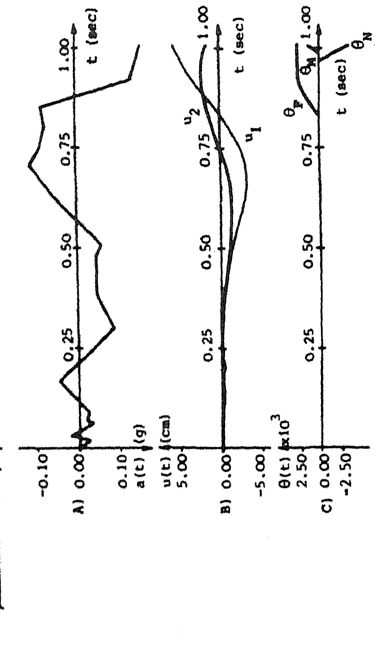


Fig.2- 1st frame: test on numerical procedures; A)ground acceleration; B) displacements; C)inelastic rotations.

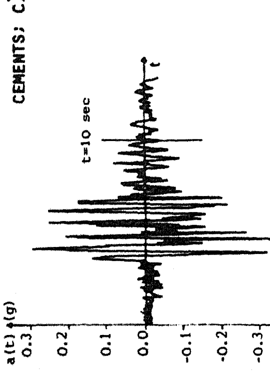


Fig.3- E-W ground acceleration, Tolmezzo, the 6th May 1976

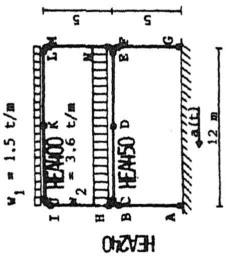
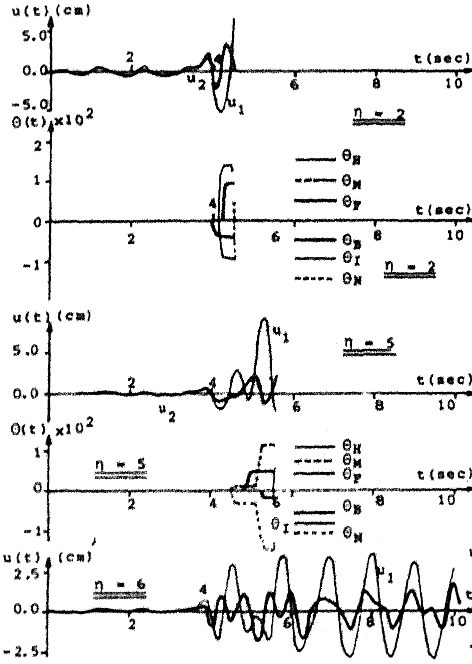


Fig.4-2nd analysed frame



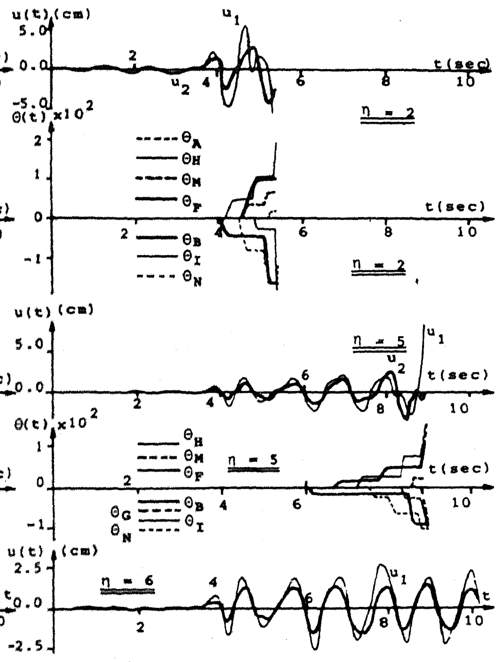


Fig.5-Results relevant to the frame of Fig.1. Fig.6-Results relevant to the frame of Fig.4.